

Are there mathematical differences in what is addressed through different modes of interaction in classrooms?

Nick Andrews

University of Oxford

Is what is addressed in lessons through classwork likely to be mathematically different to what is addressed through seatwork? And if so, is this a planning issue to which mathematics teachers might usefully attend? This paper begins to address the above questions, which arose from an explorative study in which four series of lessons were analysed by treating each as a sequence of episodes. Focusing on the mode of interaction, each episode in the study could be categorised either as classwork or seatwork. Focusing on the mathematics, a finding of this study was indeed that across each of the four lesson series what was addressed through classwork episodes was different to what was addressed through seatwork.

Classroom research, Instructional design, Tri-polar analysis.

Background

In this paper I focus on an exploratory study of four cases of classroom practice over a series of lessons, and in particular differences in the manifestations of mathematical concepts offered by the teacher over time (a phrase I will define in due course) revealed through quantitative content analysis of teacher talk and classroom tasks. I draw on and adapt ideas from comparative studies in order to inform aspects of the research design of this local study and the wider approach of detecting differences in patterns of practice in order to infer underlying pedagogical choices.

Comparative research has for many years now highlighted teaching styles of particular groups of mathematics teachers (e.g. Stigler & Hiebert, 1999; Clarke, Emanuelsson, Jablonka, & Mok, 2006). Such comparative research allows for the teaching styles, or to use the term I will adopt on the current paper *patterns of practice*, of one group to be contrasted with another. My research approach is concerned with detecting a specific difference between patterns of practice, based on the premise that the source of such a difference is associated with a pedagogical choice (or sequence of choices). This then is the research means to the goal of *collecting* pedagogical choices. Drawing attention to each such choice affords a range of possible actions that result from that choice to be subjected to critical analysis, either by practitioners or researchers.

The grain size and nature of the pedagogical choice is a function of the unit of analysis and the analytic framework of the study that is seeking to detect differences. One line of enquiry reported on by Stigler and Hiebert (1999) for example explores at the lesson level the amount of time given over to classwork and seatwork. Here classwork means the teacher ‘working with all the students and, usually, orchestrating the discussion’ (Stigler & Hiebert, 1999, p.67) and classroom talk being public. Seatwork is ‘when students work individually or in small groups on assigned tasks’

and ‘talk is mostly private — teacher-student or student-student’ (Stigler & Hiebert, 1999, p.67). Stigler and Hiebert noted:

Teachers in all three countries spent more time in classwork than in seatwork. In Japan and the United States 60 percent of the time was spent in classwork; in Germany, it was 70 percent. Although the overall percentage of time spent in classwork was similar, shifts within the lesson from classwork to seatwork and vice versa were considerably more frequent in Japan than in the other two countries.

(Stigler & Hiebert, 1999, p.67)

This finding highlights the relative allocation of time to classwork and seatwork within a single lesson as a pedagogical choice. Whether this is a choice to which teachers consciously attend when planning and teaching, or indeed whether it is useful for them to do so, would not be a concern of the collecting stage of my research. It would however be a concern of the secondary critical analysis stage.

Stigler and Hiebert’s quotation above identifies further pedagogical choices: the length and sequencing of classwork and seatwork episodes across a lesson, where by episode I mean a part of a lesson within which the mathematical purpose and nature of teacher-learner participation (or *mode of interaction*) are essentially constant. The quotation also offers a glimpse of the structured types of episodes that are reported on in some other studies of Japanese lessons. Being smaller units of analysis, observed differences at the episode level are associated with finer-grain pedagogical choices than those mentioned so far. Shimizu (1999) for example highlights four distinct teacher roles within a lesson, each with a different mathematical purpose: *hatsumon*, *kikan-shido*, *neriage* and *matome*. This identifies pedagogical choices at the episode level, not just choices around the mode of interaction but also the mathematical nature of what the teacher makes available to learners.

The previous paragraph brings to the surface the nested nature of pedagogical choices at differing grain sizes, and how the actions that result from a choice at one level shape and are shaped by the actions that result from a choice at another level. Comparative studies into teaching styles (such as Stigler & Hiebert, 1999) have tended to use the lesson as the unit of analysis, yet Clarke et al. (2006) propose a series of lessons on a topic as more appropriate for the purposes of comparison. The current paper heeds this advice and explores if there are differences in the mathematical nature of what teachers make available to learners during classwork and seatwork across a series of lessons.

Methodology

For the purposes of the current paper, a short summary of the approach taken to detecting differences between patterns of practice will be offered. A methodological paper is being worked on separately that will provide fuller details and justifications of methods employed. Essentially the approach is interpretivist and teacher-focused, and is an analysis of the text of what the teacher is seen to offer learners over the course of a series of lessons on a single mathematical topic. By text here I mean the literal text of teacher-talk during lessons, whether that be public or privately to individual learners, but also the tasks, video clips, simulations, physical resources and other teaching media that the teacher makes available. It is worth emphasising also here that by task I mean the prompt that initiates subsequent activity in a lesson rather

than that activity itself, and to highlight this I refer to given-tasks. With these definitions in place, the text that constitutes my data then is the union of teacher-talk and given-tasks.

When observing a teacher working with a class on for example linear equations, I might see the teacher offering a bar model as in Figure 1, mathematical symbols such as “ $5x + 3 = 24 - 2x$ ”, or word problems such as “My father is currently three times my age. In five years’ time the sum of our ages will be 50. How old am I now?”

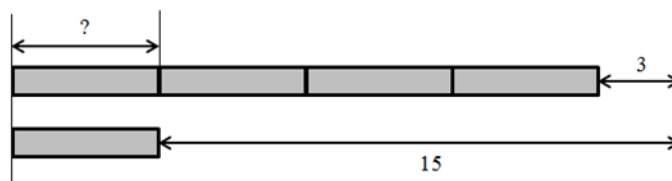


Figure 1: A bar model for a linear equation

I see each as a different *manifestation* of a linear equation, where by manifestation I mean the form of mathematical content I observe the teacher offering learners rather than the content itself. Attending to the form of content allows for comparison between episodes when the content is different and even when the topic is different. I classify the manifestation in Figure 1 as visual, the mathematical symbols as technical and the worded problem as functional (in the sense that a solution is likely to be arrived at through modelling the problem mathematically). These three components of manifestation may be combined, for example if the teacher was observed offering the worded problem above and heard saying “Solve this by forming and solving an equation,” this would be both a functional and a technical manifestation of a linear equation. In other cases, the spoken instruction may not be so explicit but nevertheless clearly implied in the context of the lesson. Either way, I would classify what is offered as functional-technical. Similarly manifestations might be classified as visual-functional or visual-technical, or indeed visual-functional-technical.

A quantitative approach is taken in order to detect differences through applying what I call a *tri-polar analysis*. Following Mason (1996) and Swan (2007), a triangular space (that I refer to as a *tri-polar space*) may be used to characterise graphically any experience of a phenomenon conceptualised as having an underlying triadic structure. Barycentric co-ordinates provide a way of representing quantitatively a point within the interior of a tri-polar space. A relative stressing of the three elements of the triad {visual, functional, technical} represented quantitatively in the ratio $v : f : t$ and satisfying the condition that $v + f + t = 1$ corresponds to the point (v, f, t) within the tri-polar space. For the current study, in order to ensure reliable coding the text of each lesson series is parsed into 30-second intervals, each of which is then qualitatively coded for manifestation. Quantitisation is achieved through each qualitative category being associated with a Barycentric co-ordinate. The quantitative code for the interval (referred to as the *interval centre*) is the mean of the Barycentric co-ordinates of all of the manifestation categories present in that interval. Each 30-second interval is also coded as either classwork or seatwork applying Stigler & Hiebert’s (1999) definitions given above (the very few intervals coded as both are removed). The *classwork centre* for the lesson series is then the mean of the Barycentric co-ordinates of all the intervals coded as classwork, and the *seatwork centre* is similarly defined. A statistical analysis of the v -values for each interval allows the stressing of visual manifestation between classwork and seatwork to be

compared quantitatively (reported here using an effect size r), and a similar analysis is conducted on the f - and then the t -values.

The current paper reports on four lesson series taught in local secondary schools in England with students aged between 13 and 16. One of the teachers, Ashley taught a series of five lessons on geometrical constructions. Bernie was at the same school as Ashley and taught a series of four lessons on ratio with one class and three lessons on linear equations with another. Courtney was at a different school and taught a series of four lessons on linear equations. In each case, the topic and the teaching approach was determined by the teacher. The four cases are used to address the question as to whether there are mathematical differences in what the teacher makes available during classwork and seatwork.

Results

Ashley's lesson series was parsed into 487 30-second intervals, 77 of which were categorised as classwork (16%) and 410 seatwork (84%). Across the series as a whole visual, technical and functional manifestation was stressed, with overall a slight foregrounding of technical manifestation. The tri-polar space for manifestation in figure 2a indicates differences in what was made available during classwork and seatwork, and further analysis revealed these differences to be statistically significant with a greater stressing of the visual element during classwork ($r = .205$) and functional element during seatwork ($r = .256$).

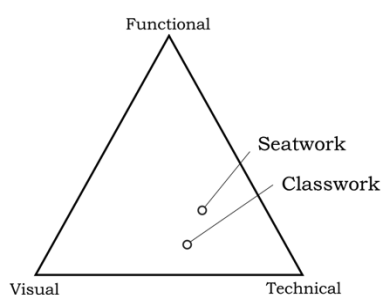


Figure 2a:
Tri-polar space for manifestation for the case of Ashley teaching geometrical constructions, indicating differences in what was made available during classwork ($n = 77$) and seatwork ($n = 410$).

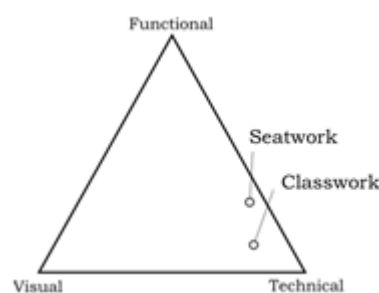


Figure 2b:
Tri-polar space for manifestation for the case of Bernie teaching ratio, indicating differences in what was made available during classwork ($n = 82$) and seatwork ($n = 310$).

In Bernie's lesson series on ratio, there were also differences between what was made available during classwork and seatwork (see figure 2b) again with a statistically significant greater stressing of the functional element during seatwork ($r = .384$).

The functional element of manifestation was under-stressed as a whole in the two lesson series on linear equations as in both series methods for solving equations were emphasised rather than using equations to solve problems, and it is interesting as an aside to note the possible effect of the topic between Bernie's two lesson series. Yet there were again differences between what was made available during classwork and seatwork in these two cases (see figure 3a and figure 3b) with a statistically significant greater stressing of the technical element during seatwork in both Bernie's ($r = .193$) and Courtney's ($r = .173$) lesson series.

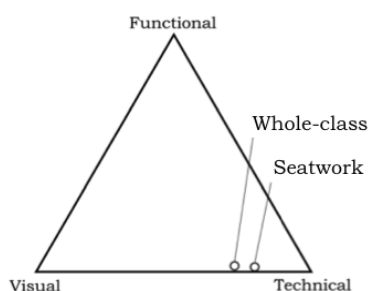


Figure 3a:
Tri-polar space for manifestation for the case of Bernie teaching linear equations, indicating differences in what was made available during classwork ($n = 81$) and seatwork ($n = 208$).

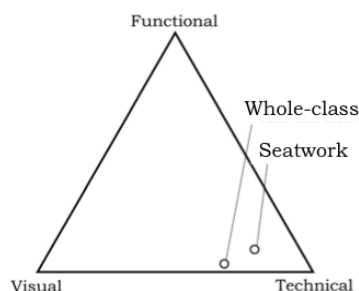


Figure 3b:
Tri-polar space for manifestation for the case of Courtney teaching linear equations, indicating differences in what was made available during classwork ($n = 146$) and seatwork ($n = 270$).

Discussion

Tri-polar analysis of manifestation at the lesson series level in each of the four cases detects differences in what the teacher made available to learners during classwork and seatwork. In turn this suggests that when planning a series of lessons, what to address mathematically through these two modes of interaction is a pedagogical choice to which the teacher could attend. Attending to this choice is an opportunity to think critically about the purposes of these different modes rather than acting out of habit. There is an opportunity to reflect on whether current enacted patterns of practice align with what is espoused.

The principle purpose of the research is to identify and collect pedagogical choices to which teachers could attend. That a difference in patterns of practice is detected is the primary finding, and so the nature of the differences is of secondary importance. The nature of differences however does suggest further lines of empirical enquiry, specifically whether the relative foregrounding of functional manifestations during seatwork and the visual manifestations during classwork noted in the current cases generalises more widely. Discussions in the BRSLM workshop led to a conjecture for example that the difference in visual manifestation may be less pronounced in a primary school mathematics classroom as more opportunities to work independently of the teacher with visual models may be offered.

References

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