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DIAGNOSTIC TOOLS AND PEDAGOGICAL CONTENT KNOWLEDGE: A RESPONSE

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This paper is a critical response to four papers given in a symposium entitled Diagnostic Tools and Pedagogical Content Knowledge at the Durham Conference. The papers (most of which are summarised in these proceedings) were given by Thekla Afantiti, Constantia Hadjidemetriou, Christina Misailidou, Julie Ryan and Julian Williams, all of the University of Manchester.

INTRODUCTION

My first comment would be that critique includes the possibility of appreciation. These symposium papers represent the latest fruits of a coherent and systematic research agenda pursued for some years now by Julian Williams and his collaborators in Manchester. I shall call them the 'Manchester Group', although 'Oxford Road Group' might be less ambiguous if more obscure. The background to this research is summarised in Williams and Ryan (2000a). To the best of my knowledge, it began with a question something like: in what ways can teachers be enabled to make use of the results of national Standard Assessment Tests in order to enhance their teaching? This led to the ITAM project (Improving the Teaching and Assessment of Mathematics), and the subsequent research stream has been both rigorous and useful, a heady combination. The usefulness is exemplified by Ryan and Williams' publication (2000b), *Mathematical Discussions with Children: exploring methods and misconceptions as a teaching strategy*, which offers 23 'prompt sheets' for teachers to use as a focus for discussion with a group of children. Each prompt sheet addresses typical misconceptions in specific mathematics topics (such as ordering decimals) within the KS2 and KS3 curriculum. The misconceptions in question were identified in earlier research, including the authors' scrutiny of pupils' responses to the National Tests for Wales. The prompt sheets are also informed by research by the authors, based on 'conflict' discussions with groups of children identified as exhibiting particular misconceptions. The sheets offer the proposals of children ('Kylie', 'Diane' and so on), and their 'teacher's' response. For example, when Kylie suggests that 73.5 is less than 73.32, 'Ms Jones' asks "Why the point 5 then point 32 ...?". Each prompt sheet is preceded by a page of material for the teacher, listing typical diagnostic errors, suggesting how they might arise, and proposing useful *mathematics-specific* strategies for the group discussion (e.g. try placing the numbers on a number line, try switching from decimals to fractions).

The research reported by Ryan and Williams in their current paper builds on this strategy for developing teacher pedagogical content knowledge (PCK). A 'regular' cohort of 74 Year 6 pupils in one school were screened with a 30-item test designed to reveal errors identified as common ones in earlier research. On the basis of their response to the test, children were grouped into fours for discussion of particular topics with a researcher acting as a quasi-teacher. The key concepts here are first *misconception* and *diagnosis*, then *conflict* and *argumentation*. The analysis of the argument follows Toulmin's classic (1958) scheme. The outcome of this analysis is a kind of map, a chart for future reference, of the argumentation 'space' associated with the particular *problematic* embedded in a restricted domain or 'conceptual locale'. These charts are perceived by the researchers as a kind of bridge or *boundary object* between research activity and classroom practice. They are tools to assist teachers in their planning for the management of such dialogues with their own pupils.

The papers by Afantiti, Misailidou and Hadjidemetriou remind (or inform) us of the wealth of accumulated knowledge from many studies of the misconceptions specific to particular topics: in this case, probability, ratio and graphs respectively.

I have devoted so much space to the above account of the grand design of this research by the Manchester Group, first because I believe that it is important not to lose sight of it in the necessary details in the four papers, and also because it will set the scene for the reflections that constitute the remainder of my response.

DIAGNOSTIC TEACHING

The idea of diagnostic teaching is not exactly new, but it might as well be. Those of us in the initial 'training' business solemnly tell our students that their lesson planning should be 'informed' by their assessment. What we intend is a kind of re-statement of Ausubel's famous dictum. The quality of assessment that we have in mind is diagnostic assessment, or 'assessment for learning'. I call to mind a book entitled *Diagnosis and Prescription*, published nearly 20 years ago (Rees and Barr, 1984). The publisher's copy reads:

> This book is designed to help teachers at school and colleges to ask the right questions and provide the right answers. It is based on the teaching and research activities of the Mathematics Education Group at Brunel University. Research studies … have shown that there are certain basic mathematical tasks which cause common misconceptions …

Ryan and Williams acknowledge the work of Alan Bell and his colleagues at Nottingham in the 1980s (the Diagnostic Teaching Project: Bell et al, 1983), in which cognitive conflict was seen as the route to developing understanding. More recently, Paul Black and Dylan Wiliam (1998) have accumulated evidence from some 250 studies that formative assessment "raises standards". They also found evidence that there is ample room for improvement, in the UK and elsewhere, in the use of 'assessment for learning'. The identification of assessment with (summative) testing is commonplace. But why is it so, some 14 years after the assessment framework for the National Curriculum (DES, 1988) emphasised four different purposes of assessment - diagnostic, formative, summative and evaluative? The work of the Manchester Group is about making diagnostic methods available to teachers through the medium of 'boundary objects', such as diagnostic assessment instruments, prompt sheets and charts of argumentation spaces - tools at the interface between research and practice.

CONSISTENCY AND DEMOCRACY

The idea of *misconception* has become a fashionable one in recent times, not least because it is central to the specification of the government 'standards' for the 'training' of prospective primary and secondary mathematics teachers (DfEE, 1998). Our students catch on to the fact that this is a Big Idea, and make sure that it appears in their assignments. The danger is in the possibility of attaching the label 'misconception' to almost any kind of error or difficulty. Hadjidemetriou and Williams address this danger in their paper:

> We draw a distinction between an error, i.e. erroneous responses to a question, and a misconception which may be a faulty cognitive structure that lies behind, explains or justifies the error.

One might say that the difference between misconception and error is the same as that between cause and effect. (Presumably, some errors are trivial - the result of nothing more serious than carelessness. I recently caused some mirth amongst colleagues myself on account of the statement 4=2x1+0 which had crept into a manuscript that I had prepared). There is, of course, something judgmental, though not unreasonably so, about the proposal of a 'faulty' cognitive structure. What might a radical constructivist conception of 'misconception' look like? Might it be re-cast in terms of communication rather than cognitive structure? Or perhaps an active construction of knowledge notable for its incompatibility, or lack of consistency, with dominant social norms of knowledge construction? I'm thinking aloud here, of course, and I'm not suggesting that mathematics would have much of a future as a social activity if its practitioners were to suggest that there was something arbitrary about mathematical 'truth'.

Argumentation and the resolution of conflict through discussion amount to a social realisation of Piaget's notions of accommodation and equilibration. There is also an association with self-awareness. We 'know' something all the better for knowing why we had misunderstood, or imperfectly understood, it previously. (Not that I believe that understanding is ever perfect or perfected: phrases like 'fully understand' make me cringe). Leaving aside the more pathological conundrums of mathematical logic, mathematics demands consistency. The resolution of conflict is the search for consistent understandings, within the limitations of human communication. This is at the heart of fallibilism; when new facts or examples come to light that seem to refute what we believed we had understood, then that knowledge and the ways we declare it is in need of revision. Mathematics is, in that sense (and many others) a *rational* domain. There is a reason for everything. For this reason, Warwick Sawyer has described mathematics as a *democratic* discipline.

> Mathematics is, or should be, the most democratic subject in the world. In mathematics there is no evidence available to the teacher that is not available to the student. (Sawyer, 1964, p. 82)

(See also Huckstep, 2001). Mathematical truth cannot be imposed from above: rather, it can only be established by argument and persuasion. The least citizen is as powerful as the greatest, in their capacity to assert, to prove and to refute. It does not often happen that I am proved 'wrong' by one of my students, but it *does* happen. In such circumstances, seniority, experience and any claim to wisdom count for nothing. How very sad it is that, as a consequence of their experience of school mathematics, so many adults come to believe that mathematics is about rules without reason, about stipulation and authoritarianism, when nothing could be further from the truth.

Adults with such beliefs include some teachers of elementary mathematics. Last summer I read Liping Ma's book (Ma, 1999), which in effect compares the PCK of two groups of elementary teachers, one American, one Chinese. I have found myself recounting one passage time and time again. Ma describes a 'dilemma' situation in which a primary school pupil says that she's discovered something about rectangles: "The bigger the perimeter, the bigger the area". The pupil's evidence is comparison of a 4x4 rectangle with an 8x4 rectangle. Ma reports that some USA teachers presented with this dilemma accepted the claim on the basis of this evidence. The majority said they didn't know whether it was true, and *would have to look it up in a textbook*. Most of the Chinese teachers, by contrast, set about investigating different ways of increasing the perimeter, and the effect on area in each case. My own research in collaboration with colleagues in the UK suggests that many of 'our' beginning primary teachers (and, no doubt, others) do not regard claims of a mathematical kind as amenable to *enquiry* on their part (Rowland, Martyn, Barber and Heal, 2001).

PCK AND THE MANAGEMENT OF ARGUMENTATION

The fundamental work reported by Afantiti, Misailidou and Hadjidemetriou is essential to the enterprise of enhancing teachers' PCK, and their enquiries are resulting in the development of important diagnostic tools. Of course, such tools are of limited value unless teachers are aware of the misconceptions that they are designed to identify. Yet Hadjidemetriou, for example, finds little evidence of such awareness. On the contrary, she finds that some secondary mathematics teachers themselves held some of the misconceptions that they might be expected to identify in their pupils. Afantiti's topic, probability, is an interesting domain for misconceptions research. For a cautionary tale - the Monty Hall Dilemma - see Chapter 6 of Hoffman (1998). Afantiti's research homes in on a cluster of related misconceptions, the so-called 'representativeness heuristic'. Other well-documented stochastic misconceptions include equi-probability, availability, positive and negative recency, conjunction, the sample size effect. I would guess that these terms do not enter the professional discourse of secondary mathematics teaching, yet the kinds of errors that they manifest might be plausible or even recognisable to mathematics teachers. If then, as Hadjidemetriou suggests, much of teachers' PCK is tacit rather than explicit, it must be doubtful whether they have the awareness necessary to draw on such knowledge in planning for teaching and learning.

I conclude with a word of caution relating to the teachers' management of argumentation with their pupils. My reading of the account in Ryan and Williams' paper, with the researcher acting as quasi-teacher, leads me to suggest that the teacher-pupil interaction has some of the characteristics of a clinical interview. Let me add two things straight away.

First, I believe that a teacher-managed argumentation space *should* have some of the characteristics of a clinical interview. The (verbal) clinical interview is characterised by the following features (Rowland, 1998): (a) the interviewer employs a *task or tasks* to channel the subject's activity (b) the interviewer constantly makes *instantaneous decisions* about the direction of the interview (c) the interviewer asks the subject to *reflect* on what s/he has said or done and to articulate her/his thoughts (d) the contingent nature of the procedure enables the interviewer to *test hypotheses* that s/he has generated in the interview, or in advance of the interview. It is not hard to see how each of these characteristics might be evident in the exploration and remediation of misconceptions.

Second, teacher-managed argumentation cannot actually *be* a clinical interview, since the purpose of the former is for the pupil to learn about mathematics, whereas the purpose of the CI is for the interviewer to learn about the subject. Nevertheless, my words of caution are these: do not underestimate the skills that teachers need to acquire in order to manage such discussions appropriately and productively. Piaget himself remarked that "It is so hard not to talk too much when questioning a child, *especially for a pedagogue*!" (Piaget, 1929, pp. 8-9, emphasis added). Doig and Hunting describe a programme for training teacher-clinicians to use interview methods of pupil assessment. They note the potential advantages of clinical approaches to assessment, but report that teacher-clinicians have been observed to "fall back on ingrained methods such as telling students, or providing direct information rather than questioning" (Doig and Hunting, 1995, p. 285). Markovitz and Even (1994) report that secondary ("junior high") teachers are more prone to these teacher-habits than elementary teachers in clinical interview situations. I am not saying that teachers cannot acquire the ability to manage conflict resolution effectively, but I am saying that this ability cannot be assumed, and that specific training in this regard might turn out to be necessary in order to maximise the effective application of tools at the boundary between research and practice.

REFERENCES

Bell, A. W., et al. (1983) *Diagnostic teaching: report of ESRC project.* Nottingham: Shell Centre for Mathematical Education.

Black, P.. and Wiliam, D. (1998) 'Assessment and classroom learning'. *Assessment in Education* 5(1) pp. 7-71.

DES (1988) *Task Group on Assessment and Testing : A Report.* London : Department of Education and Science and the Welsh Office

DfEE (1998) *Teaching: High Status, High Standards: Circular 4/98*. London: HMSO.

Doig, B. and Hunting, R. (1995) 'Preparing teacher-clinicians in mathematics', in Meira, L. and Carraher, D (eds) Proceedings of PME-19, Volume 3. Recife, Brazil: Universidade Federal de Pernambuco, pp. 280-287.

Hoffman, P. (1998) *The Man Who Loved Only Numbers*. London, Fourth Estate.

Huckstep, P. (2001) 'Mathematics and human well-being'. *Mathematics in School* 30(2)

Ma, L. (1999) *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States,* Mahwah, New Jersey: Lawrence Erlbaum Associates.

Markovits, Z. and Even, R. (1994) 'Teaching situations: elementary teachers' pedagogical content knowledge', in da Ponte, J. and Matos, J. F. (eds), Proceedings of PME-18, Volume 3, Lisbon, University of Lisbon, pp. 239-246.

Piaget, J. (1929) The Child's Conception of the World. New York, Harcourt, Brace and Company.

Rees, R. and Barr, G. (1984) *Diagnosis and Prescription Some Common Maths Problems*. London: Paul Chapman Publishing.

Rowland, T. (1998) 'The clinical interview: conduct and interpretation'. In Zaslavsky, O. (Ed.) *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education.* Vol. 4, pp. 129-136. Haifa, Israel. Israel Institute of Technology.

Rowland, T., Martyn, S., Barber, P. and Heal, C. (2001) Investigating the mathematics subject matter knowledge of pre-service elementary school teachers. In M. van den Heuvel-Panhuizen (ed.) *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education, Volume 4* pp. 121-128. Utrecht, The Netherlands: Freudenthal Institute, Utrecht University.

Ryan, J. T. and Williams, J. S. (2000b) *Mathematical Discussions with Children: Exploring Methods an Misconceptions as a Teaching Strategy.* Manchester: University of Manchester.

Sawyer, W. W. (1964) *Vision in Elementary Mathematics*. Harmondsworth: Penguin.

Toulmin, S. (1958). *The uses of argument.* Cambridge, Cambridge University Press.

Williams, J. S. and Ryan, J. T. (2000a) 'National testing and the improvement of classroom teaching: can they coexist?' *British Educational Research Journal,* 26(1), pp. 49-73.