

## CONSTRAINTS AND FREEDOMS IN MATHEMATICS AND MATHEMATICS TEACHING: DESCRIPTIONS OF PRACTICE IN YEAR 8 SECONDARY MATHEMATICS CLASSROOMS

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*We are working on a small-scale research project tracking the development of a group of four mathematics teachers who are teaching parallel year 8 groups in mixed-ability classes in the same school. We are currently exploring, through this project, the view that mathematics, mathematics teaching and mathematics research can be analysed in terms of constraints and freedoms. After setting up what we mean by 'constraints and freedoms' we analyse extracts from two transcripts of lessons. Finally we discuss the practice on the project by offering responses to the points raised in discussion during a session at the BSRLM conference in Bristol where we looked at extended versions of the two transcripts included here.*

### CONSTRAINTS AND FREEDOMS

#### **A story within mathematics**

Laurinda was interviewing for the Post Graduate Certificate of Education course. She asked the interviewee a standard question: 'What of the mathematics that you covered for your degree can you imagine sharing with the children you are going to teach? What has interested you or caught your enthusiasms?'

After a little thought the woman replied: 'It would have to be something using the 'MathCad' package. The students could look at a line (infinite) and a square in relationship with each other? We would need a question though. What about seeing what shapes could be made?'

In engaging with the mathematics Laurinda was aware of organising what she did, beginning with: 'What if the line were initially parallel to one side of the square and moved to the right across the square?' This organisation might lead to some sense of knowing that she had found all the shapes.

#### **Reflection**

In talking with Alf about the incident later in the day there was a sense that in preparing to teach the mathematics the interviewee had needed to constrain the situation in some way. Laurinda had also used constraints to get organised in doing the mathematics itself. Alf offered the following as support for the need for constraints and freedoms in any learning:

'I want to stress here a point of great importance, which is that by limiting the imagination in some directions we can very often free it to move in others, and that quite often it is only by limiting it that we can free it' (Holt, 1971 p.230).

Too much freedom (e.g. everything available that MathCad can do) and without a purpose there might be no energy to use the package. It is arbitrary where to begin. Time needs to be spent exploring to allow questions to arise. Too many constraints mean there is no space to explore (e.g. the line crosses the square in a fixed place and the shapes are recognised - a closed situation). We have talked about this before as there being no 'grit in the oyster' (Brown and Coles, 2000, p.174).

## **WAYS OF DESCRIBING PRACTICE**

What we are interested in documenting in our research are examples of practice and developments of practice that show what is possible in a classroom. Our current research project involves 4 mixed ability year 8 groups in one half year of a secondary comprehensive school. For the first time the teachers of these groups have taken their mixed ability year 7 groups through to year 8. The teachers, with their year 7 groups, are used to working for three to four weeks on problems. The idea is to continue with this model in to year 8. Alf is one of the teachers and head of department of the school. We have developed a language of description of the way of working in these year 7 classrooms through Teacher Training Agency (TTA) (see Coles, 2000) and Economic and Social Research Council (ESRC)<sup>1</sup> (see Brown et al, 2001) funded projects as well as other small-scale studies. The key ideas of this way of working will be described briefly below. In discussing examples of practice from these year 8 classes later in the paper we will be using this vocabulary.

### **Contingency:**

(When the teachers are) 'adapting their plans through their judgements about the situation ... these teachers are subordinating their experience, understanding and skills to the learning of their students. They are concentrated, not self-conscious, with no attention left over to worry about any problems – their skills are adequate to cope with the challenges at hand' (Brown, 2001, p.195).

### **Imagery:**

'In looking for what to offer their students the teachers often cite imagery in some form. Again there is a belief in the students' ability to hold something in the mind and operate on it ... e.g. the power of the mathematical notion of infinity ... which these teachers recognise ... when things go "big" or head off to a limit' (Brown, 2001, p.193).

### **Same/different:**

'All (the) teachers showed evidence in their practice of having a belief that 'pattern recognition is the basic mental process and, as such, is the ground for all that we call learning' (St. Julien, 1997, p.275). ... The nature of the activities with which students engage are seen as crucial and lessons often begin with some activity for the students to do which allows them to use their powers of discrimination. We call this strand same/different because the teachers often ask 'What is the same?' or 'What is different?' of their students and there is some evidence of students beginning to notice (and use) same/different for themselves. Same/different is a pedagogical tool through

which mathematical ideas of order, inverse, pattern and structure can be explored. Often the students describing same/different leads to algebraic activity through classification or use of notation' (Brown, 2001, p.192).

### **Algebra:**

We see algebra as 'holding the process' which, when a constraint is applied becomes dynamic – there's an action to perform such as substituting in a formula or an image is created that provides the evidence from which coding can be developed. (See 'Gattegno Anthology', Brown, Hewitt, Tahta, 1989)

### **TWO EXAMPLES OF PRACTICE**

The two transcripts below were taken from the lessons of Teacher 1 and Teacher 2 on the project as they began a three week piece of work looking at tessellations. The statement, introduced by the teachers: 'Find me a four-sided shape that will not tessellate' provided a purpose for much of the three weeks.

NB Three dots during speech indicate a pause. Three dots in brackets indicate some dialogue has been left out due to considerations of space.

#### **Teacher 1**

- We have a new topic and what I want somebody to do is to come up and to draw on our dotty paper here, a four-sided shape and the only restriction with our four-sided shape is that all the corners must be on the dots.

(A student comes to the board and draws a shape ...)

- Now, I want somebody to come and draw a copy of this shape exactly the same size as this shape and touching it in some way so that we can cover the whole sheet of paper with this shape.

(A second student comes to the board and draws a copy of the first shape ...)

- Now, with this shape what we want to think is 'Do we have enough information here so that if I give you all a piece of dotty paper we would all get exactly the same pattern if we continue what has been done there?' Hands up if you think we would all get exactly the same pattern if we wanted to cover our paper. Hands up if you think we would all get exactly the same pattern if we wanted to cover our paper. Mmm ... OK ... nobody's put their hands up. Could somebody then tell me why they think they wouldn't ... we wouldn't all get the same pattern?

- Somebody thought you don't know what to do. Jack.

~ Either down or like to the sides like that.

- OK. So would somebody like to come and therefore put another square touching this one so that we continue to cover this whole sheet of paper. Would somebody like to come out and have a go at that? Rich. Rich, you said that was easy.

## Teacher 2

- On this grid I would like someone to come and draw a four-sided shape, a shape with four sides ... someone to come and draw a four-sided shape on this grid.

(A student comes to the board and draws a shape.)

- OK. I now want someone to come and draw a copy of that shape and ... place it somewhere touching that one but we're looking for a pattern... we're looking to try to do this in a pattern. What we're going to try and do is cover the board with copies of that shape. So I want a copy of that shape that's in some sort of organised place, in some sort of pattern.

(Pen passed to Angie who comes to the board to draw.)

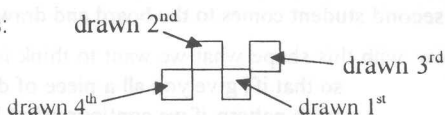
- In your mind now I want you to place a third shape that is continuing that pattern. Angie has defined a pattern ... there is only one or possibly two places where a new shape can go ... it has got to follow that pattern. In your mind place another copy of that shape in the same pattern that Angie has started. I'm going to invite someone to come to the board to do their one. But in your mind everybody place it, place it in your mind. Shaun ...

(Pen passed to Shaun who comes to the board to draw a third shape.)

(Later in the same lesson, after a fourth shape is drawn by a different student ...)

- It seems to me people are making up their own patterns. I don't think I'm going to agree with either of those two ones (the third and fourth shapes) that have been placed there because that seems to me what you are doing is you are making your own patterns from what Angie has done. Angie gave us that one and then *that* one (pointing to the first and then *second* shape), she defined a single pattern and it seems to me there's only one way that can carry on.

At this stage the board was as follows:



## INITIAL ANALYSIS

We chose to present these two transcripts in Bristol because we were struck by their similarities and differences. The teachers on the project work closely together developing lessons ideas and starting points for rich mathematical activity. This can be observed in the almost identical words used by Teachers 1 and 2 (T1 and T2) at the very start of the lesson; T1: 'I want somebody to (...) come up and draw on our dotty paper here a four sided shape', T2: 'I would like (...) someone to come and draw a four-sided shape on this grid'. Both lessons start with the creation of an image for students to look at.

However, despite these similar starts the lessons soon diverge as the teachers respond contingently to their students and impose different constraints on the situation. The difference that initially struck us was that Teacher 2 did not accept two of the students' placements of shapes: 'I don't think that I'm going to agree with either of those two ones (...) it seems to me what you are doing is you are making your own patterns'. Teacher 2 has a definite idea of what he means by 'pattern' in this context and the students are constrained to adapt to this meaning. In Teacher 1's lesson every student offer is accepted and the focus he introduces is: 'Do we have enough information here so that (...) we would all get exactly the same pattern'.

We do not imply any value judgements about the two lessons. The students did quite different things in the two lessons although over the 3 weeks of the project similar work was done. The point, for the research project, in analysing these transcripts is to extend our practice and awarenesses in a classroom, given how the teachers constrained the situation in different ways. We become more aware of our choices when teaching and their possible implications. These issues were discussed at a meeting of the teachers and researchers on the project when we worked on the similarities and differences in the videotaped lessons.

## DISCUSSION

We are indebted to all participants for points raised during discussion in Bristol of the transcripts above. We have selected a few of those points through which we aim to describe further our methodology and the practice on the project.

Anne Watson commented on the statement: 'Find me a four-sided shape that will not tessellate', which occurred later in both lessons. She said that in an analysis of the lessons looking for 'open/closed questions' she might overlook this statement yet it provoked rich activity amongst the students. In any research we have to constrain our view in order to see anything of what happens. However we see one of our tasks as teachers and researchers as staying open to what we are thereby leaving out. We take Anne's comment as an example of her becoming aware of a blind spot imposed by the constraint of looking for 'open/closed questions'. An explicit part of our research is the exploration of the constraints of any view (itself a constraint) we impose on data. We value watching videotaped lessons with others exactly because of the different lightings each person brings, both when trying to view the extract with the same constraint (eg 'open/closed questions') and when we view from different perspectives.

A number of participants were unhappy at Teacher 2's use of the word 'pattern'. Placing three shapes anywhere creates a pattern, so how can the student who placed the third shape be 'wrong'? John Mason suggested that a better word than pattern, would be 'action'. The second student defined an action, performed to the first shape, the question then becomes; 'Can anyone perform the same action on the second shape?'. We find John's reformulation helpful, particularly since it makes more explicit the link between what the students are doing and our sense of algebra.

However, whether the word action or pattern is used, the task in this lesson is for students to adapt to what Teacher 2 means by the word. When faced with new situations and contexts, we bring our own meanings for words and it is easy judge anything new as being 'wrong'. When observing lessons we attempt to describe in as much detail as possible what we see. We find judgements such as 'this use of the word pattern is wrong' get in the way of these observations. When such judgements inevitably arise we try to return to the detail from where this difference has arisen and stay open to the possibility of re-structuring an old idea and learning.

We were asked how much we involved the other teachers in our analysis of their lessons. It is an axiomatic part of our work that teachers and researchers are part of a joint culture. Laurinda has attended all the meetings of the teachers and is a frequent visitor to the school. There is a shared language that we use both to analyse what has happened and plan for the future. Our current focus on the idea of 'constraints and freedoms' has arisen from these words becoming ones that teachers on the project started (and continue) to use after discussing the transcripts above.

### CONCLUDING REMARKS

Through our work and research we aim to extend our awarenesses of mathematics teaching and learning. We believe strongly that this is only possible using detailed descriptions of practice through which we can become aware of the constraints and freedoms of what we do both as teachers and researchers.

### NOTES

1 Economic and Social Research Council (ESRC) project ref: R000223044, see [www.regard.ac.uk](http://www.regard.ac.uk)

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