# Studying the link between classroom dialogue and the implementation of rich tasks in post-16 mathematics with Underground Mathematics

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Underground Mathematics (UM) develops online resources to support teaching and learning in post-compulsory mathematics and holds a strong belief that dialogue is inherent to mathematics. To fully understand the implementation and use of its resources, the project evaluation has employed a mixed-methods approach. Three questions guide the research: How is UM implemented in post-16 classrooms in England? How is attention to dialogue essential in understanding UM? To what extent is the methodology of analysis effective at addressing the first two research questions? To answer these questions, classroom observations were conducted in A level classrooms across England using UM materials. Dialogue was analysed using the Cam-UNAM Scheme for Educational Dialogue Analysis. This paper presents insights from the data analysis and argues in favour of a methodology in which dialogue is central to research in mathematics education.

**Keywords: classroom dialogue; evaluation; post-16 mathematics; instructional resources; case study research**

## Introduction

Underground Mathematics (UM, formerly the Cambridge Mathematics Education Project) develops online resources to support teaching and learning for post-compulsory mathematics (students aged 16-18). The public website (accessible at https://undergroundmathematics.org), a free repository of rich resources that covers the content of A level mathematics, was formally launched in February 2016 and continues to add to the available resources. A key aim of UM is to help students and teachers of mathematics to experience mathematics as a creative, stimulating, interesting, and coherent subject.

Further to the project's aims, the resources are designed to be *low-threshold, high ceiling*; that is, they are widely accessible to students regardless of prior attainment and provide great potential for growth for all. The resources focus on problem solving, mathematical reasoning, and fluency. As a result, it is the project's intent that students will talk mathematically and reflect on their own understanding in completing UM tasks.

Embedded within the project is an evaluation team, including the authors of this paper. The team's remit is to provide feedback on the project to the resource writers and project funders, as well as to contribute to the community of mathematics education research at large. This paper is situated within the team's ongoing mission to do so; it explains and analyses the current methodology. At present, the evaluation project is in the early analysis stage; we therefore present here the piloting of the analysis.

## Framework

### *Dialogue*

Numerous researchers have discussed the nature and the importance of dialogue in mathematics education. Their arguments centre on the sociocultural aspects of learning in education; that is, cultural processes that transpire from interactions (Hennessy et al., 2016; Moschkovich, 2007). Adding to this, Mercer and Sams (2006) argue that dialogue in classrooms improves students' mathematical reasoning and problem solving. Boaler (2002) expands this view, asserting that reasoning, or explanation, is essential to mathematics as a discipline. Ruthven et al. (2017) also call for additional research to be carried out in dialogic teaching to better understand its relationship with effectiveness of learning. Of particular importance to this study is the position put forward in project development meetings and conversations with Martin Hyland, a principal investigator for UM, that dialogue is inherent to mathematics. This reflection served as the impetus for this analysis and its central positioning in the theoretical framework.

### *Research questions*

Based on the philosophies espoused by UM and the remit of the evaluation team, two research questions emerged as part of the team's overall task. First, how is Underground Mathematics implemented in post-16 classrooms in England? *Implementation* is taken to include a wide range of elements of a teacher's practice. It can include the tasks presented to the students, any modifications or instructions made by the teachers, any technology employed in the lesson, the organisation of the classroom, the intended aims and objectives of the teacher, the self-efficacy of students, and other elements.

Second, how is attention to dialogue essential in understanding implementation of Underground Mathematics? Essentially, are there insights to be gained from the analysis of dialogue that other forms of analysis may miss? Furthermore, could UM's philosophy even be captured without such attention?

At this stage of the research, however, a complete response to these questions is impossible. A third question thus presents itself: to what extent is the evaluation project's methodology of analysis effective at addressing the two major research questions? While this paper begins to construct a response to the first two questions, its focus is ultimately on this final question.

## Methodology

### *The case study*

As a lens into classroom dialogue and implementation of UM, the evaluation team carried out an exploratory case study design. Such an approach was chosen because of its ability to provide rich, in-depth data from classrooms, which are inherently complex and largely unsuitable for controlled trials.

Six A level (i.e. post-16, non-compulsory) classrooms in five schools across England were selected to participate, based on their prior involvement with UM. The sampling is not able to be representative of all English post-16 classrooms or even all classrooms using UM, nor was it meant to be. Of primary concern was selecting participants that would provide meaningful feedback to the project's resource developers. Potential bias in using schools that chose to opt-in to the research is compounded by their original self-selection to use UM resources. Additionally, the research questions do not seek to understand *all* implementations of UM resources, but rather to begin to build an understanding of their potential implementation. Therefore, the present study does not seek to compare across groups, teachers, or materials. The researchers recognise the limitations with this sample; however, they see it not as a weakness, but a reflection of the evaluation team's remit.

### *Coding and analysis*

To analyse the whole class and group transcripts, the researchers employed the Cam-UNAM Scheme for Educational Analysis (SEDA, see Hennessy et al., 2016). The scheme uses 33 unique codes, organised into eight clusters, to code each "communicative act" in a dialogic experience. For example, the R cluster is used for utterances that *make reasoning explicit*, and is divided into four codes: *explain or justify another's contribution*, *explain or justify own contribution*, *speculate or predict on the basis of another's contribution*, and *speculate or predict*.

SEDA, which was developed by a team of researchers at the University of Cambridge and the National Autonomous University of Mexico, is based on a sociocultural perspective. This framework holds centrally the belief that human life, and therefore education, is intrinsically social and communicative (Hennessy et al., 2016). This scheme therefore matched the project's philosophies about mathematics and the researchers' theoretical perspectives about the nature of dialogue. At the onset, the researchers postulated the 33 unique codes were well suited to highlight the types of dialogue expected at a level of detail that is both manageable and meaningful. Finally, and pragmatically, many of the scheme's developers are faculty colleagues of the researchers and were readily available to provide training, support, and feedback on the usage of the coding scheme.

Every transcript was independently coded by two members of the evaluation team using a popular qualitative data analysis computer software. Following this, the researchers examined each of the clusters in greater detail. By isolating individual clusters, and in some instances individual codes, further themes for consideration emerged; these are explored in detail below. The researchers decided to pilot the use of these considerations in analysis by initially selecting a single vignette of classroom dialogue.

### *Vignettes*

While the SEDA coding scheme and the associated themes provide a macro-scale lens of the dialogue in the classrooms, they do not provide a detailed view of the content of the lessons. Therefore, the researchers decided to select a number of smaller segments of classroom activity to gain a richer understanding of both the dialogue transpiring and the mathematical content. In the first instance, one such vignette was used as a pilot to ascertain the feasibility of selecting and analysing such a segment.

The vignette for pilot analysis was selected through multiple criteria by a member of the evaluation team. The selection was made from a later round because the researchers had developed a more nuanced, deep understanding and proficiency with the scheme by that point. From the classrooms that had participated in the fifth round of observations, one was selected that had used the same resource as another classroom. In addition, this school had two teachers participating in the study. Finally, the two coders had a very high coding agreement rates and very low disagreement rates across the clusters for this round and classroom. 81 lines of "typical" dialogue were selected from one of the groups.

The UM resource used during this lesson is *Logarithm Lattice*. This task asks students to arrange a series of logarithmic expressions in a grid, demonstrating increasing value. At the onset of the lesson, the teacher reviewed logarithms.

In the selected vignette, the group of students are early in the exercise and are working through a few particular sets of logarithmic expressions. At the onset, the students do not seem to have formulated an idea on how to approach the task. By the end of the vignette (which is neither the end of the lesson nor the task), some of the students are showing evidence of deepening their understanding of the material. For example, a few noted the relationship and difference between $\log_4(5)$ and $\log_5(4)$. One student describes the process of thinking through the expressions as "common sense," upon which the teacher, visiting the group at the time, then expounds.

## Findings

As stated above, analysis of the clusters revealed further themes of interest. These themes were an additional lens with which to analyse the dialogue. Relevant themes are explained below with regards to the vignette.

### *Disagreement, challenging, and reasoning*

In this selection, there are multiple instances where a student disagreed or challenged a peer. In two distinct examples of those occasions, there was overlap with providing explicit reasoning (e.g. "Oh no, because it equals that, doesn't it?"). In others, explicit reasoning appears shortly after the disagreement, but rarely in connection to the disagreement; instead, the conversation has moved to a new communicative act. It appears as if the disagreements had been tacitly resolved. In instances when a student voiced agreement with a concept, none were supported by a rationale.

### *Inviting talk, elaboration, and reasoning*

There are three instances when students invite others to speak and their peer's response includes an explicit rationale. In the first, the student proffers a question to the group. A second student responds, "I don't know," to which the first subsequently offers a prediction. This could suggest that the student had the prediction prior to asking the question, but perhaps lacked the confidence to state this outright. This is plausible, given that he appends, "I think" to the end of his predictive utterance. Alternatively, he could have intentionally wanted to provide an opportunity for his peer to make an attempt.

In the second instance, a student delivers a communicative act that both speculates and invites agreement, "That's going to be smaller than one, isn't it?" It is unclear how this prediction is immediately resolved, but it then quickly leads to the third instance of explicit reasoning following an invitation to talk. When another student seeks clarification, the first offers reasoning behind the original line of thought. In so doing, he realises he had made an error. As a result, he shifts his position. This moment seems to be a critical one for the group; it seems to be an "aha moment." Here, some members of the group come to an essential understanding necessary to complete the task (i.e. the relationship between $\log_a(b)$, $\log_b(a)$, and 1).

### *The teacher and authority*

The teacher makes a brief appearance at the end of the vignette; the students ask whether their work thus far is correct. Before the teacher is able to provide such feedback, the conversation is side tracked to the technical capabilities of a calculator in calculating logarithmic expressions. This is because one student, who does not seem to completely comprehend the key understanding, wants to check the group's work. In this instance, the teacher acts as an authority of this very specific, technical knowledge, but then brushes it off as unimportant. She then focuses the students' attention on a particular expression.

When the students offer analysis of the expression, the teacher begins to build on their response. She acts as the authority of how to think about that solution. She also focuses how to think about the logarithmic expressions. Interestingly, this approach does not provide an exact answer, but refines how to approach the task. One student had already indicated that this process was "common sense." It is possible that the teacher was explaining a concept which she felt the students already understood. Alternatively, she could have felt that they needed a more specific and explicit guidance about how to approach the problems. Regardless, in her words and actions it is clear she is an authority—but this does not mean she intends to provide the answer.

### *Dialogue in the group*

The utterances in this vignette were coded across a variety of clusters. Some codes were featured quite extensively, notably the E cluster, *Invite opinions/beliefs/ideas* and *Make other relevant contribution*.

In some respects, it is common to think of this cluster as a "catch-all" or for general miscellaneous comments. As such, it can be easy for researchers to disregard these codes. However, this vignette has underlined the significant value of E cluster dialogue. In the beginning, there is little use of reasoning or elaboration; the instances of disagreement appear in isolation and are largely left unaddressed. At this point in the lesson, the students are only beginning to take ownership of the material. The seemingly disjointed nature of the dialogue suggests they are struggling with the material, but this struggle is productive. It is necessary as the students begin to build an understanding of the task and concepts they face. This struggle and progression towards understanding highlights the value of dialogue within the E cluster; as an essential part of learning, its analysis should not be discounted.

Additionally, over the course of the vignette, the nature of the dialogue slowly changes. The latter part of the vignette has more elaboration and reasoning that was absent from earlier parts. As some students begin to construct a deeper understanding, they are able to provide more elaboration to their peers. While many educators would desire a more thorough explanation than "common sense," as one student uses, it demonstrates how the nature of the understandings has changed.

## Conclusions

### *The vignette*

Overall, the use of this segment of a lesson was helpful to the research. It provided a narrow focus for greater in-depth analysis of individual codes and themes that have become apparent. This analysis has begun to provide useful findings about dialogue and Underground Mathematics in classrooms. The length (approximately 80 utterances) seemed an appropriate length to provide a wide range of dialogue and a progression in students' understanding.

While many theoretical frameworks can be an effective lens to explore students' understanding, dialogue and the SEDA tool have been beneficial, particularly given the nature and philosophy of UM tasks. It has highlighted both "aha moments" as well as times of productive struggle—both of which are inherent to learning. Additionally, it is a lens to examine themes relevant to the project; such themes may not have been highlighted had dialogue been ignored. Attention to dialogue, in particular by means of SEDA, was also particularly helpful in identifying students' productive struggle. This, along with more nuanced interactions, could have been missed without an examination of dialogue. Thus, this methodology seems effective in addressing the major research questions of the team.

### *Future steps*

Keeping in mind the aforementioned limitations regarding generalisability, the researchers intend to continue employing this methodology. Future vignettes will be selected and analysed by the researchers using the same process. Based on the insights beginning to form from this pilot analysis, the researchers are confident in their ability to answer the questions on how Underground Mathematics is being implemented and how dialogue is essential to understanding learning in the classroom.

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