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# INVESTIGATING CHILDREN'S THINKING ON THE TOPIC OF 'RATIO'

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*In this paper we present three items from a diagnostic instrument which was constructed with the aim to reveal children's proportional reasoning. The items for this instrument were selected having as criterion their diagnostic value, that is, their potential as a compilation to provoke a variety of responses from pupils. The instrument contains two versions. One consists of the items presented mainly as written statements and the other consists of the same items, accompanied with models thought to facilitate children's thinking. We present data from Year 6,7 8 and 9 children and we comment on the diagnostic value and the models' influence on children's strategies.*

## INTRODUCTION AND BACKGROUND

Previous research in mathematics education (Hart, 1984; Tourniaire and Pulos, 1985; Singh, 1998) reveals that ratio and proportion are difficult concepts for pupils to learn and teachers to teach. We believe that children's errors and misconceptions can be the starting point for the effective teaching of this topic. In this study we aim to contribute to teachers' awareness of their pupils' misconceptions by developing a diagnostic instrument for proportional reasoning.

We constructed the instrument using 24 'missing-value' type, items. Some of the items have been adopted with slight modifications of those used in previous research studies and others have been created based on findings of that research. All the problems were selected having as criterion their 'diagnostic value': their potential to provoke a variety of responses from the pupils, including errors stemming from misconceptions already identified in the literature. We tried to use a variety of problems as far as 'numerical structure', 'semantic type' and 'local context' is concerned. As a result of this selection, errors indicative of common and frequent misconceptions such as the 'addition strategy' (which will be described later) were expected to occur. Furthermore, we hoped that less frequent misconceptions or even ones that are not mentioned in the research literature would also occur.

Two versions of this instrument were constructed. The first version ('W Test') contains all the items presented as mere written statements. The second version ('P Test') contains the same items supplemented by 'models' thought to be of service to children's proportional reasoning. These models involve pictures, tables and double number lines.

## METHODOLOGY

The study sample (N=232) was of Year 6, 7, 8 and 9 pupils (aged 10 to 14) from four schools in the North West of England. In each class, half of the pupils were given the W version of the test and half the P version.

Firstly, a qualitative analysis of the results was conducted. For each item, all the pupils' answers, correct and erroneous, accompanied, where possible, by the corresponding strategies were recorded. Then the results were subjected to a Rasch analysis, which, amongst other things, allowed us to examine the difficulty of the same item in the W and P form. This is possible since the two forms are equated through common items.

## FINDINGS

### Item: 'Paint 2'

This item has been created based on Tourniaire's (1986) findings and is presented in the W form of the test as follows:

> John and George are painting together.
>
> They want to use exactly the same colour.
>
> John uses 3 cans of yellow paint and 5 cans of green paint.
>
> George uses 20 cans of green paint
>
> How much yellow paint does George need?
>
> Answer:
>
> How did you find this answer? Please show your working out below.

From the qualitative analysis of the data it was obvious that almost all of the pupils that solved the item correctly used a 'multiplicative within measure space' approach: 4×5=20 so 4×3=12.

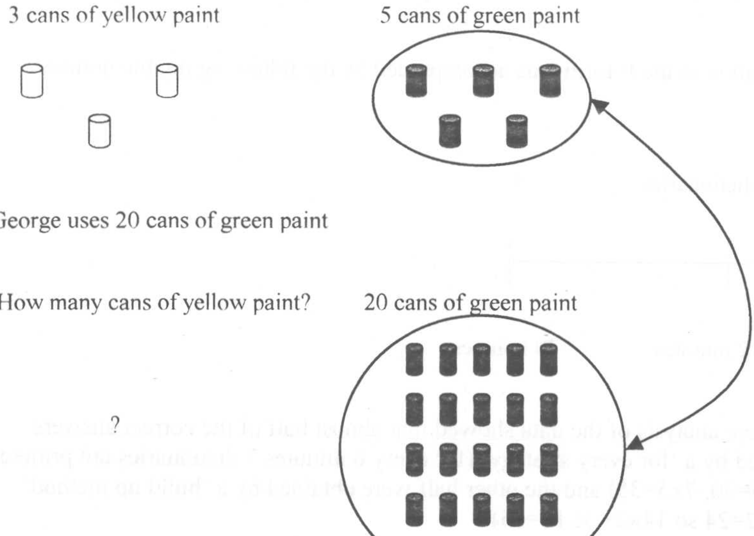
The most frequently used erroneous strategy in this task was the 'constant difference' or 'additive' strategy which has as a result the incorrect answer '18'. This answer can be obtained either by thinking that 5-3=2 so 20-2=18 or by thinking that 20-5=15 so 3+15=18. The second most frequent incorrect strategy employed was the 'incomplete strategy'. Here the pupil gives as an answer the number '3' because three are the cans of yellow paint or the number '20' because twenty are the cans of green paint. Finally, another incorrect strategy that was provoked by this item was the 'constant sum' strategy. In this case, the child thinks that the sum of John and George's cans of green paint which is 25 should be equal to the sum of John and George's cans of yellow paint: 20+5=25 therefore 3+22=25 and so the answer should be 22.

The same item is presented in the P form of the test as follows:

John and George are painting together.

They want to use exactly the same colour.

John uses 3 cans of yellow paint and 5 cans of green paint.




How much yellow paint does George need?

(You may use the pictures above to help you find the answer)

Answer:

How did you find this answer? Please show your working out below.

The performance of pupils on the W form of the item was compared with the pupils' performance on the P form using the data from the overall Rasch analysis of the items. The percentage of correct answers on the W form was 33.6% whereas the same percentage for the P form was higher, namely 48.3%

**Item: 'Printing Press'**

This item has been adopted from Kaput and West's (1994) study and is presented in the W form of the test as follows:

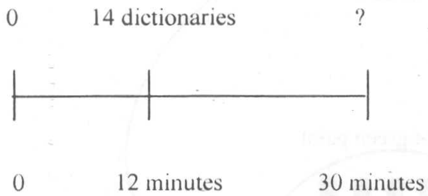
A printing press takes exactly 12 minutes to print 14 dictionaries.

How many dictionaries can it print in 30 minutes?

Answer:

How did you find this answer? Please show your working out below.

The same item is in the P form was accompanied by the following double number line:




The qualitative analysis of the data showed that almost half of the correct answers were obtained by a 'for every strategy' (for every 6 minutes 7 dictionaries are printed so since 6×5=30, 7×5=35) and the other half were obtained by a 'build up method' (12×2+ ½ 12=24 so 14×2+ ½ 14=35)

Equally frequent incorrect answers were the one derived from the additive strategy, namely 32 and the answer '28' derived by a 'magical doubling' strategy: the pupils think that by just doubling they can obtain a correct answer.

The performance of pupils on the W form of the item was compared with the pupils' performance on the P form using again the data from the overall Rasch analysis of the items. The percentage of correct answers on W form was 15.5% whereas the percentage of correct answers on P form was higher (20.7%)

**Item: 'Book Reading'**

This item has been adapted from Resnick and Singer's (1993) study and is presented in the W form of the test as follows:

Sheila reads 2 pages of her book every day.

George reads 4 pages of his book every day.

They both read in exactly the same way, each day.

After some days, Sheila has finished 20 pages.

How many pages has George finished?

Answer:

How did you find this answer? Please show your working out below.

The qualitative analysis of the data for this item showed that it was quite easy. Most of the pupils (79.3%) solved this problem by simply multiplying 20 by two or by multiplying four by 10. Hence, the data do not show any significant misconceptions connected with this item. Still we believe that the fact that this context and this number structure provoked a lot of correct responses could be used as a useful information to the teachers about what their pupils can or can not do.

The same item in the P form was accompanied by the following table:

| Day | Sheila | George |
|---|---|---|
| 1 | 2 pages | 4 pages |
| | 20 pages | ? |

The Rasch analysis shows that pupils performed better when they were given this item in its P form. The percentage of correct answers in this case was 86.2%. The fact that even an easy item became easier when accompanied by an appropriate model might deserve further research.

## CONCLUSION

Items from a diagnostic instrument as ours could be used in different ways depending on the specific needs of each class and teacher.

Items like 'Paint 2' or 'Printing Press' could be used by teachers as tools that have a good chance to provide various responses and strategies in their classes. This variety could be the starting point of a discussion based teaching session that could lead pupils in confronting and tackling their misconceptions.

A combination of items that yielded different results (like 'Paint 2' and 'Book Reading') could be used in turn by the same group of pupils. The possibility of having different responses by the same pupil to each item might provide the cognitive conflict necessary for enhancing the pupils' understanding.

Finally another way to provoke cognitive conflict could be to present the same item in two versions: one as a written statement and one accompanied with an appropriate model.

The next stage of the research will be firstly to collect more data in order to conclude with more confidence on the diagnostic value of each item. Then interviews with the pupils must be conducted in order to validate the data and finally we aim to investigate the use of our items for effective diagnostically-designed mathematics teaching.

## NOTE

Both versions of the instrument can be obtained in full by emailing Christina.Misailidou@stud.man.ac.uk

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