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CHILDREN USING TRIAL AND IMPROVEMENT METHODS:EXAMPLES FROM KEY STAGE TWO TESTS

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We consider the use of 'trial and improvement' methods by 11 year olds, drawing upon children's responses to two questions in the 2001 Key stage 2 National Curriculum tests. For both questions, the first step was to consider which solutions count as 'trial and improvement'. In doing this we also identified another category, 'spot and check' which differed from 'trial and improvement' in that only one (correct) solution was offered and tested. The next step was to identify different sub-categories within trial and improve for each question. Children using such methods generally used a relatively small number of trials and reached the correct answer. Where this was not the case, this was nearly always because of errors in calculation.

INTRODUCTION

This work arises from a wider study, carried out with the Mathematics Test Development Team at the Qualifications and Curriculum Authority. The study concerns the responses of 11 year olds to tasks which can be seen as pre-algebra. The first phase is based on the responses of children to Key Stage 2 written mathematics tests. Using the framework suggested by Mason *et al* (1985), questions were selected from Test A and Test B, 2001, which were considered to have the potential for encouraging children to display aspects of algebraic thinking. The work that follows uses the responses to two of these questions to deal with one particular issue which arose when considering categories of solution, that of 'trial and improvement'. We will start by considering which solutions fall into this category and why. We will then suggest subdivisions within the category.

BACKGROUND

The literature suggests widely differing views of 'trial and improvement' and its relationship to algebra. It has been viewed both as a step towards algebra and as an obstacle. One advocate of the former view is Bell (1996a), who calls on the work of Bednarz and Janvier (1996) to list approaches to problems with differing degrees of closeness to algebraic thinking. The first stage is arithmetic step by step problem solving, the second is trial and adjustment, the third is intermediate and the fourth is global recognition of problem structure. Bell acknowledges that there are differing views about whether the second or the fourth is closest to algebra. There may be dangers in seeing a strict hierarchy here. For example, considering a later stage of learning, Wheeler (1996) points out that although 'test and adjust' methods often precede formal methods for solving equations, they may sometimes still be an appropriate method to solve particular equations. Different categories of solution also depend on the type of question. Thus some classifications focus on the solution to word problems (Berdnarz and Janvier 1996, Bell 1996b), while others focus on 'missing number' problems (Foster 1994). What such classifications have in common is the inclusion of some type of 'direct solution' and some form of 'trial and adjustment' or similar.

Strong reservations about the use of 'trial and improvement', are expressed in a report by The Royal Society (1997). These concerns are initially expressed in the context of secondary education. Firstly they suggest that pupils who become proficient with such methods are unlikely to want to learn algebraic methods. They also suggest that trial and improvement may actually constitute an obstacle to the learning of algebraic methods. They assert that trial and improvement involves working forward from a known starting number to the unknown number, whereas algebraic methods involve working backwards from an unknown number to a known number.

Later in the report, the authors return to the same issue in relation to primary education, considering in particular the way pupils tackled a missing number problem on a 1996 key stage 2 test. Trial and improvement methods were common and comparison with French pupils found that the British pupils made more use of written trials on paper. The authors go on to say 'This emphasis on 'trial and improvement' methods, which we have found within the curriculum from Key Stage 2 throughout the pre-16 curriculum, is possibly one of the most worrying aspects of the curriculum from the point of view of developing algebraic ideas and relates to an over-emphasis on answer as opposed to method. More research needs to be carried out on whether these 'trial and improvement' methods do constitute an obstacle in the development of algebraic ideas.' (page 11)

FINDINGS

We looked at the responses of 451 children to selected questions from the 2001 KS2 tests. The two questions considered here are from Test A, a non-calculator paper. We call question 9 'Multiplication' and question 20 'Isosceles'. We aimed to categorise pupil solutions to these questions. In both cases, 'trial and improvement was a possible category. This is considered in more detail below.

Multiplication

Many children showed no working, hence it was not possible to categorise their solutions. Some children set out a division in the space around the question and solved it. Some had carried out a multiplication, sometimes evidenced by carrying figures under the question, but sometimes the multiplication was set out separately. Although these children had checked by multiplying, there was no evidence that they had tried out several possible answers, as their first attempt was correct. We therefore considered these to be 'spot and check', rather than 'trial and improvement'.

Other children used the space around the question to try out several answers. These we have provisionally classified as 'trial and improvement'. We have also identified sub-categories within 'trial and improvement'. We call these 'systematic', 'intuitive', and 'problematic' (see examples in appendix 1). The example of systematic trial and improvement given shows three properties: use of constraints, systematic trials and correct calculations. The jotting 24+1=25 suggests that the 2 has been deduced and the child remembers the 4 is also fixed. Once these constraints are in place, it is merely a case of trying out different possibilities for the hundreds. Following the calculations from left to right suggests that 5 was tested first, proving too high, then 1 which was too low. It seems4 was tested next, followed by 3 giving the correct answer. All the multiplications were completed correctly using a standard algorithm.

Some children, though less systematic, arrived at the answer more quickly, some reaching the correct answer after one false trial. We call these 'intuitive trial and improvement' It is possible that, in the example given, the child was helped by the extended recording to change both the units and the hundreds. Other examples of 'intuitive trial and improvement' showed multiplication abandoned halfway through, suggesting that the child stopped as soon as they realised the answer would be too high and modified their solution.

Some examples were classified as 'problematic trial and improvement'. These children all seemed to have difficulties. Some did reach an answer after a large number of trials, while others abandoned their attempts. In one example, the child seems to have been hampered by not fixing the units digit. However closer examination reveals that this child also made some errors in calculation. In other examples, errors in calculation hampered solution. This sometimes meant no answer was reached, but not always, with the example in the appendix being an extreme case where a child makes many errors but still gets the correct answer. Some children using trial and improvement failed to reach the correct answer because of a faulty multiplication algorithm with carrying from the tens to the hundreds being a common difficulty. There was one child who had a systematic approach and correct calculation, but still abandoned their working after two trials. However, this was an exception. In other cases where working was shown and a correct answer was not reached, there were errors in the calculations.

Isosceles

In this question, children were specifically asked to show their working in the box provided (see examples in appendix 2). As in the previous question, one category of solution was 'spot and check'. These children gave the correct values for the sides of the triangle and proved they worked. This was by far the most common method, used by 57% of children who obtained the correct answer. Another category was 'trial and improvement'. Within this category we identified different approaches. One approach was to suggest the long side, find half and adjust till the total is 20. A slightly different method was to suggest the long side, see what is left, then adjust till the third side is half of the long side. Other children chose a value for the short side, then adjusted. In most cases adjustments seemed to be informed by previous errors. Some children using trial and improvement approaches started by dividing 20 by 2 and then making adjustments to ten. A less common approach was to start by dividing 20 by 3 and then adjust the answer. Children using trial and improvement usually needed two or three attempts and then reached the correct answer.

DISCUSSION

The main difference between the two questions was that 'trial and improvement' methods were sometimes unsuccessful in 'multiplication', but not in 'isosceles'. There were similarities between 'trial and improvement' approaches to both questions. 'Spot and check' approaches where only one answer was tried were more common that 'trial and improvement' using multiple trials. In both questions, 'trial and improvement' was used alongside aspects of deduction, often in order to narrow down the range of possible answers tried. Most children using trial and improvement used a small number of trials. Where children did use a large number of trials or where they failed to reach an answer, this was nearly always because of calculation errors. Our findings do suggest that for both questions, children showing a method preferred to show that their answer worked rather than showing explicitly how they got from the information given to their answer.

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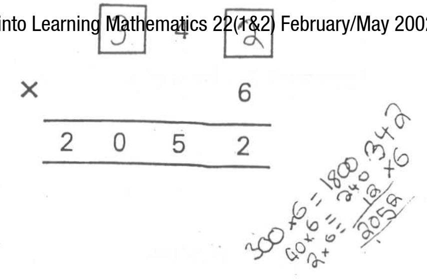
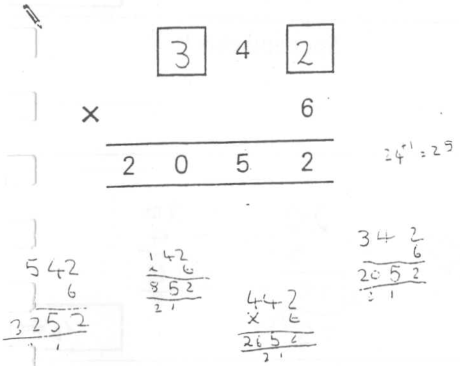
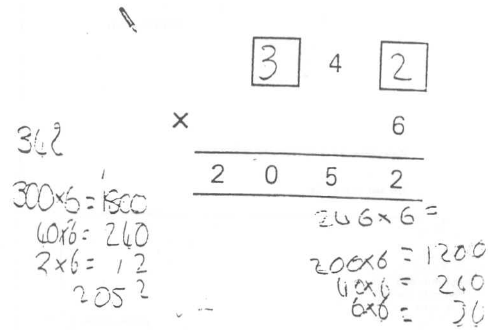


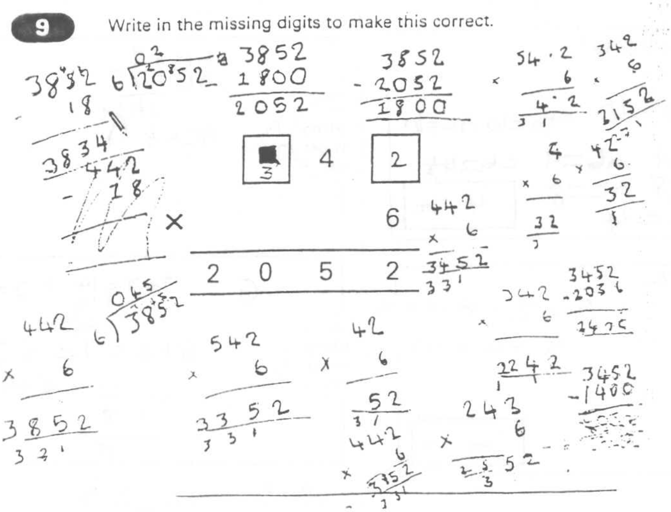
Spot and check



Systematic trial and improvement



Intuitive trial and improvement



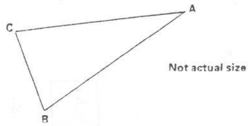
Problematic trial and improvement

Appendix 2 – Isosceles

Copy of question

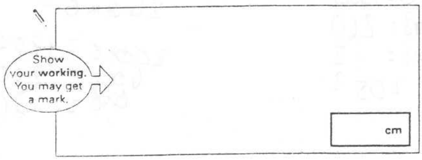
21 Triangle **ABC** is isosceles and has a perimeter of 20 centimetres.

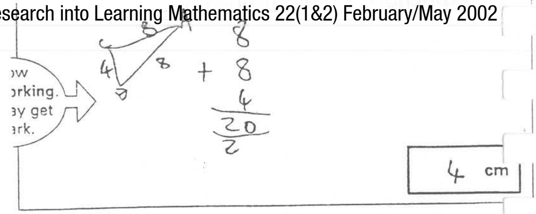
Sides **AB** and **AC** are each **twice** as long as **BC**.

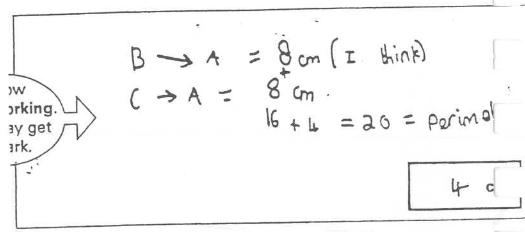


Calculate the length of the side BC.

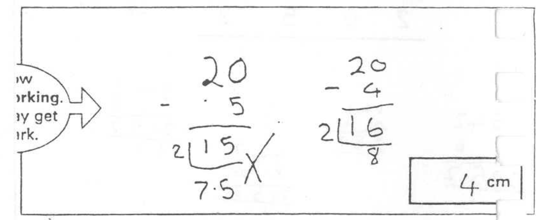
Do **not** use a ruler.



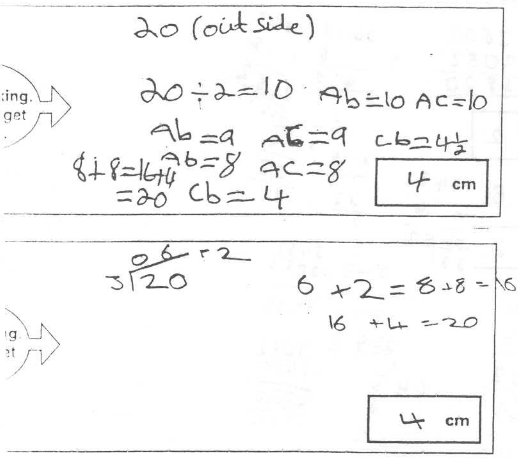




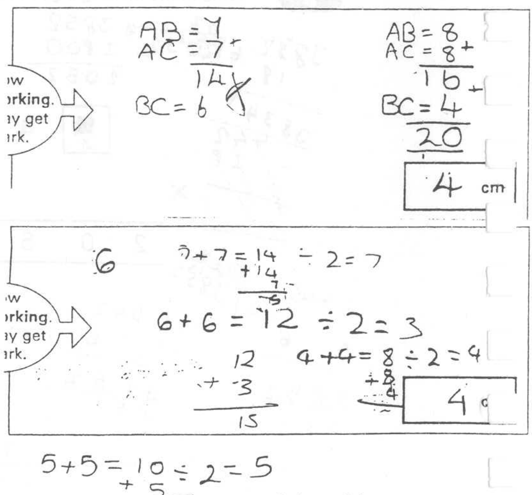
Spot and check



Trial and improvement
Starting with short side



Trial and improvement
Starting with division



Trial and improvement
Starting with long side