A case study to explore approaches that help teachers engage with students' development of mathematical connections

Nicola Trubridge and Ted Graham

Plymouth University

This research study considers the Collaborative Connected Classroom (CCC) model and how it might be implemented within a school via a programme of sustained CPD that incorporates: research sharing; engagement with activities that bridge theory and practice; active collaboration and exploration of ideas. This paper reports the findings when looking at which aspects of the CCC model engaged teachers themselves and then which of these tasks were used with their learners to develop mathematical connections.

Keywords: CPD; connections; conceptual understanding; secondary teachers

Introduction

It is accepted that pupils' understanding of mathematics can be developed by exploring connections between concepts and different representations (ACME, 2011; Swan, 2005). However, there is a shortage of mathematics specialists and many reports (ACME, 2002; Cockcroft, 1982; Smith, 2004) contend that one of the most effective ways to raise the quality of mathematical provision is to expand continuing professional development (CPD) for teachers of mathematics. This study considers the Collaborative Connected Classroom model (Trubridge & Graham, 2013) which was implemented via a programme of CPD that was designed to support a group of teachers; the aim being to develop a more connected approach to school mathematics. This paper evaluates which approaches engaged teachers as they took part in the CPD programme.

Project outline

Trubridge and Graham (2013) identified the nature of mathematical activity within a Collaborative Connected Classroom to be as shown in Figure 1 below.

Nature of Mathematical Activity	1. Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema.
	2. Tasks either connect different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
	3. Links are made between procedures and concepts
	    a. meaning is built for procedural knowledge before mastering it
	    b. procedures are evaluated to promote conceptual understanding
	4. Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
	5. Application tasks are presented as challenges that may be problematic and need to be reasoned about

Figure 1. Nature of mathematical activity in the CCC model

There were many aims of the CPD, with the focus being to develop teacher's pedagogical content knowledge through collaborative working. The 'subject expert' led an initial session sharing research papers and findings that informed the development of the CCC model. This was followed by sharing activities that were 'challenging and inspirational' to model each of the aspects of Figure 1; showing what they might look like in the mathematics classroom, hence bridging theory and practice. The next aim was for teachers to explore and develop these new ideas through sustained active classroom experimentation and collaboration with action research.

Methods

The case is a 'typical' mathematics faculty within an 11-18 college. Semi-structured interviews were carried out with all eleven members (A, B, C, D, E, F, G, H, I, K, L) of the faculty on four different occasions throughout the longitudinal four-year study. Additional data was collected to enable triangulation in the form of: book scrutinies, learning walks and presentations by the teachers of their action research projects.

Teacher development model

As the study advanced, it became evident that teachers progressed along a continuum (although not necessarily in a linear way). The first phase was the 'awareness' phase where teachers learned about the elements of the CCC model. They then experimented and used resources that were provided by the 'subject expert' which is named the 'guided exploration' phase. The biggest change in practice was as teachers moved to the 'independent exploration' and then 'independent development' phases where their use and development of ideas went beyond those provided in CPD sessions. Some teachers moved to the 'transformation' phase whereby the new way of working became the norm in their practice. Data was coded in the form of a two-way table where each cell had a description assigned to enable consistency in coding the data to the most appropriate stage.

Findings

Figure 2 shows most data at the 'independent exploration' phase with approximately equal amounts in 'guided exploration' and 'independent development' phases.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation	Total
Conceptual structure	6	16	17	5	0	**44**
Connect areas of mathematics	16	5	24	8	0	**53**
Multiple representations	19	22	64	30	4	**139**
Procedures and concepts	17	29	23	20	2	**91**
Comparisons	5	11	51	19	3	**89**
Application tasks	4	3	21	5	0	**33**
Total	**67**	**86**	**200**	**87**	**9**	**449**

Figure 2. Strands of CCC model mapped against phases of teacher development (number in cell is count of references coded)

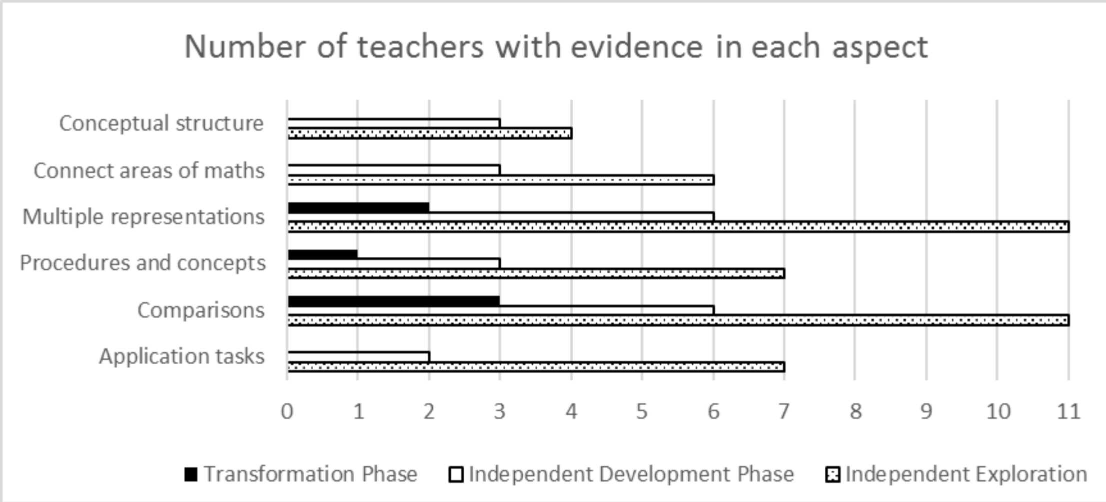

Figure 3. Number of teachers at the independent exploration/development and transformation phase for each aspect

Figure 3 shows that all teachers had independently explored using multiple representations with six of these teachers that had moved to the independent development phase and beyond. Fraser's books revealed examples that introduced trigonometry by making links with the unit circle and explicitly mentioned similar triangles and links to the graphical representation of the sine curve. Charlotte had designed a task making links between function machines, tables and graphs, whereas Kate had found and adapted resources that matched improper fractions, mixed numbers and visual images of fractions. Elliot used visual images to help explore percentages. Heidi used multiple representations in great depth to support the derivation of fraction calculation procedures. Louise also explored in depth and wrote her PGCE action research project on the question 'Can presenting a mathematical concept in different representations help pupils make links and develop a conceptual understanding?'.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Conceptual structure		A*, I, K,	B, C, D, F, L	E, G, H*	
Connect areas of mathematics	A*, D*, F*, I*		C, G, K,	B*, E, H, L	
Multiple representations			A*, B, D*, G, I,	C, E, K, L	F*, H
Procedures and concepts		F, I	B, D, E, K, L	C, G	H
Comparisons			A, B, F, G, I,	C*, K, L	D*, E*, H*
Application tasks	K		A, B, C, D, F, G, I, L	E*, H,	

Figure 4. Teachers positioned in cell with highest evidence (* no triangulated evidence)

Whilst the 'making comparisons' strand of the CCC model had two elements to it (exploring similarities and differences and efficiency of method), there was a marked discrepancy as to which was trialled and developed. The strategy of exploring similarities and differences was embraced by everyone at least at the independent exploration phase with six teachers (C, D, E, H, K, L) taking things further to come up with their own examples. Figure 4 shows evidence from interviews that three teachers (D, E, H) believe they have transformed their practice using the aspect of making comparisons in the form of looking at similarities and differences although triangulated evidence was difficult to gather due to the nature of the tasks, being discussion-based with less written evidence. When Elliot was questioned how often he used the strategy his response was “once a day at least and at other times it would be all the lessons that you would say those words” (2016, Final interview). Heidi provided a similar response:

> What I do a lot of in my lessons is similarities and differences. I have started to introduce it at the beginning of each topic to encourage students, to give them the overall picture and to see how the topic will look. (2016, Final interview)

The similarities and differences aspect was also explored in depth by Daniel who comments “I embedded that in a lot of my other subjects and in my maths .... now that is becoming just off the cuff, what’s the same and what’s different is becoming more everyday speak for me in my maths lessons” (2016, Final interview).

The final aspect where there was evidence of teachers making changes to practice was making links between procedures and concepts. This progress however was restricted to two curriculum areas; fractions and volumes of prisms. The teachers themselves engaged with the Play Doh task (Trubridge, 2015) that was shared at a CPD session as it was engaging and inspiring “I love it and think it is exactly what volume needs really, to have the resources” (Fraser, Mid-study interview) and five teachers used this with their learners. Developing a greater understanding of fraction calculations was chosen as an action research project by Heidi, Ian and Georgie.

Why was progress made in multiple representations?

Early interviews show that most teachers were already familiar with using multiple representations and the use of the area method to help explain multiplying algebraic expressions. Building and extending these ideas to activities such as completing the square or using visual methods for proof could be seen to be a natural extension for both learners; “we started linking back in the work they had done on quadratics, they had already seen the various representations of quadratics” (Fraser, Presentation) and for teachers “so they came a bit more naturally for me to stretch myself to prepare more things” (Kate, 2016, Final interview). When teachers themselves were exposed to several proofs that would require sophisticated algebraic skills but found that they could be explained to a much lower age of audience using visual representations there was an incentive to engage with the use of multiple representations across the curriculum. “I did a Pythagorean proof …it was surprisingly useful how it worked and did grab attention from the lowest ability. The Eureka moments were evident and lovely” (Fraser, Mid-study interview).

One of the tasks that was modelled for the faculty, completing the square, was seen to be new pedagogic knowledge for teachers with Georgie saying “I would never have thought about coming up with an activity like that and seeing the way that the students made the connections after I had made the connections in a similar way was inspiring” (Georgie, Mid-study interview).

Another reason for progress in the aspect of multiple representations was that it is an approach where there are readily available resources. Fraser drew on lots of resources from the Further Mathematics Support Programme, Kate from research papers, Louise from Swan (2005) and these were then adapted for use in the classroom.

Why was progress made in similarities and differences?

There were perhaps a couple of reasons why this aspect was so successful with the faculty. The strategy was seen to be relevant to all topics so could frequently be used "probably most lessons I say what is the same and what is different" (Georgie, Mid-study interview). It was portrayed as an 'easy' thing to implement that didn't take much time to plan however 'value added' relative to the time to plan was great.

> It took five minutes to set up with the equations where you had to think a little bit carefully but then it is a good hour's lesson of good discussion going on between the students to find out what is actually going on. (Charlotte, Mid-study interview)

Elliot (Mid-study interview) commented "It is probably the easiest one to implement as a teacher because you are just asking a simple open question and I think it can have a big effect".

Teachers were motivated by the responses from their learners. Daniel commented on the importance of the strategy enabling learners to generate ideas so they derived knowledge themselves that gave them a deeper understanding of the concepts being covered and on the need for "getting those really good leading questions that you can bring into maths to start churning out those really good discussions" (Mid-study interview). This was reiterated by Charlotte "when they look at the differences and similarities I sort of point out: look, I have taught you nothing and you have worked it all out" (2016, Final interview). This was echoed by Elliot "it gets them thinking for themselves and they are not relying on you telling them" (Mid-study interview).

There was another unplanned aspect to the research study that also moved teachers forward in the aspect of similarities and differences. A slide referring to effect sizes of different pedagogic approaches was shared at whole staff training which gave additional evidence from another source that similarities and differences would have a greater effect on gains in learning than areas such as repetition and practice or cooperative learning. This was after the faculty had moved to the active experimentation phase of the CPD model and reassured Daniel and Elliot that they were doing the 'right thing'.

> I was really surprised; well not surprised, but happy to see the similarities and differences when the assistant principal did his presentation and saw that it was a 1.6 effect size, …….it was good to see that using similarities and differences, which is something that we all have embraced as a department, has such a big effect in the development and progression in understanding of students, so that was really good to see. (Elliot, Mid-study interview)

Daniel (Mid-study interview) also referred to the same slide being shown and commented "I was always going to try and extend what I had started last year but I thought if that is going to have that effect size then that is something I really do want to discover a bit more about". He then went on to explore this further for his action research CPD project "it was all kicked off by Marzano's research on effect sizes knowing that had a huge effect size if you get it right. That's why I wanted to investigate it really" (2016, Final interview).

Why was progress made linking procedures and concepts?

Heidi's progress in this area, and Georgie's to a lesser extent, was largely due to extensive research looking at readily available literature and other people's ideas to help formulate ways to use visual representations to support the development of fraction procedures from concepts. Heidi provided a range of academic references within her presentation to the faculty. She also articulated her own opinions as to why this was important for her to study further “this is one of the topics that most teachers and students understand only instrumentally” (Heidi, Presentation). Heidi had a personal desire and commitment to improve the teaching of fractions and challenged herself with the support of her academic reading to explore and transform her practice in this area.

Georgie referred to the legacy within the faculty of using the trick of ‘times and twiddle’ (for dividing by fractions) and how she wanted to move on from this collaboratively;

> I am probably more prone to using tips and tricks because I have had to and now I feel like I have got the rest of the department saying if we are going to start this in year 7 then there doesn’t have to be tips and tricks. (Mid-study interview)

Progress in developing a more conceptual understanding of volumes of prisms was due to teachers themselves being inspired and enthused by tasks that were modelled in the CPD sessions and recognising that if time was dedicated to teaching these concepts more effectively then they wouldn’t need to consider every case of prism within schemes of learning.

Conclusion and further questions

This study has shown that the aspects of the CCC model that engaged teachers were: the use of similarities and differences, multiple representations and linking fraction procedures and concepts. Whilst in this study teachers were quick to use the strategy of ‘what is the same and what is different’ with their learners on a regular basis, reporting an increase in class discussion and deeper thinking, further study should be carried out to research the impact on learners within the mathematics classroom. Another area to explore is the challenge of moving teachers from guided to independent exploration and it raises the question how do you enable teachers to be more independent thinkers rather than relying on what someone else has thought about?

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