IMPROVING CHILDREN'S FLEXIBILITY IN ELEMENTARY ARITHMETIC

Hazel Howat

Mathematics Education Research Centre, Warwick University

The first phase of the numeracy lesson aims to rehearse, develop and sharpen mental skills to give children flexibility with the way they deal with numbers. However, it appears that many children have not responded to the initiative in the way desired. Such children do not seem to acquire the benefits of mental approaches but continue to focus on perceptual or figural representations of number and the procedures associated with them. This paper reports on an investigation that presented these children with an alternative focus - the arithmetical symbol. A constructivist teaching programme was designed to examine whether such a change in focus may improve the sophistication of the approach children use for mental arithmetic.

INTRODUCING A PERSPECTIVE

This paper considers a teaching programme aimed at remediating the lack of flexibility that some children have in mental arithmetic. This programme is considered in the light of initiatives implemented within English primary schools over the past decade. In doing so, it focuses particularly on children who, despite these initiatives, have difficulty recalling number combinations to 20. The National Numeracy Strategy (DfEE, 1999) was introduced into English schools to raise standards of numeracy for all children.

> The Task Force have made recommendations for a National Numeracy Strategy that we believe will improve standards and raise expectations in primary mathematics. (DfEE, 1998[1], p. 2)

The strategy recommends a daily three part numeracy lesson, where the first phase of the lesson is designated for whole class teaching of oral and mental work with the aim of rehearsing and sharpening skills (DfEE, 1998[2]). In the context of elementary arithmetic the government initiatives have placed an emphasis on the ability to recall basic facts and to use them to derive others. The latter feature lies at the heart of a strong emphasis on the development of flexible mental approaches to the calculation of two and three digit number combinations. However, research by Gray and Tall (1994) shows that children across the ability range think in a qualitatively different way about mental arithmetic. They suggest that children within the lower quartile of arithmetical achievement operate at a procedural level in the sense that they attempt to use a step by step routine such as counting to obtain solutions to elementary number bonds. This would suggest that there are children who are not able to take advantage of the whole class teaching involving mental arithmetic. Therefore, this numeracy initiative may not raise the standards of numeracy of all children.

This paper presents evidence to illustrate how a teaching programme was designed to focus on the proceptual nature of symbolism to encourage more flexible understanding of number and therefore improve their readiness to participate successfully in the first phase of a numeracy lesson. The programme focuses on the equivalencies inherent within numerical symbols but removes the need for lengthy counting procedures through use of a graphical calculator. The intention was to realign the children's focus on procedures such as counting to a focus on symbols giving access to the flexibility of processes and the thinking power of procepts.

A CONTEXT FOR CALCULATORS

The teaching programme was based on an exploratory study (Gray and Pitta, 1997) that made use of the graphical calculator. The contribution that calculators can make towards enhancing children's mathematical thinking has been well documented within reports of the Calculator Aware Number Project (CAN) (Shuard, Walsh, and Worcester, 1991, Rowland, 1994). Set up in 1986 the CAN project was designed to develop a mathematics curriculum that featured open access to calculators. The outcome indicated that CAN children showed a greater flexibility with number than children in non-project schools. They demonstrated that they understood and were able to deal with very large and very small numbers at a much younger age than they would by following a traditional maths curriculum. The calculator was not used to replace mental and written arithmetic but was used as a teaching tool to develop understanding of number. However, the CAN literature rarely explicitly mentions changes in achievement by under-achievers. It is a hypothesis of this study that such children would benefit from a teaching programme that used a calculator in such a way that it could change their interpretation of arithmetical activity and thus enable them to benefit more fully from the oral phase of their numeracy lessons.

The advantage of using graphical calculators is that both the input and output can be seen at the same time, giving children the opportunity to link them together. In addition, three equations can be shown at once which allows the children to easily use a trial and improvement method of generating a number sentence.

METHODOLOGY

The teaching experiment was modelled on constructivist teaching experiments (Steffe and Kieren, 1994) to examine whether a change in focus would lead to a more sophisticated perception of number. The participants for this study came from an inner city primary school where 74 per cent of the pupils are entitled to free school meals. This is well above the national average (OfSTED, 1999). The school was chosen because of its poor Key Stage test results in 1999: only 35% of the 11-year-old pupils at the school reached level 4 and above in maths. The participants were eight low achievers in mathematics from year 5 (aged 9 to 10), four from each of two parallel classes, selected by their teachers using data from class tests and teacher assessment (one child left during the study leaving a group of seven). Exploratory observations of the children during the first phase of the numeracy lesson suggested that the lower achievers appeared to be focusing on mathematical procedures rather than known and derived facts, the transformations that were focus of the teacher's discussions during each mental and oral phase. In individual interviews (replicating questions given by the teacher) it appeared that the children were unable to answer a majority of these questions. Often, those that were answered involved a considerable period of time that would not fit the notion of 'rapid recall'.

Before the teaching programme the children completed three mental arithmetic tasks to assess their ability in elementary arithmetic. Their responses stand as a record of their achievement and flexibility before the teaching programme. The tasks required the children to present within four minutes as many number sentences as they could to provide a given number in four minutes. The numbers given were 10, 7 and 20.

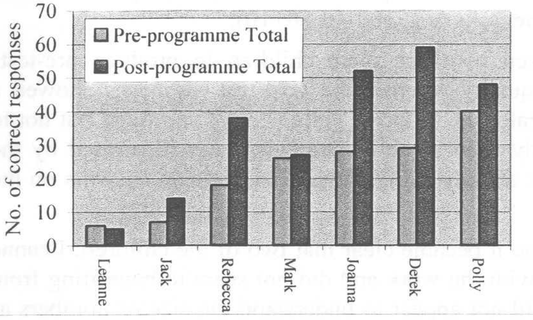
A ten week teaching programme followed. This aimed to decrease children's focus on the interpretation of numerical symbols as actions and increase their focus on knowing and using arithmetical facts. In each session of the teaching programme the children began by thinking of ways to make a number, such as 8, without the calculator. They then used the calculator to generate several ways of making the same number and then they were asked to generate ways of making the number starting with a chosen number (e.g. 10). The completed work was discussed either individually or within the group to provide children with an opportunity to reflect upon their observations and give them the opportunity to abstract from their activity the relationships and patterns they had observed. Each child was then asked to write down what they felt was an important observation.

At the end of the ten week programme the children were then post-tested using the pre-test mental arithmetic tasks to assess changes in achievement. This paper reports on these changes and provides an indication of children's commentary.

RESULTS

The children's responses to the mental arithmetic tasks can compared by looking for changes in the quantity and quality of the responses given by each child. Figure 1 gives the total number of correct responses given by each child to the pre-programme and post-programme mental arithmetic tasks. To further examine the quality of answers given the responses on each sheet were examined to see if there was any evidence of pattern being used to derive answers. (Use of pattern to derive answers was assumed if three or more

Figure 1 Children's Responses to tasks

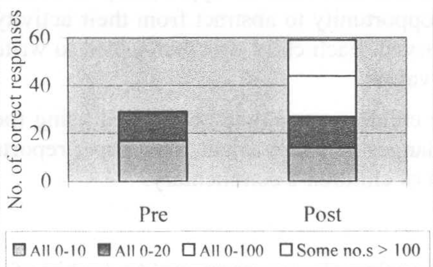


sums near to each other contained pattern in the number system, such as place value. For example, 80-70, 70-60, 60-50). Responses containing more than one operation or more than two numbers in a sum were also noted.

It can be seen from figure 1 that there is a clear difference between the pre-test and post-test totals. After the teaching programme, the total number of responses given by the sample increased for all children but one. From looking at the quality of responses it is clear that the increases in post-programme totals were due to the fact that the children often used pattern rather than time-consuming counting procedures to produce the majority of their responses. They therefore had opportunity to produce more responses in the time given.

The comparison of pre- and post- programme mental arithmetic tasks leads to the conclusion that the teaching programme has significantly improved the elementary arithmetical skills of 3 of the children (Joanne, Derek and Rebecca). There was a change in both the quantity and quality of their responses. This is exemplified by the following analysis of Derek's responses. Derek has:

Figure 2 Derek

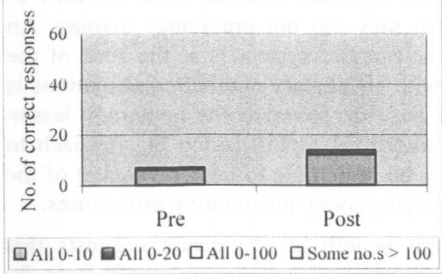


- doubled the number of responses on the post-test compared to the pre-test.
- used a wider range of numbers. On the pre-test he only used numbers up to 20 but after the programme he used numbers as large as 900,000,000.
- used no pattern on the pre-programme tasks but used pattern to derive some responses on every sheet after the programme (e.g. 300-300+10, 200-200+10, 9000-9000+10).
- given one response using more than two numbers on the pre-programme tasks but gave 21 responses on the post-programme tasks (e.g. 106-10+1+1-2).
- given no responses using more than one operation on the pre-programme tasks but gave 18 on the post-programme tasks (e.g. 100-50-50+10).

To summarise, the strategies used by these three children to produce pre-tests responses were all of a similar quality whereas the post-test responses showed a range of qualitatively different strategies. A fourth child (Polly) improved but not to the same extent, since she already used some of strategies later illustrated by the other three. For example, she was already using patterns to generate answers on the pre-test (9+11, 8+12, 7+13).

After two weeks of the programme it became clear that two of the children, Leanne and Jack, were having difficulty with the work and did not seem to benefiting from the teaching programme. They did not appear to understand the size of numbers as they could not order them without counting through the number sequence and they had no understanding of place value. This lack of understanding made it difficult for these children to use the trial and improvement method on the calculator without individual help. Jack appears to have made a slight improvement in his ability to answer simple arithmetical problems successfully. The teaching programme has also improved his confidence to participate in the first phase of the lesson, according to his teacher. Leanne appears to have shown no improvement in achievement or confidence. The difficulty these children had with the teaching programme suggests that there is a pre-requisite amount of knowledge needed for this teaching initiative to work. The graph of Jack's pre-test and post-test responses shows the lack of change in the quality of the responses and therefore the strategies used. Jack has:

Figure 3 Jack



- doubled the number of the responses to the pre-programme tasks. (However, his post-test score remains well below the average pre-test score.)
- only used numbers between 1 and 20 on both pre and post-test.
- uses no pattern to derive answers either before or after the programme
- only used more than two numbers in a sum (5+5+5+5) on one post-test sheet
- not given any response using more than one operation in one sum on any task either before or after the programme.

The final child, Mark also made very little progress, though the reason for this is unclear. Throughout the teaching programme his attention wandered and he had to be continually brought back on task. According to his class teacher this is typical behaviour and he is currently taking part in special needs assessment. The quality of his responses to the written section of the sheet was different to those for whom the programme appears to have been successful. His responses were rarely about the patterns in the number system but more about the size of the numbers used, although he did use pattern to produce some answers.

To summarise, unlike the first 4 children, the strategies used by the final 3 children to produce both pre-tests and post-test responses were all of a similar level. The greatest improvement was by those who began to use a wide range of strategies. Less improvement was shown by those whose responses remained quantitatively similar in both tests. Post- teaching programme interviews with the teachers suggested that, with the exception of Mark and Leanne, the children had become more confident and responsive within the mental/oral phase of the numeracy lesson.

DISCUSSION

The initial findings suggest that the teaching programme can be successful at improving some children's flexibility in mental arithmetic. Those who appear able to respond to the initiative have used numbers on their worksheets that they would be unable to utilise without the calculator. Their explanations demonstrate that they are focusing on the patterns in their number sentences. In the post-programme tasks their use of numbers over 20 has increased. These numbers would be difficult to access through the use of the counting procedures that dominated the children's approach prior to the teaching programme. The use of patterns in the post-programme responses indicates that the focus on number symbols enabled these children to access patterns with the number system that they did not previously display. An understanding of the patterns inherent in the number system is at the root of the flexibility in mental arithmetic. Flexibility with elementary number combinations is necessary for a child to take a full part in the first phase of the numeracy lesson within year 5. The hope is that with this improved sophistication in approach to mental arithmetic these children will be now be more able to take advantage of the mental/oral phase of the lesson, rather than relying solely on counting procedures.

However, the teaching programme may not benefit all. Evidence suggests that children may need a fundamental level of numerical competence to benefit from the teaching programme. This teaching programme needs further study and would benefit from a deeper analysis of the students' understanding of number prior to the teaching programme and a comparison group to verify whether the changes apparent in the group are due to the teaching programme rather than other factors.

REFERENCES

DfEE.:1998[1], *Numeracy Matters: The Preliminary Report of the Numeracy Task Force*. London: DfEE.

DfEE.: 1998[2], *The Implementation of the Numeracy Strategy: The final report of the Numeracy Task Force*. London: DfEE.

DfEE.: 1999, *The National Numeracy Strategy*. London: DfEE.

Gray, E.and Tall, D.: 1994, 'Duality, ambiguity and flexibility: A proceptual view of simple arithmetic.' *Journal for Research in Mathematics Education*, 25 (2), 115-141.

Gray, E. M., & Pitta, D.: 1997, 'Changing Emily's Images.' *Mathematics Teaching*, 161, 38-51.

Rowland, T.: 1994, *CAN in Suffolk. The First Six Months of a Calculator-Aware Number Curriculum*. Cambridge: Homerton College.

Shuard, H., Walsh, A., J., G., & Worcester, V.: 1991, *Calculators, Children and Mathematics - The Calculator Aware Number Curriculum*. Simon and Schuster.

Steffe, L. P. and Kieren, T.: 1994, 'Radical Constructivism and Mathematics Education.' *Journal for Research in Mathematics Education*, 25 (6), 711-733.