Completing the square: the cultural arbitrary of Oxbridge entrance?

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I report on part of a research project which investigated how independent schools might give their students an advantage when they apply to elite universities. The case study approach draws on qualitative methods and examines a preparation programme, in an independent school, for students applying to study mathematics at elite universities. I draw on a Bourdieusian conceptual framework to analyse teachers' social and cultural capital and show how this can provide experiences that are thought to confer distinction. As an insider researcher and mathematics teacher on the programme, I was able to examine my mathematics sessions in detail. Here I look at some of the skills which I think could be argued to become arbitrary, rather than functional, in practice and to what extent a student who acquires these skills gets 'use' and/or 'exchange' value from these skills.

Keywords: capital; Bourdieu; elite universities; independent schools; advantage

Introduction

It is widely believed that students who attend independent schools are advantaged compared to those in state schools, partly due to the fact that students from state schools are underrepresented at elite universities (Hoare and Johnston, 2011). The admissions process for a mathematics undergraduate course in 2017 at the University of Oxford, the University of Cambridge, Imperial College London and the University of Warwick required an admissions test, either the Mathematics Admissions Test (MAT) or the Sixth Term Examination Paper (STEP) in addition to an interview. Many other elite universities required an interview. The aim of the admissions process, both interview and test, is to enable the universities to select the best candidates. However, independent schools invest time and resources to help prepare candidates for the admissions process at such universities, for example, teacher advisors (Dunne, King & Ahrens, 2014) and sessions on the application process (Donnelly, 2014). This paper looks at how the mathematics lessons of a preparation programme in an independent all-boys school might give students an advantage in the admissions process over those in state schools.

Theoretical framework

In this study, I have adopted a Bourdieusian framework to understand what capital is at work in the admissions process. Drawing on Bourdieu, I have taken cultural capital to mean cultural goods, educational qualifications and skills, and social capital to mean the value associated with connections in people's social networks due to the capital that these connections provide access to (Bourdieu, 1986). This paper focuses on the cultural capital that might be transmitted from teacher to student through the programme. Intrinsically linked to capital is a person's habitus, a structured system of dispositions which predispose a person to act in a particular way, consciously or unconsciously (Bourdieu, 1990). In this research, I identified capital that the teachers were aware of, as well as, through my reflections on my own practice, capital that had been ingrained in my habitus. I realised that I had been buying into the doxa - “Doxa is the relationship of immediate adherence that is established practice between a habitus and the field to which it is attuned, the pre-verbal taking-for-granted of the world that flows from practical sense” (Bourdieu, 1990, p.68) - thinking that I was doing particular things in the lessons to make my students into better mathematicians, but in fact there was an element of particular things giving the students an advantage in the admissions process. Habitus exists in relation to a field or fields: if a person's habitus is better aligned with a particular field then that person holds more power in that particular field. One might say, knowing the rules of the game allows one to do better. Practice is produced by the interaction of habitus and capital with field (Bourdieu, 1984). In this study, power is associated with knowing how to succeed in the admissions process and how to impress university admissions tutors at interview. Whilst some activities in the programme might produce ‘use value’ in mathematical competence (Williams and Choudry, 2016), they also produce ‘exchange value’ in the admissions process.

Methods

I adopted a case study approach to help me to examine one particular preparation programme in an independent school. There were twenty teachers responsible for delivering the different subject lessons, of these seventeen took part in the data collection. There was usually one or two teachers per subject. I was one of the teachers responsible for the mathematics lessons. All sixth form students were able to opt into the programme if they wished to. Lessons were given at lunch time or after school once per week and students would attend the lesson in the subject that they wished to study at university.

Data collection consisted of three stages: a reflective writing task, lesson observation and semi-structured interview. The reflective writing task allowed me to gather teachers’ perceptions of what they were doing through the programme lessons by asking them to write a paragraph, or more if they wished, considering some, or all, of the following points:

- What do you do in the programme sessions which is different to within normal lessons or beyond A-level teaching?
- What do you think the purpose(s) of the sessions is/are?
- How do you think you achieve this purpose or purposes?
- You may wish to give an example of a typical session or an example to illustrate how you think you have achieved this purpose.

The lesson observations, with three participants, allowed me to see what was happening in practice. These were followed up with semi-structured interviews to explore further anything mentioned in the reflective writing tasks which might not have been observed and anything of interest in the observations.

As a mathematics teacher on the programme, I did the reflective writing task as a participant, but before gathering any other data to ensure I was not influenced by the other participants’ data. As I analysed the data from all participants, I became aware of things I was doing in the lessons that had been deeply ingrained in my habitus. My reading of Bourdieu also allowed me to consider the extent to which I was choosing particular activities because they had use value for the students as mathematicians, and the extent to which I was choosing the activities because they would give the students an advantage in the admissions process. As an insider researcher I was able to understand the mathematics involved in a way I could not with other subjects. My own reflections on what I was doing in the mathematics lessons has allowed me to become conscious of things that had been deeply ingrained and previously misrecognised. My reading of Bourdieu allowed me to understand that a lot of what I might be doing in the programme lessons might be unconscious and parts of it I might be misrecognising as making my students into better mathematicians when in fact it was part of the game of getting them into elite universities. This process of reflexivity made me conscious of certain activities. So although there are dangers of being an insider researcher, here there were advantages as I was able to really understand and recognise some of the things that were happening in the mathematics lessons, which I could not with the other lessons.

Key findings

Through my reflections, I identified activities which might be culturally arbitrary and have exchange value in the admissions process. The examples I consider here are the technique of completing the square and the skills which help answer longer questions.

Completing the square

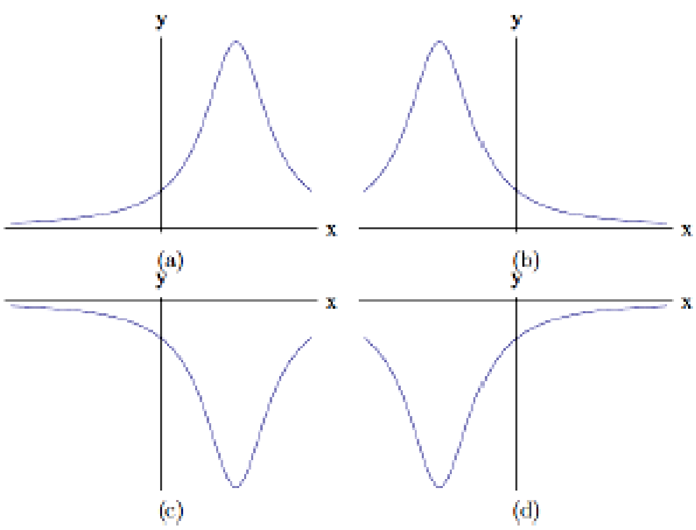
This is an example of a question taken from an A-level paper which instructs students to complete the square.

> Express $3x^2 - 12x + 5$ in the form $a(x-b)^2 - c$. Hence state the minimum value of y on the curve $y = 3x^2 - 12x + 5$. (OCR MEI, 2013).

Students will have been taught the method and will know to use it as they are instructed explicitly in the question.

The next question is from an Oxford MAT paper:

> Which of the graphs below is a sketch of $y = \dfrac{1}{4x - x^2 - 5}$?



(Oxford University, 2008)

The students are not instructed in this question to complete the square to help them with this question, yet if they do then they will be answering the question more efficiently than if they try, for example, differentiating to find the minimum point. In the lessons, I trained my students to complete the square if they saw a quadratic. In this example it would not necessarily be obvious to an A-level student that completing the square is an appropriate method to use, unless they had been specifically taught by someone at some point or, you might argue, they were a natural mathematician. The use of the method of completing the square I have termed a 'scholarly trick'; scholarly suggests there is merit to the skill to develop a better mathematician and trick suggests that there is an arbitrariness in the admissions process/game.

The next example is taken from a Cambridge STEP:

> A small ring of mass m is free to slide without friction on a hoop of radius a. The hoop is fixed to a vertical plane. The ring is connected by a light elastic string of natural length a to the highest point of the hoop. The ring is initially at rest at the lowest point of the hoop and is then slightly displaced. In the subsequent motion the angle of the string to the downward vertical is ϕ. Given that the ring first comes to rest just as the string becomes slack, find an expression for the modulus of elasticity of the string in terms of m and g.
> Show that, throughout the motion, the magnitude R of the reaction between the ring and the hoop is given by
> $$R = (12\cos^2\phi - 15\cos\phi + 5)mg$$
> and that R is non-zero throughout the motion.
> (Cambridge Assessment Admissions Testing, 2012)

The latter part of this last example - show that R is non-zero throughout the motion - makes use of the method of completing the square, a method which I would expect A-level students and teachers of mathematics to associate with pure maths, yet this is a mechanics question, on the mechanics section of the STEP paper, so students are expected to apply techniques they associate with pure maths to help them to solve a mechanics problem.

Longer questions

The examples above use a straightforward trick, which led me to reflect on what it was that made the longer questions more difficult. I reflected that this was partly due to the abstractness of the questions, but also that the questions mix different areas of mathematics and do not necessarily guide you through each part as an A-level question would. This is an example of a longer question taken from an Oxford MAT paper:

> For a positive whole number, n, the function $f_n(x)$ is defined by $f_n(x) = (x^{2n-1} - 1)^2$.
>
> (i) On the axes provided opposite, sketch the graph of $y = f_2(x)$ labelling where the graph meets the axes.
>
> (ii) On the same axes sketch the graph of $y = f_n(x)$ where n is a large positive integer.
>
> (iii) Determine $\int_0^1 f_n(x)dx$.
>
> (iv) The *positive* constants A and B are such that
> $$\int_0^1 f_n(x)dx \le 1 - \frac{A}{n+B} \text{ for all } n \ge 1.$$
> Show that $(3n-1)(n+B) \ge A(4n-1)n$,

and explain why $A \le \frac{3}{4}$.

(v) When $A = \frac{3}{4}$, what is the smallest possible value of B ?

(Oxford University, 2009)

For an A-level student, there is no context to this question and at first glance it looks to be quite complicated, mixing several areas of mathematics that at A-level tend to be distinct, i.e. functions, integration, inequalities. With these sorts of questions, there might be lengthy algebra needed to reach a solution compared to the few lines of algebra required of an A-level question. Students can find this off-putting and think that they are doing something wrong when they have not reached the answer after a few lines of working. In the lessons, I will teach my students to persevere with the lengthy algebra. In addition, with the longer questions, I will teach my students that they will often have to use previous parts of the question to aid them in later parts, something which if students are not taught they will not necessarily think to do, especially in a pressured situation. At A-level, the use of previous parts of a question is often signalled by the words "hence" or "therefore". These words are not in this question, but I would hope that it is pretty obvious that part (iv) follows on from part (iii), but I often find as a teacher you have to state what you think is obvious. Solving part (iv) involves working with two algebraic fractions - students often forget that working with fractions is the same whether numerical or algebraic, there is also care needed with negative signs in inequalities as well as persevering with the algebraic manipulation. In part (v), after putting A=3/4 back into the inequality and doing some algebraic manipulation we have to return to part (iv) and use the fact that $n \geq 1$ and hence what we have found must be true for n=1.

The skills that the students develop through these lessons include perseverance, not being deterred by complex-looking mathematics and knowing to use previous parts of the question; skills which are not necessarily taught in normal A-level lessons.

Conclusion and Discussion

When I started my research I would argue that by teaching students to be adaptable with methods and to be confident to use them when not instructed to do so would make them into better mathematicians. However, I could now argue that I was buying into the doxa and although I believed what I was doing was making better mathematicians it might in fact have been giving the students an advantage in the admissions game. Here, what I argue is that the skill of completing the square could be functional in practice for a mathematician and have use value for solving harder mathematical problems, but it might also be argued that for the students on the programme it is a cultural arbitrary because it helps them perform better in the admissions tests (MAT and STEP) for some elite universities and thus has exchange value. The programme facilitates a transmission of capital from teacher to students, capital which might not otherwise be transmitted.

In conclusion, if students are taught these tricks, for example, to complete the square when they are presented with a quadratic, then they have an advantage in the admissions tests over those students who have not been taught this, allowing them to efficiently reach a solution and move on to other questions more quickly. This means that the method of completing the square can become a cultural arbitrary in practice if it is used to discriminate in the selection process for elite universities; as such the method or skill has exchange value in the admissions process. But we can argue that the method is functional in practice if it is used to help solve harder mathematical problems and as such has use value. These admissions tests are aimed to discriminate between the applicants for mathematics degrees at elite universities. The universities know that students at independent schools are prepared for these but perhaps this preparation means students from independent schools perform better in the tests aimed to discriminate rather than actually being better mathematicians. I have simply tried to illuminate that this is happening. It might contribute to the discussion on widening participation. I am not claiming to know the answers regarding questions of how to make the admissions process for universities fair.

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