Re-analysis of observations of lessons of students in Chile working on mathematical tasks

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This paper presents a re-analysis of the behaviour of Grade 8 students (aged 13–14 years) in Chile within mathematics lessons where they are engaged in their usual mathematics tasks and in a mathematics modelling task for the first time. Observations and re-analysis of the teacher's and students' behaviours from an enactivist perspective showed that patterns emerged, such as interval of waiting, in the interaction between the teacher and students.

Keywords: Enactivism, mathematical modelling task, actions, behaviour

Introduction

When I first observed students and their teacher in Chile, they were working to convert decimal numbers into fractions, doing operations with fractions and recurring decimals. I noted how the students changed both terminating and recurring decimals into fractions. They were using a procedural technique that is usual in the Chilean context, for example:

$$0.\dot{7} = \frac{7\text{-}0}{9} = \frac{7}{9} \qquad 2.\dot{5} = \frac{25\text{-}2}{9} = \frac{23}{9}$$

In addition, I was not only observing the change from decimals into fractions, I was also observing how the students arrived at this change and what they and their teacher were saying about that process.

However, when I was reviewing and analysing the data, I noted that I was not looking, for example, for how the mathematical procedural technique enabled the students to convert decimals into fractions, nor was I focusing the research on a specific concept or method; rather, I was observing actions that began to recur in the data.

As a participant in a British Society for Research into Learning Mathematics (BSRLM) discussion group, I showed participants three transcripts. Here is one of them, lightly edited, related to a conversation between a teacher and students about solving a budgeting modelling task, in which the students need to create a carnival game for 175 players, and plan how to spend a certain amount of money between prizes and materials.

C*n* refers to the number of the contribution in this excerpt from the transcript (text translated from Spanish):

> C1: Student 1: How [will] I know [how] to divide the prizes? [addressing the teacher] could be a way to change this sentence? […show how your plan for the game will work for 175 children]
>
> C2: Teacher: You make the game.
>
> C3: Student 1: And if we buy only smaller prizes, we can stay within our budget.

> C4: Student 2: The restriction says [the game] cost $150. We cannot exceed that amount. It must be exact.

I asked the audience in the discussion group to tell me what actions they observed in this transcript. The participants came up with different responses; for example, the teacher is shifting the responsibility back to the learners; the students have a responsibility to provide reasoning and justification for their actions that could not be seen otherwise; one student asks a question (contribution 1) but then, the answers are different (contributions 3 and 4) opening up the space.

These different observations were not a surprise. Our personal history and the experiences that we are living cause us to focus on different actions in the observation as Maturana & Varela (1992, p. 27) say, "where we came from and where we are going".

When I was analysing my observations of these actions, I was not just observing the questions and answers, I was observing the chain of events happening between the questions and the answers allowing me to see in more detail the interaction between teacher and students.

By recognising that there can be different approaches from the one used by the observer who is doing the research, can the different observed actions work together? This can be accomplished by analysing the interactions between teacher and students and noting the pattern of change in those actions.

To explore the observed actions between the students and their teacher when they were engaged in a mathematical task, I adopted an enactivist perspective in which learning happens in the interaction with one's surroundings (based on Proulx & Simmt, 2016). This allowed me to observe and re-analyse the actions of the students and their teacher when they were working on mathematical tasks (including a mathematical modelling task that was new to them due to recent changes in the Chilean mathematics curriculum).

From this re-analysis, which is part of my ongoing doctoral project, I will present a pattern that emerged from the behaviour I observed in different tasks (including the mathematical modelling task): the interval of waiting.

Theoretical Framework

According to Maturana and Varela (1992, p. 29), "knowing is effective action"; thus, by interacting with our surroundings in a specific moment we are learning.

I would like to make a distinction based on my position as an enactivist: the "existence" of the task occurs when the teacher and a student (in this study) interact, because "without action, there is no "world" and no perception" (Stewart, 2010 p. 3). Therefore, the world is not pre-established and neither is knowledge. Students and their teacher are in the world interacting, learning and doing mathematics.

In order to achieve "the mathematics of doing" in a task, the observer plays a crucial role, bringing forth the event observed in that particular time when students and their teacher are interacting and students are learning mathematics through the process of the interaction. This avoids the possibility of "making assumptions of what the participant may or may not know mathematically" (Maheux & Proulx, 2015, p. 216), *i.e.*, the observer does not impose a preconceived idea about how the interactions should be; rather, he or she is living what is happening between the actions that are being observed.

Therefore, I was observing the lesson at a "basic level" of categorisation of the objects in which I "recognize attributes of the object" through its actions (based on

Rosch, 1978, p.32, p.35; Varela, Thompson & Rosch, 1993, p.177). Naturally, I was not teaching the lesson; instead, I was there observing, interacting through my writing, taking notes, recognising attributes of questions that students had after engaging in the work of converting decimals to fractions. In short, I was observing and living the interaction from a researcher position.

In addition, I was also observing at the "superordinate level" of categorisation in which I can "recognize the particular attributes into the action as a set of actions" (based on Rosch, 1978, p.32, p.35). In that set of actions, I could note that working, for example with the algorithm to convert recurring decimals into fractions, generated a discussion and questions about the number 9 in the denominator, and I wrote about that in my fieldwork notes.

Therefore, bearing in mind the role of the observer in the research, in the next section, I will present the analysis of part of my study, in which the action of making a distinction at the start of an episode allowed me to see an interval of waiting.

Analysis

This study took place in a school in Chile over the course of 2.5 months in the classroom of a mathematics teacher of 23 Grade 8 students (aged 13–14 years).

The collected data consisted of eight 90-minute mathematics lessons, five audio-recorded and three video-recorded; four interviews with the teacher and two interviews with a group of four and five students, respectively. The number of students differed because the participants were selected based on the frequency of the interactions observed in the lessons.

The students worked to solve problems involving word problems, exponents and powers, rational numbers, squares and percentages in their usual way and then they engaged in a mathematical modelling task that was new to them, given the addition of modelling to the Chilean National Curriculum (Bases Curriculares 2012, 2013).

In analysing the data from an enactivist perspective, I observed the change in the students' and teacher's interaction across the different tasks from the original state (a first start) to another state (a new start when they made a shift from the original state) (based on Maturana & Varela, 1992, p. 40; Maturana, 2000, p.461). This is illustrated in the excerpt from the transcript below (referred to in the introduction previously).

In the mathematics lesson, groups consisting of a maximum of four students were solving a budgeting modelling task in which they needed to plan how to allocate a budget in order to create a carnival game for 175 players, with five water bottles and five beanbags and to also award three kinds of prizes: small, medium and large.

> C1: Student 1: How [will] I know [how] to divide the prizes? [addressing the teacher] could be a way to change this sentence? […show how your plan for the game will work for 175 children]
>
> C2: Teacher: You make the game.
>
> C3: Student 1: And if we buy only smaller prizes, we can stay within our budget.
>
> C4: Student 2: The restriction says [the game] cost $150. We cannot exceed that amount. It must be exact.

In terms of the students' contributions, contribution 1 is the first action that I observed, which was a question from a student. Later, the teacher replied (contribution 2). After that, student 1, who started with the question related to correcting the sentence, made a shift, a change from the original state (the question), and moved on to a suggestion related to the mathematical problem "and if we buy only...". The shift in the actions in contribution 1 allowed me to observe a change from the original state, which was the first question (contribution 1), to the other suggestions that were observed (contribution 3).

When the new start begins with a suggestion (contribution 3), the first question or the first start is transformed into the background context because the student is no longer referring to correcting the sentence; he/she has moved on to the smaller prizes. The background is important because it allows one to see if there is another change in the action and, therefore, if another distinction could be made.

Next, by observing the last two contributions of the transcript (C3, C4), it can be seen that the two students are engaged in the same actions, speaking about the budget and the restriction, literally, of doing mathematics. No change is observed between these two actions; therefore, there is no distinction between a new start and another background.

Observing the change in the action when a teacher and the students are doing mathematics it is possible to distinguish between any start in discussing mathematics and the background by taking into account "the coherence and the capacity to understand details of how the discourse unfolds as a complex weft of multiple voices" (Towers & Martin, 2015, p. 252). This will be a way to begin to identify details about the actions that are observed (on the basic and superordinate levels), which could show a pattern of the behaviour thereby acknowledging how the students and their teacher were doing mathematics as seen through my observations.

Let us return to the modelling task transcript. As a first approach to these actions or "basic level" observations, it is possible to observe that a student posits a question related to the prizes and some possible changes to a sentence in the task, as seen in contribution 1, which may require an answer. Thus, we could possibly see the action, such as questions and answers or another observation, as illustrated in the introductory section of this paper.

However, if we go into more detail and re-analyse the interactions between the teacher and the students, the students' questions can, perhaps, generate a closed answer from the teacher or other students. Nevertheless, the reply that happened was a sentence, "You make the game", as seen in contribution 2.

After that sentence, I observed an *interval of waiting*, a space provoked by the suggestion, "You make the game", and the new start (contribution 3). In this interval of waiting, the sentence stated by the teacher can trigger multiple answers, one of which was the student's reply: a conditional suggestion, with the use of 'if', to the other students that were participating in the dialogue (see contribution 3).

This contribution 3, the new start, does not generate an interval of waiting between contribution 3 and contribution 4; there is no evidence of a shift between these two actions (C3 and C4). The answer from student 2 (contribution 4) was a closed answer referring to the problem, "the restriction says…"; maybe this was due to the use of the conditional "if" in the intervention on contribution 3.

I would like to present another transcript from the start of a lesson in which the students were working with powers, exponents and square roots. The symbol (/) represents a one-second pause and the symbol (//) represents a more-than-two-second pause, italics means emphasis in the voice. The number between parentheses shows recording time and C*n* refers to the contribution number from the participants in this study:

> (4: 40_4:47) C1: Student 1: Teacher, I would like to ask (/) if two to the power of ten plus two to the power of ten [$2^{10} + 2^{10}$] is equal to two to the power of eleven [2^{11}]?
>
> (4:49_4:55) C2: Teacher: Yes…I agree with you, yes, but could $2^{10} + 2^{10}$ be 2^{11}? Hold on a second (/), Why [would this be the case]?
>
> (4.56_5.15) C3: Teacher: Look [at] pupils, the proposal here (//) was what Thomas asked me, *is it possible* that $2^{10} + 2^{10}$ equals 2^{11} (//). I said yes, but actually I want him to tell us why he can do this.
>
> (5.17_5.32) C4: Student 1: Because, I have (/), because my (//), 1024 plus 1024 is [equal to] 2048 and two to the power of eleven is equal to 2048.

As previously discussed, in a 'basic level' observation this transcript could be a series of questions and answers between a teacher and a student. However, if we look closely at the details, the first start from this action (contribution 1) generated another question from the teacher, "…$2^{10} + 2^{10}$ could be 2^{11}?". In this contribution (C2), the question, "Why [would this be the case]?" I observed what I call an "interval of waiting", which is a space triggered in the question pursued by the teacher, that can generate different re-actions, another reply from the student or, as in the case of what happened here, a new start, a shift from the initial state (now background) which was the question from student 1 (contribution 1), to a new start, which was the question from the teacher (C3).

This shift allowed me to observe the other question from the teacher which sought to know more about the proposal made from the student at the start of this action (see contribution 1). Contribution 3 does not generate an interval of waiting given the features of the question, which seeks to obtain a direct answer from the student in order to explain the mathematical concept.

Discussion

Although there are some recognisable actions that students working with a mathematical modelling task are likely to engage in, such as "Formulating, Solving, Interpreting and Evaluating" (Burkhardt, 2017), it is known that a "teacher and students arrange instruction very differently depending on the structure and demands shaped by tasks enacted in the classroom" (Shimizu, Kaur, Huang, & Clarke, 2010, p.1); therefore, different actions can be observed.

Taking the position of the "basic level" and "superordinate level" of the observations (based on Rosch, 1978; Varela, Thompson & Rosch, 1993) allows one to recognise the kind of observations that can be reached in a lesson by the observer's personal history because "where we came from and where we are going" (Maturana & Varela, 1992, p.27). However, to analyse the observation of the learning in the action, we need to go further and consider detail between actions because learning happens in the students' interactions with their surroundings (Proulx & Simmt, 2016).

Within the actions when the students and teacher were working on a mathematical task (including a mathematical modelling task) in a classroom, we can recognise for example an instruction, an answer or, perhaps, a question. However, to re-analyse the data using the distinction between the start and the background of the interaction between a teacher and their students who are doing mathematics allows us to see shifts in how the mathematics lesson is being carried out.

The shift is triggered by the kind of intervention that opens up possibilities; therefore, the questions are not closed questions, and the teacher or the students do not give “yes” or “no” answers. Instead, as evidenced by the present study, what occurs is an action or actions that allow(s) one to observe an effective interaction that is a kind of intervention (*i.e.*, a sentence, a question). Therefore, this intervention (an action) provokes an interval of waiting, a space between a new start and the background. Taking this waiting interval into consideration could create an option to observe the process of doing mathematics from another perspective and not just focus on questions and answers, offering insights into studying the actions observed and therefore the emergence of learning in the interaction between the teacher and students.

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