

“I’ve done this before, but I can’t remember how to do it!”: Improving retention of learning in mathematics

Emma Baker

Caldicot School, Wales

A frequent complaint of mathematics teachers is that students cannot recall their prior learning, thus hindering efforts to build on this, to make further progress. This session reports on an action research project that aims to improve Year 7 students’ retention of mathematical learning and memory through recall and application of knowledge and skills. The research draws on the work of pioneering psychologist, Ebbinghaus. His work [1] in memory included the discovery of the forgetting curve, demonstrating how newly gained information is lost from memory over time when no attempt is made to retain it. This research involves a study of two similar ability Year 7 classes over two academic terms, implementing retrieval practice strategies through carefully designed homework tasks. Initial findings indicate that it is not only the act of retrieval that is important but also the associated feedback.

1. Memory: A Contribution to Experimental Psychology Ebbinghaus, H. (1913).. (H. Ruger, & C. Bussenius, Trans.) New York, NY: Teachers College

retrieval practice, task design, memory

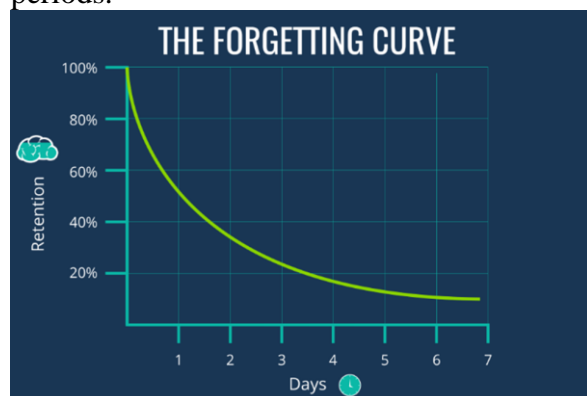
Introduction and background

Students of secondary mathematics classrooms are often heard to say that they have learned a topic before but cannot remember how to do it. Some may believe that this is because they have not truly learned it, merely undertaking an algorithm or repeating a process in order to reach a solution rather than having formed a deeper conceptual understanding. Kirshner, Sweller and Clark’s definition “learning is a change in long term memory” demonstrates that retention of knowledge and skills is an important indicator of learning. Development of long-term memory is critical in mathematics since

Expert problem solvers derive their skill by drawing on the extensive experience stored in their long-term memory and then quickly select and apply the best procedures for solving problems. Kirschner (2006) p.76

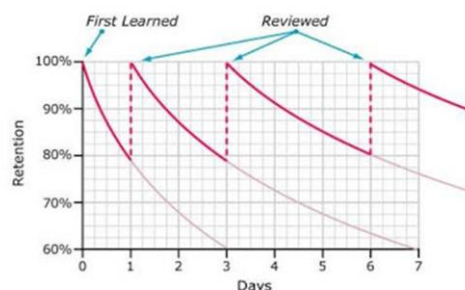
This study is based upon the seminal work of pioneering psychologist, Ebbinghaus. His work in memory included the discovery of the forgetting curve, demonstrating how newly gained information is lost from memory over time when no attempt is made to retain it. The work of memory specialist, Robert Björk, places emphasis on the importance of forgetting for learning. This work is centred on the act of recalling previously learned information in order to improve long term retention. This is an important idea to consider in improving retention of new knowledge and skills. There is an abundance of new research in the field of recalling prior learning to rehearse skills or revive knowledge intermittently over a longer period. This process of retrieving or recalling prior learning, henceforth known as ‘retrieval practice’,

enables development of long-term memory. Initial ideas in the development of this research project are based on the frequency of retrieval practice activities and how much time should be left between them. This is referred to as ‘optimal spacing’ periods.



<http://www.growthengineering.co.uk/what-is-the-forgetting-curve/>

Typical Forgetting Curve for Newly Learned Information



<https://www.marketing91.com/forgetting-curve/>

In considering retrieval practice in order to form memory, one may also consider what activities are best for students to undertake in order to embed learning better in memory. Daniel Willingham states that “memory is the residue of thought”

memory is the residue of thought—describes how memories are formed. What remains in your memory from an experience depends mostly on what you thought about during the experience. Given that we typically want students to retain meaning, we will mostly want students to think about what things mean when they study. It would be nice if you could simply tell your class, “When you read your textbook, think about what it means.” Naturally, you know that’s not the case. The instruction to “think about meaning” is difficult to follow because it is not specific enough. A better strategy is for students to have a specific task that will force them to think about meaning. Willingham (2008) p.20

The notion that for students to remember what they are learning they need to be engaged in thinking for it to begin to embed, implies that if activities are insufficiently challenging they will be more readily forgotten. For activities to begin to develop memory, there has to be some level of struggle. Put simply, how hard the students are working on a task is directly related to how well they will remember it. Task design is an important consideration with care taken to ensure tasks remain accessible whilst not being so challenging that learners disengage. This principal idea is described as the Zone of Proximal Development, a concept introduced by psychologist Vygotsky (1896-1934).

This initial research prompted a desire to further explore ideas and findings relating to retrieval practice and aim to answer the question ‘Which method of retrieval practice is most effective in improving retention of mathematics subject knowledge?’

Context and Methodology

This research took place in an 11-18 comprehensive school, located in South Wales. The school has in excess of 1300 pupils, very few pupils are from minority ethnic or mixed race backgrounds or have Welsh as a first language.

The work focuses on Year 7 students. Methods trialled have involved two, comparable, lower ability groups, working at Welsh Government National Curriculum levels 4 and 5. These learners have seven timetabled one-hour

mathematics lessons per fortnight and some are in receipt of additional intervention sessions to improve their basic number skills. At the start of year 7, students are placed in mixed ability teaching groups and, from September, have all of their mathematics lessons in this class until being regrouped by ability at the beginning of December. The curriculum students follow throughout the year is a planned sequence of mathematical topics. An example student overview is shown here:

Autumn Term

Half term 1	Half term 2
Area, Volumes and Perimeter	Handling Data – Collecting and Representing
Expressions and Formulae	Factors, Multiples and Primes
Sequences and Patterns	Decimals
Fractions	Money

In designing the model for retrieval, further practice of all topics that had been previously studied was planned. The intention being to cycle through topics in the first instance and then begin to repeat the cycle and potentially introduce practice that was a mixture of topics in order to keep skills ‘on the boil’. This ‘interleaved practice’ has been deemed important in improving mathematical learning

an approach known as *interleaved practice*, problems from the course are rearranged so that a portion of each assignment includes different kinds of problems in an interleaved order. Interleaved practice requires students to choose a strategy on the basis of the problem itself, as they must do when they encounter a problem during a comprehension examination The solution to a mathematics problem requires students to choose a strategy, not only execute the strategy, and students often find the choice of strategy to be more challenging than its execution.... Students need not learn to choose a strategy when every problem within a practice assignment requires the same strategy – an approach known as blocked practice.

Rohrer (2015) p.1

Task design

In planning retrieval tasks, the design of questions was critical, considering the idea that “memory is the residue of thought”, there needed to be an element of cognitive challenge for students and the level at which to pitch the tasks was a key factor. Whilst current practice in this department does include opportunities for students to apply their skills in context, it was a priority to extend these opportunities so that students were effectively prepared, not only for external assessments but also for further study of mathematics and, ultimately, the world of work. In order to achieve this aim, the tasks designed needed to not only check pupils were able to recall the skills needed to answer basic procedural questions but also apply these to questions set in real-life contexts. They also needed to answer problems that required a deeper understanding of concepts including multi-step problems, reverse problems or those that require proof or justification of solutions.

These ideas linked with the assessment objectives used by WJEC for GCSE Mathematics and WJEC Mathematics-Numeracy. With this in mind, the tasks designed for retrieval practice were split into three sections requiring pupils to Recall – Apply – Think. ‘Recall’ focussing on simple questions, ‘Apply’ requires pupils to use skills in context or reverse problems and ‘Think’ involves explaining or justifying and use of reasoning skills. These activities are known as Retrieval R-A-Ts. The use of these retrieval R-A-T tasks was implemented with one of the year 7 classes in the

early part of the Spring term following reorganisation of classes into ability set groups. The intention being to use the other class as a control group. Retrieval tasks were set as homework activities and reviewed in class following submission. Examples of the R-A-T tasks are shown below:

Algebra 1

RECALL
Simplify these expressions:
a) $a + a + a + a$
= _____
b) $2m + 3m$
= _____
c) $5d - 3d + 4d - 2f$
= _____
d) $7g + 3h - 4g + 3h$
= _____
e) $a \times a \times a$
= _____
f) $2 \times g \times h$
= _____

APPLY
Find the perimeter of these shapes.
[Diagram of a triangle with sides 3b, 3b, b]
[Diagram of a pentagon with sides d, d, d, d, d]
What is the perimeter of the rectangle in terms of d?
[Diagram of a rectangle with length 3d and width d]
What is the area of the rectangle in terms of d?

THINK
Sarah and her children Megan and Tom all have the same birthday.
Today, Sarah is 32, Megan is 4 and Tom is 1.
How old will Sarah be when her age is equal to the sum of Megan and Tom's ages?

Area and Perimeter

RECALL
Find the area and perimeter of these shapes:
[Diagram of a 3x3 grid]
Area = _____
Perimeter = _____
[Diagram of a rectangle with length 5cm and width 3cm]
Area = _____
Perimeter = _____
[Diagram of a house-shaped figure with a 3cm wide base, 3cm high walls, and a 2cm wide roof]
Area = _____
Perimeter = _____

APPLY
The area of Mrs Baker's rectangular lawn is 280m² in length is 7m. What is the perimeter of the lawn?
Mrs Baker wants to paint this wall.
[Diagram of a wall with a door and a window. The wall is 7 metres high and 2m wide. The door is 2m wide and 1m high. The window is 1m wide and 1m high.]
The tin of paint covers 17m². Will she have enough paint?

THINK
Can you work out the perimeter of one small rectangle?
[Diagram of a large rectangle divided into four smaller rectangles, each 3cm high and 1cm wide]

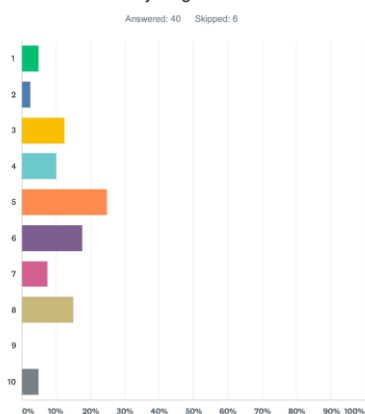
Findings

Research methods took place throughout the spring term, which provided a relatively short period to trial approaches. Students completed an e-survey to establish current perceptions and following this, the R-A-T tasks were set as homework activities.

Initial results

Forty students completed the initial survey. Students responded to questions about their mathematics lessons and approaches to homework, for example, did they receive help with homework and who provided this help. Further questions asked for students' thoughts on how well they remembered and were able to recall their mathematics learning and how they felt about their problem solving skills.

Q21 When we learn mathematics we learn skills that we need to remember, how well do you find you remember what you learn in mathematics? 10 = I always remember things well, 1 = I struggle to remember anything in mathematics



Q22 When we learn mathematics we learn skills and then we use these skills to help us solve mathematics problems. Which statement describes best how you feel about solving mathematics problems?



Students were enthusiastic about the research and attempting the new tasks but whilst the majority of students were submitting them on time, some did not. This had a knock-on effect to the spacing of tasks. This slowed the process down, as there was reluctance to proceed until nearly all students had completed tasks. This affected the number of practice tasks completed through the trial period but also had an impact on the immediacy of feedback. It was not possible to go through solutions to homework on receipt as some did not have it with them or had been absent when it was initially set. All needed to have completed tasks prior to whole class feedback so this meant that sometimes there was a delay of more than a week for feedback.

Initially this was concerning, as previously it has been understood that the more immediate feedback, the more likely learners were to engage with it. However, further research revealed that delayed feedback was not detrimental to learning and actually has its own benefits.

The finding that delayed feedback led to better retention than immediate feedback undermines the conventional idea that feedback must be given immediately to be effective...the benefits of delayed feedback might represent a type of spacing effect: the phenomenon whereby two presentations of material given with spacing between them generally leads to better retention than massed (back-to-back) presentations.

Roediger (2010) p.4

This suggests that the whole activity of feedback can itself be a form of retrieval practice in itself. Methods for feedback throughout the trial have varied throughout with the teacher modelling solutions on the board, students receiving written feedback or collectively reviewing a learner's work under a visualiser. Further work is required in this area, after further tasks are completed, the intention is to interview pupils to establish which method they feel is most valuable to their understanding.

A further finding has been that students are less likely to tackle the questions as they move across the page. On receipt of students' completed homework it is generally found that the 'recall' section is usually completed by all, most will attempt the 'apply' questions but few tackle the 'think' problems and those that do state they have received help with this from another adult. This issue raises questions regarding level of challenge. Whilst Rohrer states that "students often find the choice of strategy more challenging than its execution" this refers to interleaved practice, whereas the retrieval RAT tasks would be considered as blocked practice as all questions are based on one topic area. Roediger explores the idea of how complex a task should be discussing 'errorless retrieval' as some psychologists fear that if errors are made, these will be learned, creating a paradox.

If retrieval occurs under 'easy' conditions in which errors are less likely to be made, the impact of such retrievals on long-term retention might be undermined. Thus, a practical question is whether a strategy exists for retrieval practice that precludes making errors and at the same time permits the type of difficult retrievals that produce better long-term retention.

Roediger (2010) p.3

This also makes a case for further developing pupils' skills of perseverance and resilience. Homework may not be the most appropriate method for completing this type of task as pupils could be supported and prompted by teachers if completed during lesson time.

Implications for future work

Further study and trials are necessary in order to establish optimal spacing of tasks. The trial period for this research has, at this point, limited findings, and should be extended to fully explore the effects of spacing retrieval tasks.

Research comparing different schedules of practice provides additional evidence that repeated retrieval of information immediately after study,....., produces poor retention.....the best retrieval schedules are those that provide wide spacing of retrieval attempts, ... , expanding retrieval provides better retention after short delays, but equal interval retrieval produces better retention after long delays.

Roediger (2010) p.3

Moving forward, in this school, it is deemed necessary to expose students to a more structured approach to interleaved practice rather than the further blocked practice that R-A-T tasks provide. This does not necessarily have to be via R-A-T style questioning and may well initially focus on purely 'Recall' aspects to embed skills fully before application. Putting regular low-stakes tests in place is under consideration, using digital solutions such as online, self-marking, multiple-choice questions. These activities would need to be led carefully by teachers in order to ensure students receive effective feedback as

The need for feedback is critical after any type of test, but is particularly important for recognition tests (e.g. multiple choice, true/false etc.) because test-takers are exposed to incorrect information.... The danger is that because students learn from tests, taking a multiple-choice test might cause them to learn incorrect information and believe that it is true. Roediger (2010) p.3

Whilst this option removes opportunities for application of skills to problems, this needs to be less time intensive, as it will need to take place whilst other, new, topics are being taught. These may be undertaken as starter activities. Application of skills to contexts and problem solving will be taught through quality first teaching and can be revisited through ongoing internal assessments taken throughout the year.

A suitable way forward is to embed regular retrieval activities into the curriculum plan / scheme of work and these activities could vary between blocked practice and interleaved practice. Scheduling delayed feedback of practice activities will be another opportunity to embed retrieval practice into students' learning experiences.

References

- Ebbinghaus, H. (1913) *Memory: A Contribution to Experimental Psychology*. (H. Ruger, & C. Bussenius, Trans.) New York, NY: Teachers College
- Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational psychologist*, 41(2), 75-86.
- Roediger III, H. L., & Butler, A. C. (2011). The critical role of retrieval practice in long-term retention. *Trends in cognitive sciences*, 15(1), 20-27.
- Rohrer, D., Dedrick, R. F., & Stershic, S. (2015). Interleaved practice improves mathematics learning. *Journal of Educational Psychology*, 107(3), 900.
- Willingham, D.T. (2008). What will improve a student's memory? *American Educator*, Autumn 2008-09, 17-25