Developing a consistent concrete, pictorial, abstract approach to the teaching of mathematics in upper key stage two.

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This research explores the use of the Concrete, Pictorial, Abstract approach, sometimes known as Singapore Maths. The approach is supported by a wealth of academic research.

Concrete mathematical resources are generally much less common in upper Key Stage Two classes than in lower years. However, I believe they still have an important role and in this action research project within my own classroom, I designed an intervention to promote the use of concrete resources. My planning for the use of concrete resources is meticulous and learning focuses on developing understanding of key concepts as opposed to mathematical processes. After spending time experimenting with manipulatives such as Cuisenaire rods, number counters, bead bars and numicon, learners are moved towards a pictorial representation, commonly the bar model. Although the approach has been adopted with all children, the research has focused upon the progress of four children. The research triangulates video analysis, summative assessment and pupil voice.

Keywords: Singapore maths; concrete, pictorial, abstract; mathematics pedagogy.

Introduction and Background

It is widely recognised and understood that a concrete, pictorial, abstract, (CPA) approach to teaching mathematics supports children to develop a deep and secure understanding. A fundamental part of this is the use of manipulatives to support the teaching of different concepts. Yet, very often the teaching models observed in classrooms fail to represent underlying concepts coherently (Ofsted, 2012). This paper accepts the premise that good teaching must focus on developing learners' understanding of concepts.

This paper reports on the implementation of an approach whereby teaching focused on using concrete resources to develop children's understanding of key concepts. A variety of manipulatives have been used ranging from Cuisenaire rods to number counters and numicon pieces, depending upon the area of learning.

Literature Review

A wealth of existing research highlights the importance of manipulatives in the concrete stage of the concrete-pictorial-abstract approach. Stein and Bovalino (2001 p. 357) note that,

> Manipulatives can be important tools in helping students to think and reason in more meaningful ways. By giving students concrete ways to compare and operate on quantities, such manipulatives as pattern blocks, tiles, and cubes can contribute

> to the development of well-grounded, interconnected understandings of mathematical ideas.

It is also understood that manipulatives support learners to link ideas and different areas of mathematics. Clements (1999, p.48) calls this "integrated concrete knowledge" and notes that manipulatives help students to connect and integrate knowledge.

In order to support learners to achieve this greater level of understanding, it is important to select the correct manipulative. Skevington (2016) underlines this, explaining that manipulatives that can be easily changed by learners to represent different concepts and mathematical ideas are the most valuable as they allow children to 'bridge' and transfer their understanding of different concepts. Skevington offers examples of this type of manipulative such as multilink cubes. Sharma and Connor (2017) conducted action research into a very specific area of learning (negative numbers) and taught the concept using manipulatives. They discovered that using the CPA approach in this very specific area had a significant impact on the achievement of pupils across a range of different abilities in test scores. The implications for both of these research projects are very important. In order for pupils to develop mastery they must link concepts and ideas.

Other researchers, such as Jo Boaler (1998) have produced a plethora of research which indicates that discussion and collaboration support children enhancing their ability to develop understanding and transfer learning. As a learning behaviour, I believe that the discussion and exploration of ideas is very important and mathematics manipulatives will support this.

Research by Marshall and Swan (2008, p.346) notes that, " … for maths manipulatives to be effective, they must form part of a carefully planned programme of maths. In particular, teachers' own knowledge of the mathematics, the children and the manipulatives need to be sufficient so as they can assist children to connect the use of the manipulatives to the concept being developed." This has significant implications as I will need to ensure that my work is carefully planned and that I understand what I am trying to achieve.

As well as developing consistency in the way in which pupils use manipulatives, it is also important that I develop a consistent approach when moving pupils towards pictorial representations. Hoven and Garelick, (2007), note that the Singapore bar model is a particularly effective variant on the draw a picture problem solving strategy because students are aware of what picture they should try and draw. This supports the student to draw their understanding graphically and use the information to solve the problem. The four basic types of bar model which are adapted by Erie 2 Maths, (2012) will be used. They are the part whole addition/ subtraction model; the comparison subtraction model; the part-whole multiplication/division/fraction model and the comparison: multiplication/division model.

Intervention

I began my research with a pre-test. This took place at the beginning of January. The results allowed me to compare the progress of a focus group of four lower ability students who have struggled to catch up with their peers despite intervention. I used our standard school assessment tests which four similar pupils in the parallel year five class would also sit at the beginning and end of every term. I then started to alter my pedagogy so that I was beginning every unit of teaching using manipulatives.

Initially, this required time to be spent developing my professional knowledge and re-planning lessons. Although I wanted to focus on the progress of the four learners, all learners in the classroom engaged in the change of pedagogy.

Every unit began at a very low entry point to ensure that I was developing understanding. For example, when teaching fractions, I began with open ended tasks such as asking learners to make as many fractions as possible using the Cuisenaire rods. Then, I started to target and develop my questioning to prompt learners to investigate the area of learning which I wanted to focus on. In one instance this meant asking them what happens when you add fractions. Often as part of this exploration, learners were forced to encounter and discuss things that they did not understand, for example, improper fractions. Then, as learners developed a secure understanding of fractions, we moved to more challenging areas such as dividing fractions. Leaners were forced to investigate and prove answers using the rods.

As pupils made progress in different areas, they were prompted to move towards the bar model representation. At this point, the evidence collected in work books began to increase significantly. Learners who were still finding concepts challenging continued to use concrete manipulatives for support. Others, who had made good progress, moved towards an abstract method.

For other concepts, such as number work, I used place value counters (0.001 – 1,000,000). This allowed pupils to explore calculation methods which they had learned in previous years. This was successful and helped learners to develop their understanding of concepts such as long multiplication.

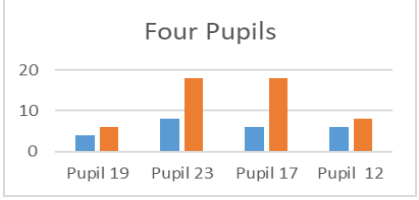
At the same time as I was adjusting the type of learning which I was presenting to the children, I also tried to use manipulatives to make my teaching more explicit. For example, I repeatedly used a bead bar to model mental calculation strategies. I also developed a working wall where I displayed a concrete representation followed by a pictorial and an abstract representation of each area of learning. Although this didn't form part of my initial planning it seemed to be a natural evolution in the way in which I could support learners.

Data was also gathered from pupil interviews and video analysis. The short video clips I gathered focused on the same four pupils and allowed me the chance to try and evaluate the quality of their discussion. I tried to take one clip a week for around five minutes as I knew that excessive footage would be very time consuming and difficult to analyse. Pupil voice offered pupils the chance to answer open questions about their opinions of their learning. The post intervention test presented the same concepts as the pre-test using different questions in a different order.

Results and Discussion

Progress Tests

Progress tests between January and May suggest that the four pupils in my intervention group made accelerated progress compared with the equivalent pupils in the control group. Two pupils in particular made very strong progress.



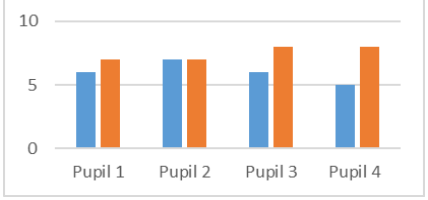


Figure 1 *Intervention Group* *Control Group*

Pupils twenty-three and seventeen have made enough progress to sit in an average ability bracket. Pupils in the control group were selected because they had a similar profile, to the pupils in the intervention group. They had all made a limited amount of progress in recent years despite intervention.

Children would be expected to make progress, even if that progress is slow, during any given time period. Significantly pupils in the intervention group made 18.75% progress on average with one pupil making 34.2% progress. Pupils in the control group made just 4.5% on average.

Pupil Interviews

Pupils' opinions were collected at the end of the intervention using open questions so that I could gain a sense of how they felt their learning had progressed.

One pupil in the intervention group noted that:

> The number counters really helped me with place value. Like when we have to make the largest number possible and when we have to take one tenth away from a thousand and stuff. It helped me to see and play with what I was doing and then I think I got that. I also liked it with perimeter using the rods. It helped me remember how it was different to area.

Another pupil, who has expressed negative attitudes towards mathematics in the past noted:

> I liked the work that we completed on fractions. I could see what I was doing and it made sense then. I also like working with my friend. When we were practising for the test I could remember how to use the bar model to help show that I had done it.

The power and importance of comments such as these should not be underestimated, as they highlight increased engagement and motivation.

Developing Understanding

Trying to develop pupils' understanding of concepts using manipulatives has led to profound changes in the way that I plan and teach. It is far from simple to just say 'start using manipulatives'. Before I started any topic or area of learning, I had to think carefully about how I could use a manipulative to support and develop understanding. This required me to spend a considerable amount of time researching the best manipulatives to use. Despite the fact that pupils were in year five, I began almost all topics at a much lower entry point. At this point the manipulatives were very powerful, showing learners how different areas of mathematics link together and how topics are progressive. As a teacher, I was constantly thinking about what I wanted pupils to learn and understand as opposed to focusing on modelling a method.

Removing the emphasis away from completed tasks in workbooks also altered the environment within the class. The fact that learners were able to experiment using manipulatives, removed a sense of failure which many lower ability pupils feel before they even begin their work. This reduced anxiety and improved behavior and many of these pupils felt a sense of achievement in mathematics for the first time. Meanwhile, more able pupils who often had a good understanding of concepts were able to progress onto more challenging concepts. For example, the picture below

shows how a pupil started to generate his own questions adding fractions with different denominators.

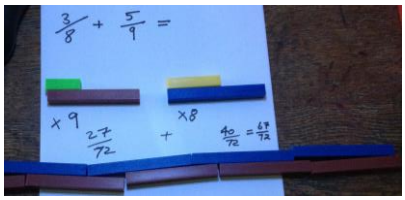


Figure 3 *Pupils Work*

Examples of learning like this not only demonstrate learner making progress but learners making generalisations. This is an important part of mastery, linking ideas and concepts together.

Bar Modelling

As well as introducing manipulatives I also sought to introduce the bar model as the pictorial method. Guides online produced by the NCETM were invaluable here. When I guided pupils and demonstrated using the bar model it had a significant impact upon attainment. However, when working independently, the impact was more variable.

Quantifying the Quality of Discussion

Attempting to quantify the quality of discussion which pupils engaged in was going to be both time consuming and challenging. Therefore, I decided to develop a very simple analysis frame where I looked for pupils explaining their understanding. Often pupils simply provide an answer or tell their friend what to do.

The video analysis which I viewed and analysed linked consistently with the work of Jo Boaler (1998) and was very positive. On numerous occasions, the pupils in my focus group were discussing and debating the questions which I presented them with. Frequently, pupils were trying to explain a problem using several different solutions. Research suggests that this will have supported pupils to develop a secure understanding.

Implications

In a very short space of time I tried to make significant changes to the pedagogy in my classroom. Although I feel that my findings are at the emergent stage, my evidence builds a picture of pupils making good progress as a result of the teaching and learning strategies employed; a concrete-pictorial-abstract approach. I recognise that much of the work that I have started needs to be refined and developed further. However, manipulatives clearly helped pupils to move past a surface understanding of concepts. Pupils who have struggled to engage fully in mathematics lessons found success for the first time and more able pupils enjoyed representing challenging concepts.

In order to develop the approach, I had to spend time developing my professional knowledge. This has implications for other practitioners if they are going to move away from a more traditional style of teaching. I invested in many guides and books which provided ideas and strategies for introducing manipulatives.

When I reflect upon how my teaching changed during the research, I recognise that the approach forced me to continually think about how I could represent areas of learning using manipulatives in a meaningful way which built upon learners' understanding.

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