Translating research into practice through collaborative planning: The case of the so called grid method

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Drawing on research that informs transformative teacher education, this paper will report on an ongoing study that develops mathematics teachers' knowledge and practice collaboratively. This paper accounts the experiences of a group of Welsh secondary school educators participating in collaborative classroom enquiry designed to develop GCSE students' understanding of linear and quadratic algebraic expressions. The paper identifies the potential to disturb and improve learning through the use of enactive and iconic representations of algebraic concepts, whilst identifying tensions that arise in the act of changing the context for learning in a secondary school classroom.

Teacher Education, algebraic expressions, collaborative enquiry

Debate about the nature of transformative teacher education is not a new phenomenon, but has, more recently, been informed by comparisons with international systems that have a sustained and transformative effect on teachers' professional knowledge. In particular, Darling-Hammond (2017) reports that professional learning opportunities that have an impact on practice are connected to teachers' collaborative work in professional learning communities. Alongside this collaboration is the need to translate research into practice. The on-going tension between theory and practice in teacher education was captured over fifty years ago by Stenhouse:

> … [the theorist] must try to produce theory which articulates the values, understandings and information and techniques which support various approaches to the work of the classroom. Such theory allows the teacher to act independently and creatively. (1967, p.152)

I think that it is my role, as a teacher educator, to work with teachers to articulate approaches to teaching mathematics that are research informed and that have the potential to transform learning in the classroom. This paper reports on a model of professional learning that has been developed collaboratively with several secondary school mathematics departments in order to centre professional learning on the classroom and on the experience of understanding how the students learn mathematics. The model is distinctive because of its co-constructed design and because the professional learning encounters are immersed in the school, at the site of learning for both the teachers and the students. In my experience, professional development courses that I have taught are well received, but translating the principles of the professional learning opportunity into practice is often limited. The professional learning remains situated in the university classroom or training centre and is not applied in order to transform the learning in the school classroom. To address this, a collaborative model has been developed that allows expertise in the theory and research that informs mathematics learning to be aligned with expertise in the context of the school classroom and the teachers' knowledge of their learners. A version of the teacher research group (TRG) has been developed that is influenced by the work of Zeichner (2003), Darling-Hammond (2017) and Swan and Burkhardt (2014), illustrated in figure 1.

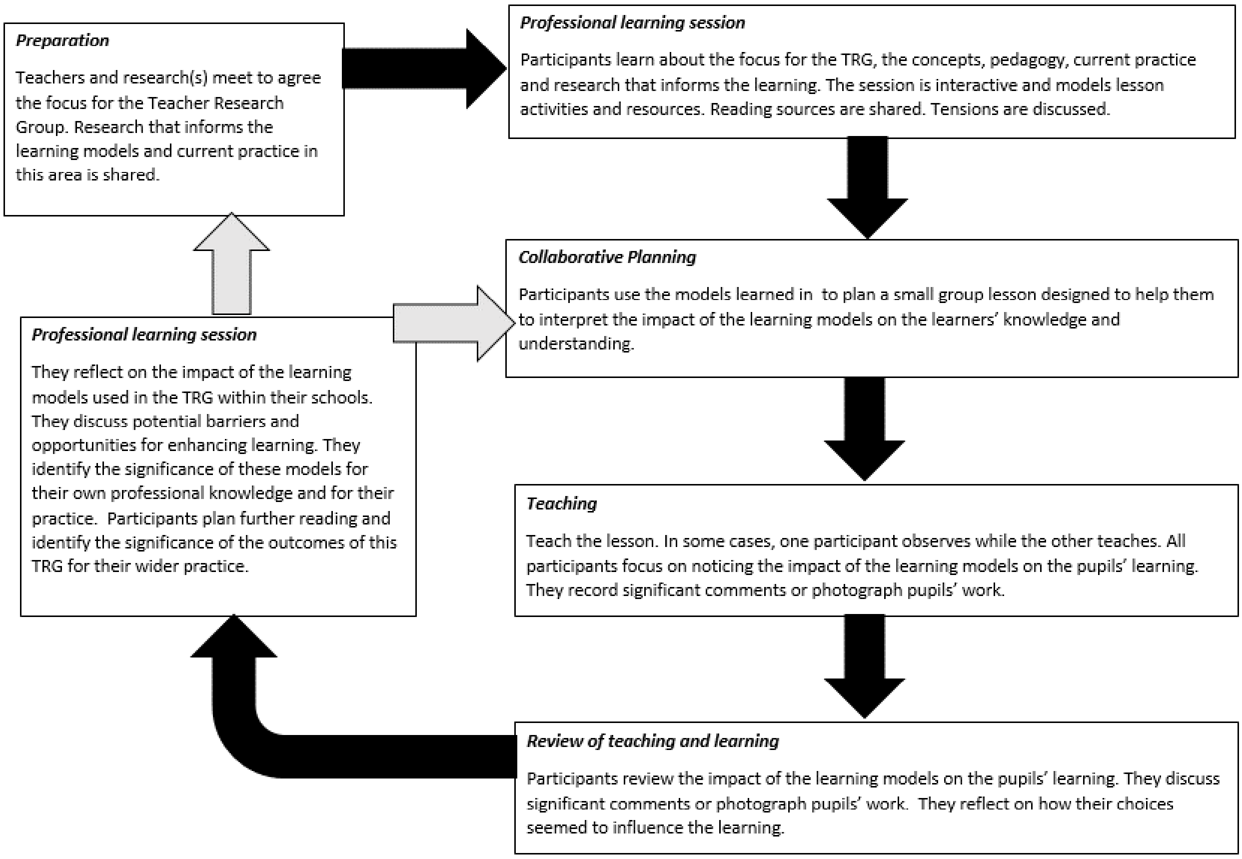

Figure 1: The professional learning cycle used for the teacher research group

One teacher research group (TRG) cycle took place in a Welsh secondary school, working collaboratively with a diverse group of secondary school educators. The preparation stage of the cycle allowed me to work with the mathematics curriculum leaders to agree that the focus should be developing GCSE students' insight into equivalent expressions for linear and quadratic expressions. This decision was informed by diagnostic use of the school's assessment data and provided the stimulus for the professional learning sessions that followed. Teachers shared samples of students' work that demonstrated varying degrees of fluency in using largely symbolic approaches to manipulating expressions. Teachers were seeking an approach that had the potential to allow more students to gain fluency in finding equivalent expressions. I shared resources with the teachers that demonstrated enactive and iconic representations (Bruner, 2006) of the expressions. In particular, the use of algebra tiles as an enactive representation of expressions was discussed, together with the connections that could be made between iconic and symbolic representations. Leong, Ho and Cheng (2015) illustrate this in their discussion of how concrete, pictorial and abstract representations of algebraic expressions are modelled in Singapore, highlighting the origins of this approach in Bruner's enactive, iconic and symbolic representations of knowledge. Figure 2 is Leong et al.'s diagram illustrating an approach to representations of factorisation that are similar to the models developed in the TRG:

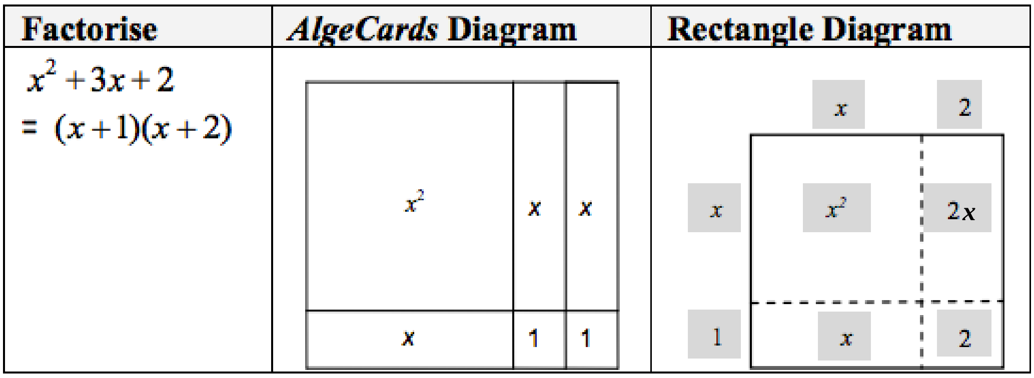

Figure 2: Leong, Ho & Cheng's (2015) three representations of quadratic expressions aligned to Bruner's symbolic, enactive and iconic representations of knowledge.

Teachers noted similarities between Leung et al.'s rectangle diagram and the approach that they described as the grid method. However, there was little connection made between the grid and the enactive representation of the concept. Figure 3 illustrates some of models used by the teachers.

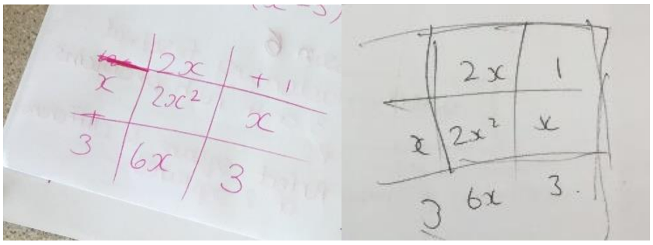

Figure 3: Teachers in the TRG used a grid as an algorithm and not an iconic representation of an array.

The grid method grew in popularity in schools in England and Wales from 1999 as a model for multiplying numbers and algebraic expressions. As an enactive representation of a multiplication array, the method could provide students with a stimulus for connecting physical arrays, grids that image the array and symbolic multiplication algorithms. When discussing changes to the national curriculum for England in 2014, Education Minister Elizabeth Truss suggested that the method did not have a place in the new curriculum:

> We are clearer about the required standard […] There will be marks for children who use efficient methods of calculation - such as long division and multiplication, or adding and subtracting in columns (as opposed to so-called 'chunking' or 'grid' methods) […] We know the best-performing places – like Singapore or Shanghai - have high expectations for every student […] Classes are 'taught to the top' - and then struggling students are given extra support to keep up. (2014, p.152)

To me, the so-called grid method had the potential to provide a stimulus for making connections between the structure of multiplication of numbers and expressions and the product of the multiplication. However, my experience of working with many teachers in professional development courses suggests that connections between the representations were missing. The sections in the grid were not connected to a physical representation of 27 rows of 38 or x+2 rows of x+1. The absence of this connection was apparent in the teachers' models in figure 3, where teachers talked about the grid method as no more than an algorithm for manipulating expressions. Through collaborative planning we were able to design a sequences of lessons that had the potential to stimulate connections between each representation and to pose problems that allowed students to understand the structure of the expressions and their equivalents.

Teachers were concerned about how GCSE students would be introduced to a new representation of expressions. The TRG lessons were to be taught to two higher tier GCSE classes in Year 10 and one foundation tier class in Year 11. We were aware that students would be likely to resist a change in the context of learning (Bruner, 2006), but were committed to implementing the models because most teachers believed that the enactive and iconic representations had the potential to stimulate connections between equivalent expressions and to allow more students to access a deeper understanding of the concepts. Students were introduced to the representations through a matching task that allowed them to connect the image of the array in the grid to statements like '5 rows of *x*+2' for the linear expressions and 'x+3 rows of *x*+2' for the quadratic expressions. The algebra tiles were available for students to connect each image to the symbolic expression 5(*x*+2) or (*x*+3)(*x*+2). During the first lesson the students were taught by their usual class teacher, with those observing focussing on the students' responses to the planned learning models. We were clear that we were not conducting an observation of the teacher, but an observation of the impact of the planned learning model on the students' learning. Figure 4 illustrates some of the students' responses to the tasks.

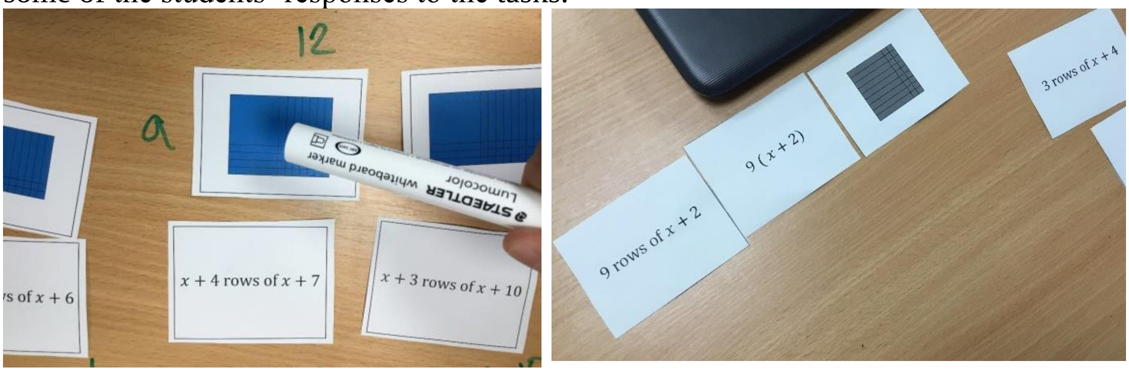

Figure 4: Year 10 and 11 students use of arrays to connect expressions.

This was followed by problems that allowed students to interrogate the structure of the array, such as 'which image has an area of 108 when x is 5?' and then to connect the area with the value of the expression when x is 5. The images, tiles and symbolic expressions provided the stimulus for discussion that helped students to make connections between one expression that represents the dimensions of the array and another that represents the area. As predicted, some students were reluctant to use the algebra tiles initially and were encouraged to draw images of the tiles on their whiteboards when the teacher asked students to convince her of the equivalence of two expressions using an image or array. Some students were able to share their images and were clearly able to connect the different representations. However, several students were not able to make connections and were distrustful of the proposed purpose of using the tiles. Without the collaborative community of the TRG, it would have been easy for the teacher to discard the tiles when the students resisted. However, using data from other projects that have used these representations, teachers were able to persevere with the models in subsequent lessons. It appeared that the explanations from the students who made connections between the images and expressions were a persuasive factor in encouraging more students to attempt the connections.

It was through a sequence of three lessons that the students became more adept at connecting the representations, and were able to make a choice between the image or algebraic manipulation to match equivalent expressions. Further planning allowed teachers to introduce factorisation as a process of '*make a rectangle with no gaps using all of the tiles given*'. The tasks were designed to provide students with a physical stimulus for connecting the area of the array with the dimensions of the rectangle, seen in figure 5.

Figure 5: Year 11 students' use of arrays to factorise $10x+5$.

These lessons exposed several tensions for teachers. Firstly, the additional time that it had taken to plan and teach the lessons in a manner that connects representations was not aligned with the school's learning programme. Teachers involved in the TRG were aware of the impact of the TRG on students' understanding, but were also aware that there was a risk that the students would not translate what they had demonstrated in the lessons into success in GCSE questions. Theories such as Skemp's relational and instrumental understanding (1976) underpin the principles of the design of the Singapore curriculum, the success of which presents a persuasive argument for teachers who are currently being exposed to the strengths of mathematics education in Singapore. Skemp argued for the need to spend time teaching in a manner that fostered relationships between concepts so that the depth of understanding gained meant that students were not forced to continually revisit concepts year on year. Nonetheless, additional lesson time was a feature of the discussions that followed the TRG. Secondly, teachers were aware that the activity and discussion that characterised the TRG lessons did not produce much work in the students' books. Figure 6 demonstrates the notes in one student's exercise book at the end of the lesson.

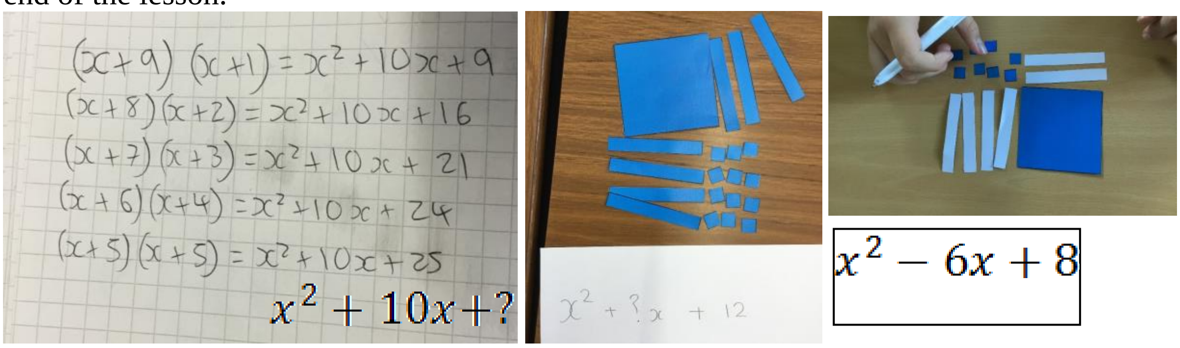

Figure 6: Year 10 students' use of arrays using algebra tiles in finding expressions that factorise.

At first glance the image from the exercise book could signify a limited amount of work in "expanding brackets" or factorising. However, the student had been engaged in discussion and enquiry to find possible coefficients of x in the quadratic expression to ensure that the expression factorises. The notes in the book summarise the outcome of the discussion and enquiry but fall short of capturing and representing the richness of the discussion and the route to the solutions. Thirdly, teachers were aware that the lessons had been designed using resources that I had provided. This approach is aligned with the work of Swan and Burkhardt (2014), who acknowledge the need for carefully designed tasks in classroom enquiry. Teachers raised concerns about how they might design tasks that allow students to reason from and connect between multiple representations of the concepts that they teach.

Again, comparisons were drawn with the Singapore curriculum, in which enduring learning models that embrace images alongside symbolic representations of concepts are commonplace, supported by Ministry of Education guidance (cited in Leong et al., 2015), together with enduring professional learning opportunities and carefully designed tasks within textbooks that serve as a guide to teaching. Despite the teachers' perception that these features were absent in their current practice, we were all aware of the strengths of the collaboration within the TRG and, in particular, the trust that was apparent between the participants.

Unlike earlier approaches to professional learning that we had experienced, this TRG was immersed in the school, at the site of learning. This feature, alongside the trust developed between participants, is particularly significant because we were able to translate research into practice in a manner that allowed us all to respect the teacher knowledge that we each brought to the TRG and were able to ensure that the enquiry was entirely focussed on the students' experience in the classroom. These aspects of the TRG are crucial to developing a more democratic, collaborative and transformative model for teacher education and is at the core of the on-going design of the projects that we undertake within this study.

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