Design for teaching and learning that supports answering GCSE maths questions – the case against "hey diddle diddle"

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There is an increase in assessment questions that probe students' reasoning in the latest design of the English mathematics curriculum. Analysis of this new genre of General Certificate of Secondary Education (GCSE) questions, that demand students engage in reasoning, provides evidence of the need for students to have knowledge of key ideas, fluency with basic procedures and contextual conceptual understanding. As part of an ongoing design research that seeks to support teaching and learning towards answering such questions this paper illustrates the issues at stake through one exam question. Focusing on the particular context of statistical measures and representations this paper explores what we mean by reasoning and how procedural understanding embodied in rhymes such as "hey diddle diddle, the median's the middle" fails to provide students with the necessary grounding for success at GCSE – let alone statistical competence to make sense of the world in which we live.

GCSE mathematics, statistics, algorithmic reasoning, creative reasoning

Introduction

In February 2013, the then Secretary of State for Education, Michael Gove, stated that the exams and qualification system was broken (Gove, 2013). Following this, a revised Mathematics GCSE, for examining from June 2017, was introduced. Alongside this change came the requirement that all 16 to 18 year olds with a near pass (previously grade D, but now grade 3) GCSE must continue to study for a re-sit (Department for Education [DfE], 2014a).

It is within this context and as part of an action research project to develop resources and lessons for teachers of GCSE re-sit students that this paper has been written. Here I report the investigation and review that took place prior to the design of lesson resources. As these resources are part of an ongoing randomised controlled trial the designed materials will not be shared in this paper. Instead, the focus is on understanding some of the changes to the assessment of GCSEs that have taken place and how this has informed the subsequent lesson and professional development design.

Changes to Mathematics GCSE exams

One of the English exam boards, Oxford Cambridge and Royal Society of Arts (OCR), summarised the changes in GCSE mathematics as

> A revised set of Assessment Objectives, with an emphasis on decreasing rote learning and more problem solving, that will often require multi-step solutions. Questions in assessments will be less clearly structured and more open ended, frequently set within real world contexts. (OCR, 2014)

Here I investigate the claims made in this summary by carrying out a systematic analysis of the newly formulated GCSE assessment and hence the relative importance of developing problem-solving and reasoning skills. The first claim considered below, albeit briefly, is whether exam papers have become 'less clearly structured and more open ended'. Second, I consider the claim that exam papers include 'more problem solving'. Finally, changes to the assessment objectives are exemplified by looking at one sub-objective to provide insight into the mathematical reasoning demanded of students.

Analytical framework

To inform our understanding of the changing nature of GCSE examinations an analytical framework was used that had previously been employed by the research team in its work on Evaluating Mathematical Pathways during the period 2006-2010. This framework attempts to provide detail of assessment in UK examinations by examining at item level details of structure, focus of task in terms of content and process, complexity and familiarity of task, as well as whether the task is situated in a pure context or whether it requires application of mathematics in authentic or artificial contexts. Further details of the analytical framework can be found in Drake, Wake & Noyes (2012).

Less clearly structured and more open ended

One indicator of whether the new GCSE has less structure is to look at the change in the number of marks per question. The charts in Figure 1 illustrate the aggregation of the Pearson Edexcel GCSE Mathematics exam papers in June 2016 (1MA0 - prior to the introduction of new GCSE papers) and June 2017 (1MA1 - following the introduction of the new GCSE papers) for both Foundation and Higher tiers.

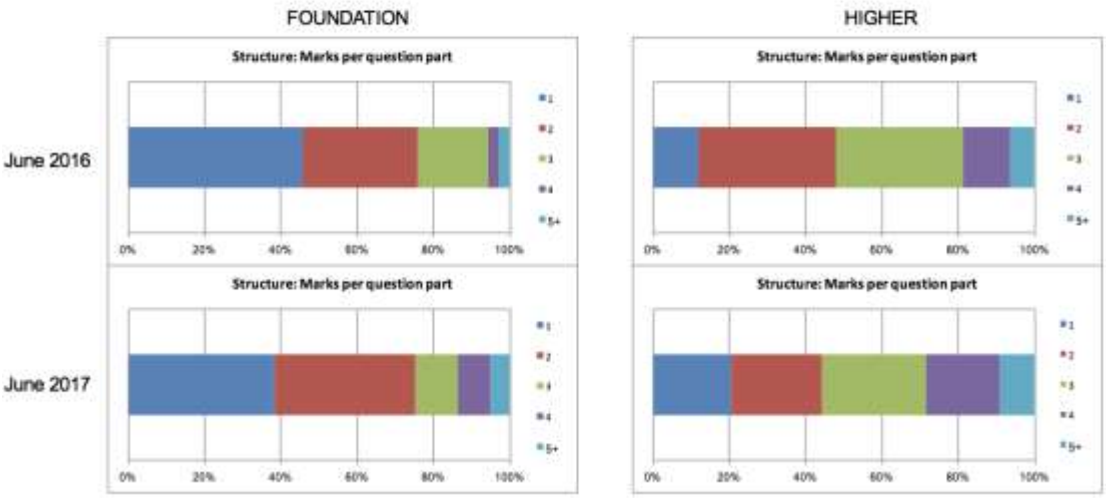

Figure 1: Number of marks per question part.

Given limited space here I draw attention only to the increase in 3, 4 and 5+-mark questions for both tiers. Hence, using this measure it does appear a reasonable claim that there is an increase in open ended questions, or at a minimum less structuring of assessment items.

More problem solving

The second summary claim is that there is an increase in problem solving in the new assessments. Figure 2 shows the comparison between June 2016 (1MA0) and June 2017 (1MA1) papers at both Higher and Foundation for Pearson Edexcel based on judgement of the nature of each mark in terms of process skills. Here, process skills have been identified in line with those outlined in the PISA framework that defines mathematics as a domain of study (Organisation for Economic Co-operation and Development [OECD], 2013).

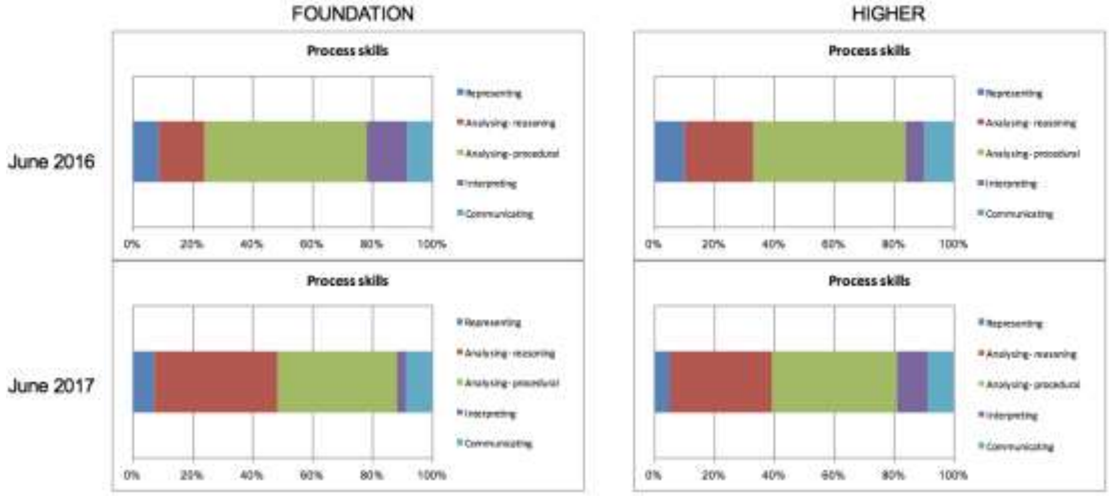

Figure 2: Pearson Edexcel - process skills

Again, space restricts commentary, but both Foundation and Higher assessments point towards a reduction in questions categorised as requiring only ‘anlaysing-procedural’ understanding and an increase in questions that require ‘analysing-reasoning’. Note that this is not the same as making a judgement on the reasoning methods employed by students - that would require access to a sample of student scripts. Rather, judgement has been made as to whether the question might rely on recall of straightforward procedure as opposed to engagement with more demanding conceptual understanding.

Assessment Objectives

Finally, the changes to the assessment objectives, and how this has affected the assessments, were considered. Here I exemplify our analysis by focusing on just one question that would usually involve students engaging in mathematical reasoning. Questions categorised as “analysing-reasoning” take many forms – and only one particular type (based on Assessment Objective classification) is considered below. One sub-assessment objective states that students should be able to “assess the validity of an argument” (DfE, 2013, p.13). This is particularly pertinent as it is both new to the curriculum and critical in helping equip students to make sense of the world in which we live.

At this point I draw further attention to the important distinction between the two categories ‘analysing-procedural’ and ‘analysing-reasoning’. Fundamentally this depends on whether the question can be answered by memorising a routine procedure or not. What this classification does not show is the method of reasoning that the students have adopted. Lithner (2008) defines reasoning as “the line of thought adopted to produce assertions and reach conclusions in task solving” (p.257). Students may attempt questions using a variety of reasoning styles – including Algorithmic Reasoning (AR) and Creative Reasoning (CR). One condition of AR is that the strategy chosen to answer the question is simply to "recall a solution algorithm" (p.259). In contrast, students that use CR have "arguments supporting the strategy choice" (p.266) and arguments "anchored in intrinsic mathematical properties of the components" (p.266). Hence, in this analysis a question may be classified as 'analysing-procedural' but students may have answered the question based on either AR or CR.

To illustrate the prevalence of AR that was detected in our analysis a comparison is made between two parts of one question that appeared on the Pearson Edexcel 1MA1/3H GCSE paper in June 2017 (Pearson Edexcel, 2017a). Figure 3 and Figure 5 show parts (a) and (b) of question 3 taken from the Higher paper. (The identical question also appeared on the Foundation paper but the Higher paper has been referenced here). Question 3(a) was classified as 'analysing-procedural', whilst part (b) was classified as both 'analysing-reasoning' and as part of the assessment objective of assessing the validity of an argument.

The table shows some information about the dress sizes of 25 women.

Dress size	Number of women
8	2
10	9
12	8
14	6

(a) Find the median dress size. **(1)**

Figure 3: Question part (a) from the 1MA1/3H, June 2017 Pearson Edexcel paper.

	Overall percentage of all candidates scoring each mark total	
	0	**1**
Foundation	64.6	35.4
Higher	27.1	72.9

Figure 4: Overall percentage of all candidates scoring each mark total for part (a).

Figure 4 shows the percentage of students that scored zero or one mark at both Foundation and Higher entry level on Question 3(a) (Pearson Edexcel, 2017b). As with all summaries of responses it is possible to gain an overview of behaviours but to only make reasonable speculations about the underlying thought processes (Vinner, 1997). According to the Examiners' Report (Pearson Edexcel, 2017b) the most common wrong answers in finding the median were 11 and 7. Both of these answers indicate that these students are not really considering the context of what the table represents. Instead, they are carrying out AR in a procedural calculation possibly based on a memorised sequence such as 'hey diddle diddle, the median's the middle'. Whether this exact ditty is used is not the point – the point is that many students have learnt a procedural, or algorithmic, reasoning process.

It is difficult to know what the reason for the difference between students entered at Foundation tier and those at Higher tier is. Have the students entered at the Higher level developed CR as opposed to AR? Alternatively, have the students entered at Higher level simply been able to remember a more extended form of AR than those entered at Foundation level? Some clues may be found in the second part of the assessment question shown in Figure 5. (Note that students are required to access information provided in the table of part (a) to successfully answer part (b)).

3 of the 25 women have a shoe size of 7.

Zoe says that if you choose at random one of the 25 women, the probability that she has either a shoe size of 7 or a dress size of 14 is $\frac{9}{25}$ because $\frac{3}{25} + \frac{6}{25} = \frac{9}{25}$.

(b) Is Zoe correct? You must give a reason for your answer. **(1)**

Figure 5: Question part (b) from the 1MA1/3H, June 2017 Pearson Edexcel paper.

	Overall percentage of all candidates scoring each mark total	
	0	**1**
Foundation	96.6	3.4
Higher	91.4	8.6

Figure 6: Overall percentage of all candidates scoring each mark total for part (b)

The first observation to draw from Figure 6 is that students at both tiers were extremely unsuccessful at this question. It appears that students preferred to answer this part of the question by considering whether the calculation should be a multiply or an addition rather than by looking at the contextual concept of mutual exclusivity (Pearson Edexcel, 2017b). The students have failed because they have focused on algorithmic understanding ('and' means multiply, 'or' means add) rather than on arguments anchored in a contextual conceptual relationship. In other words, for this part of the question, the majority of students that attempted it did not receive a mark as they focused on a calculation (indicative of AR) and not on the context (indicative of CR).

Secondly, the narrower difference between the two tiers could be a further indication of the adoption of AR. The students entered for the Higher tier did not have to apply a longer algorithm than the students at the Foundation tier - as could be argued on part (a) – and so were not more successful. It appears likely that AR with a focus on the calculation has led to the failure at the Higher tier.

Concluding thoughts

Helping students to develop creative reasoning skills is more important now than ever – both in terms of exam expectation and in terms of understanding the real world (DfE, 2014b).

These creative reasoning skills are best fostered in a teaching environment with a focus on context and the understanding of mathematics. As the Office for Stadards in Education, Children's Services and Skills (Ofsted) comment "a key issue for the majority of schools was to improve pupils' understanding of mathematics by focusing more on concepts…and by relying less on teaching 'rules'." (2012, pp.24–25).

Phrases such as 'hey diddle diddle the median's the middle' and "'and' means multiply, 'or' means add" are examples of rule based procedural teaching. This style of teaching leads to algorithmic reasoning which in turn means students are blind to understanding concepts within a context. In contrast, teaching aimed at contextual understanding develops creative reasoning and helps students make sense of both exams and the world in which we live.

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