The variation of emotional affect in subject knowledge enhancement (SKE) students when undertaking problem solving exercises.

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We examined the emotional affect of a small cohort of nine postgraduate students enrolled on a subject knowledge enhancement (SKE) course on a teacher training programme in the northwest of England as they undertook a series of three substantial mathematical investigations. We monitored the emotional affect of the students at three points during each session using the short Positive and Negative Affect Schedule (PANAS) questionnaire and then related these scores to the students' output from the sessions. We noted a relationship between creative attempts at the problems and an increase in anxiety in the students. The results from the study appear to show that some students undergo substantial emotional changes, both positive and negative over the course of an investigation, despite the maturity of the cohort of students. This suggests that some additional thought should be given to the emotional wellbeing of students when planning intensive, investigative, mathematics lessons.

Keywords: Problem solving; emotional affect; subject knowledge enhancement; anxiety; creativity

Introduction

Effective mathematical problem solving is underpinned by several distinct criteria: subject knowledge; familiarity and effective strategy. This idea of a multifaceted approach to problem solving is explored in detail in Schoenfeld (1992) and Resnick (1988). Training non-specialist mathematicians to be effective mathematical problem solvers is an issue of growing importance as only 51% of mathematics teachers have a degree in mathematics and 12.8% of mathematics teachers have no qualification in mathematics beyond A-Level (Department for Education (2016)). Added to this is a drive towards problem solving through the new GCSE mathematics curriculum. To this end, as part of the subject knowledge enhancement course (SKE), we have six sessions devoted to investigative mathematics and problem solving. Within these six sessions, three are the focus of this study. The cohort of the SKE course comprised of eight postgraduate students, all of whom had studied least A-Level mathematics. Some of the students joined the course directly from their undergraduate degree, whilst others had diverse industrial backgrounds and were switching to teaching later in their careers. As a consequence, the ages of the students varied between early twenties and early fifties.

During these three sessions, the students were given a mathematical problem to investigate. At three points during each session, the students were asked to complete a Positive and negative affect schedule (PANAS) questionnaire (Watson, Clark and Tellegan, 1988) to assess their emotional affect. The PANAS questionnaire returns a value for the positive and negative affect of between 10 and 50. A high score on the positive words represents a high level of positive affect, similarly a high score on the negative words represents a high level of negative affect. The population mean score for positive affect is 29.7 and the population mean score for negative affect is 14.9.

At the time of this study the cohort of SKE students had participated in two sessions on problem solving, in which we discussed strategies for coping with open mathematical problems and outlined some useful strategies for problem solving. Once the three sessions had been completed, we attempted to link the students' output from the sessions to their emotional affect by considering their work and their PANAS score in conjunction.

Methodology

To anonymize the work of the students, each student was given a randomly generated three-digit ID number. This was used to relate the questionnaires to the output from each session. We shall only refer to individual students by using this ID number.

We chose the PANAS questionnaire to assess the students' affect, because, although it may not assess characteristics of affect directly relevant to problem solving, it does give us an accurate snapshot of affect with a robust statistical validity (Watson et al. 1998). The PANAS questionnaire, with its general assessment of affect, also gives us an interesting starting point for a more targeted follow up study.

We chose three distinct mathematical problems for the students to investigate. In the first instance, these were chosen for their own pedagogic worth and to provide the students with enjoyable experiences in investigative mathematics. They were also chosen for the relative obscurity of their solutions in that no student would be likely to succeed in quickly searching the internet for a solution. Finally, the questions were chosen because each question will produce results of note through multiple different approaches. Outlines of the three problems are given below.

The students initially received input on the problems, and were then presented with a PANAS questionnaire. They then worked on the problems for 45 minutes. At the end of the 45 minutes, they drew a line in their work, to indicate this moment, before being given a second PANAS questionnaire to complete. The students then worked on the problem for a further 45 minutes, before being given a third PANAS questionnaire.

Investigation 1: The Doubling Map.

Let $f(x)$ be defined on the closed interval [0,1] by

$$f(x) = \begin{cases} 2x, & 0 \le x \le \frac{1}{2} \\ 2x-1, & \frac{1}{2} < x \le 1 \end{cases}$$

Given that x is a rational number,

- Determine the set of all x such that $f^n(x) = 1$, for some n. (Note that here, we are defining $f^n(x) = (f \circ \ldots \circ f)(x)$ to be the composition of f, n times.)
- Determine which fractions $\frac{a}{b}$ create orbits, that is, determine the fractions, $\frac{a}{b}$, for which there exists $n > 1$ satisfying $\frac{a}{b} = f^{(n)}\left(\frac{a}{b}\right), \frac{a}{b} \neq f^{(n-1)}\left(\frac{a}{b}\right)$
- Determine what fractions satisfy neither previous category.

Supporting examples were given. For example, $\frac{1}{7} \to \frac{2}{7} \to \frac{4}{7} \to \frac{1}{7}$ shows that $\frac{1}{7}$ generates an orbit. Whereas $\frac{1}{8} \to \frac{1}{4} \to \frac{1}{2} \to 1$, showing that $\frac{1}{8}$ converges to 1.

Investigation 2: Rolling dice.

Suppose a die is oriented with a 1 on its top surface, a 2 on its front face and a three on its right face. (This is a unique orientation, possible with a Western die). Then suppose it is rolled along a path to a final location. Determine the new orientation and how this orientation is dependent on the path taken.

Both supporting examples and a suggested notation were given to assist the students in this investigation.

Investigation 3: The Collatz Conjecture.

Let $f(n)$ be defined on the natural numbers as follows:

$$f(n) = \begin{cases} \dfrac{n}{2}, \text{if } n = 2k, k \in \mathbb{N} \\ 3n + 1, \text{if } n = 2k + 1, k \in \mathbb{N} \end{cases}$$

The Collatz conjecture states that, given n, there exists k such that $f^k(n) = 1$.
The students were told to investigate this problem, describing any patterns they found.

Results

The affect of the students throughout the study is summarized in the tables below. The cohort of students was very small (n=8). However, as each student supplied a questionnaire on three occasions per session, on three separate sessions, we have an interesting set of data to work with.

Positive Affect Scores.

	Session 1 (SD)	Session 2 (SD)	Session 3 (SD)	Mean
Questionnaire 1	33.85 (5.37)	32.88 (3.48)	28.9 (7.85)	31.87
Questionnaire 2	38 (6.81)	35.13 (7.97)	29.13 (7.40)	34.08
Questionnaire 3	34.43 (6.63)	30.13 (7.00)	28.25 (7.57)	30.93
Mean	35.43	32.71	28.76	

Summary of positive affect scores by questionnaire and session.

Negative Affect Scores.

	Session 1 (SD)	Session 2 (SD)	Session 3 (SD)	Mean
Questionnaire 1	11.42 (2.30)	12.62 (2.88)	12.00 (2.45)	12.01
Questionnaire 2	14.29 (6.40)	14.25 (4.71)	13.38 (4.34)	13.97
Questionnaire 3	14 (4.97)	12.63 (3.11)	11.87 (1.13)	12.83
Mean	13.24	13.17	12.4	

Summary of negative affect scores by questionnaire and session.

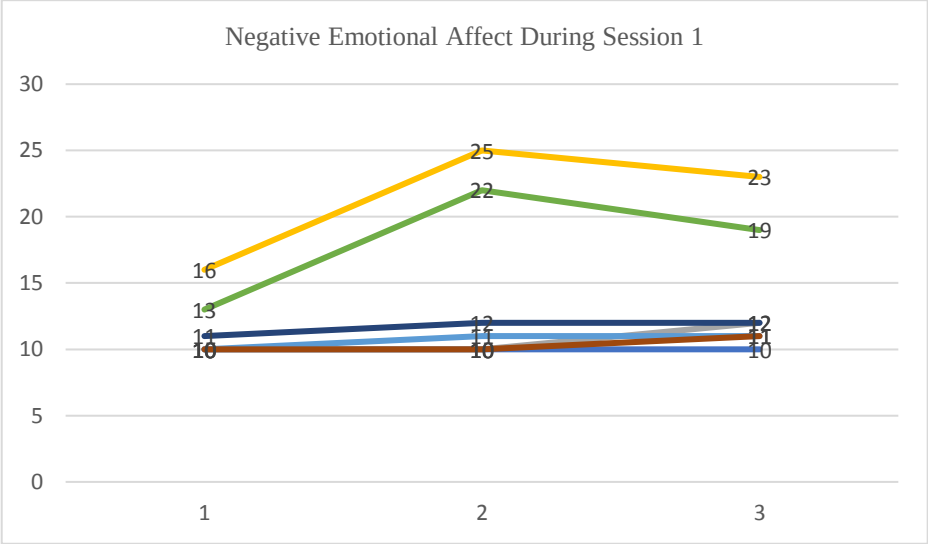
We can see that in general,

- The cohort exhibits an above average positive emotion affect (General population, $\mu = 29.7$, SD = 7.2) meaning that they are generally feeling more positive than the population at large
- The cohort exhibits a low score for their negative emotional affect (General population, μ = 14.9, SD = 6.2) meaning that they are generally feeling less negative than the population at large

The only exception to this is for the positive scores in session 3. This suggests that either the students were getting weary with the sessions or that there is some other factor to take account of.

In all sessions, we can see an increase in both positive feelings and negative feelings towards the middle of the sessions, before the affect of the students returns to close to the baseline at the end of the sessions.

The graph below shows the negative affect of each student during the first investigation as recorded on all three of the PANAS questionnaires. Each line graph represents a different student.



Graph of negative affect during investigation 1. The top line graph represents student 383, the second line represents student 612.

We note that two students have particularly high levels of negative affect. Moreover, their negative affect is undergoing substantial change during the session, whilst the rest of the cohort have considerably lower levels of negative affect and their affect is much less volatile. When we look at the output of the students from the session, we discover that these two students produced by far the most creative work.

During the first task, we see that student 612 is the only student to attempt a further abstraction of the problem by labelling the function $2x$ as a and $2x - 1$ as b and then considers the orbit of a point, x, as a word expressed in terms of a and b. This method could produce meaningful results if pursued effectively, particularly if the student had considered the preimages of these functions. Unfortunately, this was not done! This student also introduced new terminology, using the word "snared" to describe a point entering an existing orbit. For example, the fraction $\frac{1}{6}$ is "snared" by the orbit generated by $\frac{1}{3}$. Various diagrams are used to represent the fractions and their orbits, this is of interest because graphical representation of this problem were notable in their absence from the work of the rest of the cohort. Despite this more creative approach, progress with the problem eventually stalls and this I think is reflected in the

substantial increase in negative affect from 13 up to 22 and then back down to 19. For brevity, we will denote this: (13-22-19). The student's positive affect also increased, though less significantly (30-35-34).

Student 383 uniquely considered decimal expansions of the fractions and noted the divergence in the orbits of two numbers with a small initial difference. That is the student noted the sensitivity to initial conditions of the doubling map (without using this key phrase!). We see over the course of the task, a substantial increase in negative affect (16-25-23) and a corresponding decrease in positive affect (34-26-29).

Similar patterns present themselves in investigation 2 and 3. In both cases, students 612 and 383 exhibit the highest and most volatile scores in negative affect and produce the most creative work.

We see in investigation 2, student 612 separates distinct paths on the basis of "parity", which appears to refer to whether the paths had even or odd lengths. The paths were further partitioned into "straight lines", "diagonals", "square paths" and "even groups totaling 4 in any direction". There a clear process of conjecture and testing taking place within this student's work. During this investigation, we see the greatest variation in positive affect (30-42-24), whilst the student maintains the second highest negative affect in the cohort throughout the investigation.

Student 383 produced interesting work during investigation 2. Despite the sessions preamble establishing a useful algebra for describing paths, the student attempted to produce their own notation, this was problematic as they used an additive notation and implicitly assumed that the rolling of a die was a commutative process. That is the student assumed that rolling a die forwards and then right is the same as rolling the dice right then forwards. This assumption resulted in the student incorrectly asserting some distinct paths were equivalent. The progress of the student was fundamentally limited by this approach until they returned to the suggested notation. This appears to be reflected in the changes in their negative affect (18-25-20). Their positive affect remained relatively high throughout the session (33-31-29).

Investigation 3 is distinct from the two preceding investigations in that it is an unsolved problem. This had an immediate impact on the emotional states of the cohort. The range of positive affect scores on the first questionnaire was 26, in comparison to 11 and 16 for investigations 1 and 2. This is also reflected in the standard deviation (7.85) in comparison to investigations 1 and 2 (5.37 and 3.48 respectively). Student 383 made good progress with this problem, using an Excel spreadsheet to produce tens of thousands of results. The student initially had the lowest positive affect recorded in the study (15). This was maintained through the second questionnaire, but at the excel sheet produced results, their positive affect increased markedly to 32, a change of 17 over the course of the 90 minutes. Their negative affect was also initially. the highest of the cohort (17), but decreased (to 14) over the 90 minutes.

Conclusion

Throughout this series of investigations, there appears to have been an association between creative or novel thinking and comparatively high levels of negative affect. A possible explanation is that negative affect increases with the repeated failure of strategies and with a sense that insufficient progress is being made. Negative affect then falls when a strategy starts to yield results. Further to this positive affect appears to be largely unaffected by these failures, but changes when significant successes are

achieved. If this is the case, then it would follow that a creative mathematician is likely to experience high level of negative affect when tackling an unfamiliar and difficult problem as they will have many strategies available which could reasonably fail to make progress. Equally, a creative mathematician is likely to be rewarded with a substantial increase in positive affect when successes are found. This mirrors the author's experience of attempting a difficult problem, during which feelings of anxiety flourish as methods fail, but this negativity is almost completely forgotten when success is achieved.

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