# Enactivism and professional noticing: In dialogue as mathematics teacher educators

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Laurinda and Salvador both work in Mathematics Teacher Education (MTE) in different institutional contexts and with different perspectives. We are working, together with colleagues in our universities, dialoguing about how the different aspects of enactivism and professional noticing approaches can help us learn as mathematics teacher educators about prospective teachers' learning. We consider how activities in mathematics teacher education shape the way in which prospective teachers reason about teaching situations.

**Enactivism; professional noticing; prospective teacher learning; mathematics teacher educator learning**

## Contexts, research groups, theoretical perspectives and a dialogue

This paper is organised as a dialogue that is taking place between two mathematics teacher educators about the details of their practices. We have different institutional contexts and theoretical perspectives. We were seeing our work as a complementarity, which is a "relationship or situation in which two or more different things improve or emphasise each other's qualities" (https://en.oxforddictionaries.com/definition/complementarity). As time passed, the complexity of our undertaking led us to feel more intertwined than separate, as our perspectives were shifting fluidly as we interacted, adapting to how we heard the other. However, our contexts and original theoretical perspectives could not be more different:

Salvador works with a Spanish language primary teacher education programme, where each student takes four years to become a teacher. The mathematics teacher educators do not visit the prospective teachers when they are in school to see them teach. In Alicante, mathematics teacher educators work with the idea of professional noticing, which is a way to become cognizant of mathematics teaching events developing knowledge-based reasoning processes about practice registers, providing a structure for prospective teachers to understand and act in particular contexts.

Laurinda works with a one-year secondary education programme where the prospective teachers already have a degree related to mathematics. 24 out of the 36 weeks that the prospective teachers have to complete what is called the Post-Graduate Certificate of Education (PGCE) course are in school. In Bristol, the mathematics teacher educators (MTEs) visit the prospective teachers to observe them teach. The MTEs work with the enactivist view of cognition, which helps them to design activities in prospective teacher education or professional development to support prospective teachers or teachers in modifying their practice when they return to their own classroom.

We are both interested in mathematics teacher education. Laurinda is currently a member of the team that organises a thematic working group (TWG18) for the

Congress of the European Society for Research in Mathematics Education (CERME11 takes place in Utrecht in February 2019). Salvador is editing Volume 2 of the revised International Handbook of Mathematics Teacher Education (IHMTE), which will be launched at the 43rd annual meeting of the International Group for the Psychology of Mathematics Education (PME43) in Pretoria, South Africa, July 2019. We have known each other for nearly 10 years, meeting in Ouro Preto after PME34 in Belo Horizonte, Brazil and enjoy sharing our different perspectives. We have a joint chapter in Volume 3 of the IHMTE. In this paper, we will share a dialogue related to issues that arise when we write or talk together.

In the sections that follow, we are going to establish a dialogue about matters that focus our attention as MTEs, such as how prospective mathematics teachers learn about how to be teachers. Becoming mathematics teachers is understood as acting and thinking about teaching as teachers. This dialogue is a way of inquiring into what is important for us from our own current practices.

**What is learning? How is it done?**

Our contexts are different at an institutional process level. In Alicante, the group brings practical artefacts into the university through, for instance, videos and student answers. In Bristol, the prospective teachers are supported by dialogue and discussion of action in school. In both groups, what is central is a focus on the learning of the prospective teachers.

*Salvador*: Learning is a change in how prospective teachers notice and talk about teaching situations. So, the changes in what and how they notice and talk about practice are understood as the process of appropriation of existing knowledge about teaching. Existing knowledge is considered to be a resource to be used (and transformed) when making sense of the teaching situation. Prospective teachers learn to analyse artefacts from practice such as teaching video-clips, student answers, and so on. We place the emphasis on the construction of knowledge when prospective teachers engage with expert knowledge from the literature of mathematics education to think about practice in a process of making meaning with others.

A case of students' answers (adapted from Buforn, 2017), displaying some characteristics of how students are learning mathematics, is used to illustrate prospective teachers' learning from artefacts of practice. Table 1 introduces the problem and two students' answers and this is followed by what three prospective teachers write about the students' answers.

<table>
<tr><td colspan="2">Problem</td></tr>
<tr><td colspan="2">In a new building, there are lofts of three different sizes for sale:<br>a) 7.5m by 11.4m;<br>b) 4.55m by 5.08m;<br>c) 18.5m by 24.5m<br>Which one is more square?</td></tr>
<tr><td>Student answer 1</td><td>Student answer 2</td></tr>
<tr><td>7.5/11.4 = 0.65<br>4.55/5.08 = 0.89<br>18.5/24.5 = 0.75<br>So, the squarest loft is the second one since the quotient is close to 1.</td><td>As the sides of the square have the same measure, the squarest loft is the loft in which the difference between sides is smaller (closer to 0).<br>11.4 - 7.5 = 3.9<br>5.08 – 4.55 = 0.53</td></tr>
</table>

| | |
|---|---|
| (Student 1 writes the three ratios and makes the division and then interprets the loft whose ratio is closer to 1 will be the squarer.) | *24.5 – 18.5 = 6.0*<br>*So, the squarest loft is the second one since the difference is smallest.*<br><br>(Student 2 uses an additive strategy making subtractions between sides and choosing the difference closer to 0.) |

**Table 1: The answers of students 1 and 2 to the loft question.**

The following extracts show how prospective teachers attend to what they consider to be key aspects in the students' answers and interpret students' understanding from the meaning of ratio as an index of measure. What follows are some prospective teachers' answers displaying different ways in which they talk about the students' answers (adapted from Buforn, 2017). Prospective teachers A, B and C are different in the ways in which they talk about students' answers, particularly in using characteristics of students' learning of ratio gleaned from expert knowledge derived from the literature of mathematics education.

Prospective Teacher A provides judgements but not details about why the student solved the problem in that way. Prospective Teacher B provides more details in his description of what the students do, but he does not provide reasons. Finally, Prospective Teacher C provides more details and reasons about why the students seem to act in this way. In the case of student 1, Prospective Teacher C provides the meaning of "measure of being square" and points to the evidence from the student answer that allows him to infer this:

> *Prospective Teacher A*: Student 1 solves the problem well, because he understands and uses the mathematical concepts required correctly. Student 2 doesn't solve the problem well, because he doesn't use the idea of a ratio correctly, so he does a subtraction.
>
> *Prospective Teacher B*: Student 1 solves the problem well, because he thinks that it is a problem of ratio as a measure and compares ratios. He chooses as a result the ratio closer to 1. Student 2 doesn't understand that he has to compare ratios, so, he does a subtraction. In this case, he considers that the right answer is the loft where the sides are more similar in length because in a square the sides are equal. So, the reply, the sides of 5.08 and 4.55 is the more square.
>
> *Prospective Teacher C*: Student 1 understands that the "measure of being more square" is the ratio of the length of the sides. In a square this is 1. So, he calculates which ratio of the lengths of sides is closer to 1. Student 2 uses an incorrect additive strategy. He does not understand the meaning of "measure of being more square". This student seems to think that, as in a square, if the lengths of the sides are equal, with difference 0, then a rectangle seems more square if the difference of the length of their sides is 0. This strategy is incorrect since two quadrilaterals that are very different, such as one with lengths of sides 1.1m and 0.1m and the other with lengths of sides 20 and 21, have differences of the lengths of the sides equal to 1.

These different discourses illustrate differences in how prospective teachers appropriate the linguistic tools of the culture of being a teacher. In this case, the words and linguistic terms used by the prospective teachers to communicate meaning can be considered to be different uses of the culture's (of the literature of mathematics education) linguistic categories (Wells, 1999) for thinking about the situation. So, from these extracts, we can notice different ways in which prospective teachers make meaning, giving different illustrations of their learning.

*Laurinda*: The MTEs in Bristol use enactivism to frame learning and how learning is done. Learning is a behavioural change when prospective teachers are trying out new actions in their classrooms. How is the behavioural change done? In the university, the prospective teachers share their different experiences in different local cultures from the schools they are placed in. In school, the observing university tutor may trigger new actions in feedback. For instance, in a visit to a prospective teacher in school, she seemed to fill all the spaces in the classroom both physical and mental. After the lesson, she talked about how the children were not learning anything and how shocked she was at what was written in their books. It was definitely their "fault" they did not listen. I, as university tutor, wanted her to teach a lesson where she was silent throughout. She looked shocked, "You mean after I've started them off?" I said, "No, throughout", and left.

Some weeks later, on my next visit, she played a game with the children that was used by the departmental team. The game is played with the teacher being in silence. I especially remember her eyes being so prominent as she threw them about the room. She was smiling and attending to the children, seeming larger than life, more theatrical than previously. To decide what to do rather than what to say involved her in responding and attending to what the pupils were doing and, although there were no words, she learnt to listen in that session.

**How do we try to recognise the learning of our prospective teachers?**

*Salvador*: We can identify different aspects of prospective teachers' learning because we see how they provide different details of their engagement with the mathematics content, say fractions and ratio, and how they justify their interpretation of student answers. We see what prospective teachers tell us, in particular how they reason about why aspects of the students' answers are not correct. Prospective mathematics teachers' discourses show differences in the appropriation of existing knowledge about teaching, which is considered as a resource to be used when making sense of students' answers.

*Laurinda*: When we observe our prospective teachers in the school context, we recognise their learning when they adapt to an incident without relying on habits from previous experience, such as their own schooling.

**What theories have informed our practices?**

*Salvador*: Professional noticing (Mason, 2002; Jacobs, Lamb, & Philipp, 2010; van Es & Sherin, 2002) is a knowledge-based process of reasoning that allows prospective teachers to recognise relevant aspects in a mathematics teaching and learning situation, giving meaning to justify what to do. To make that visible, we have been using hypothetical learning trajectories (Sztajn, Confrey, Wilson, & Edgington, 2012), which are focused on a topic area, or issues-related writing linked to what happens in video clips. We describe professional noticing as a knowledge-based reasoning process involving four processes: describing the practical event; attending to key aspects and differences in the situation; providing an account of them in respect of mathematics learning (for example, mathematics learning objectives); and, interpreting by labelling the event as an example of a general phenomenon. These processes support the justification of a set of alternatives for teaching.

*Laurinda*: My meeting with David Reid in 1995 led to being able to articulate the practices on the Bristol course using an enactivist worldview, which is biologically based (Maturana & Varela, 1992; Reid & Mgombelo, 2015). The kernel

principle is that "All knowing is doing and all doing is knowing". An enactive view of cognition involves: describing when staying with the detail of experience to allow access to implicit awarenesses and the possibility to change them; attending to differences and patterns as a biological way of being; being open to being triggered by the world and allowing new basic-level categories to emerge; and, leading to new actions arising with the possibility of post-hoc intelligent awareness articulating reasons behind them.

## Common themes

In our recent writing for the IHMTE, four themes have arisen that we recognise from our different perspectives: Attending; describing; labelling/basic-level categories; and interpreting (!). We are both linking theory and practice and what we get from the other is a different way of seeing. For instance, focusing on attending, we illustrate our process with an emerging new common awareness.

### *What are we attending to as MTEs in our different contexts?*

*Salvador*: When prospective teachers see the answers of students to a problem, which aspects of those answers are relevant to understanding more than what the correct answer is? What differences can be observed? Prospective teachers need to identify different aspects of students' answers because, initially, the prospective teachers see students' answers as dichotomously bad/good. The differences in students' answers can be seen, instead, as examples of progression in learning.

*Laurinda*: When prospective teachers first go into a classroom, they do not see as teachers. Their attention is taken by all sorts of happenings. When observing in other teachers' classrooms they can feel that the teacher is doing things wrongly because they do not fit with their prior experiences. The process of describing what happens in detail, identifying times when they feel uncomfortable, perhaps, supports the process of moving away from judgements to attending to students in their own classroom and acting contingently out of new awarenesses. As MTEs we can attend to this process in the prospective teachers as they move to acting like a teacher. In sessions at the university the prospective teachers explore multiple school contexts and multiple perspectives on learning mathematics through discussion with other prospective teachers in the group. Exploring the range of experiences again moves the prospective teachers away from dichotomies.

In the sharing of what we are attending to, another category emerges, that of supporting prospective teachers to move away from the right/wrong or bad/good dichotomies to have a more complex reading of their students' answers or theirs or other teachers' actions.

## Final comments

Our ways of acting and thinking about terms such as "attending to" and "labelling" is part of our learning as MTEs, leading to new labels such as "moving away from dichotomies" that in turn lead to new actions in our practices. We are not pointing to professional noticing and enactivism as being the preferred theoretical perspectives to hold, but more that, if two or more people are using different perspectives in action then looking together at the details of their practices, no matter how different they seem on the surface, will be a fruitful way to continue to develop new awarenesses and consequently new practices. We are currently working on the biological basis of

being alongside sociocultural approaches, such as use of tools and language, but this is work in progress.

As a participant in the session, Caroline Ormesher, wrote, after the event:

> It feels important to be with the sameness and difference in your two worlds. It makes me think of other work across contexts and how, by the time the findings reach their target audience, they have become a set of teaching tips and commands - do it like this because that's best. Your presentation made me aware of how a catalyst for change comes by being involved in some way. The different programmes of study for prospective teachers had different ways for the people to be deeply involved in their work - for me, that was the similarity.

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