# Teachers' perception of mathematical tasks: what teachers 'saw' in a published task, and how this changed following enactment of the same task.

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What do teachers perceive when presented with a mathematical task? In a dissertation study for the MSc in Teacher Education (Mathematics) I analysed three teachers' lesson planning notes and interview responses to investigate their intended use of a published task, before and after their observation of teaching using the same task. Teachers had distinct plans, and each responded differently following enactment of the task. I relate this pedagogical decision making to teachers' perception of the task, described in terms of teachers' beliefs, knowledge and goals. Teachers formed a relationship with the task, supporting, challenging or complying with the task based on their differing perception, leading sometimes to significant changes in the intended lesson. From a mathematics teacher educator perspective I suggest the Japanese practice of kyozai kenkyuu as a means to inform teacher decision making before teaching the lesson.

**Keywords: mathematics teacher education; perception; mathematical tasks; teacher beliefs; teacher knowledge; pedagogical decision making.**

## Introduction

I was intrigued by secondary mathematics teachers' responses in a professional development session, part of my work as a Local Authority adviser. Given the prompt "65 + 37 = 67 + 35 : True/False? What is the purpose of asking students this question?", suggestions ranged from addition of 2-digit numbers to partitioning and place value, adjustment and compensation strategies to the meaning of the 'equals' symbol. This much was not a surprise; mathematics is a highly interconnected subject, and teachers' ability to 'unpack' the subject has been identified by some as an identifiable type of "specialized content knowledge" (Ball, Thames, & Phelps, 2008, p.400) which is of particular use in teaching the subject. I was struck however that different teachers had had different initial ideas, and wondered what variation in teaching might have arisen had we not had the opportunity to discuss our thinking.

This first noticing developed into a study undertaken with the same group, informed in particular by the work of Remillard (2005) on teachers' relationship with curriculum, and Schoenfeld (1998) on teacher decision-making.

## Mathematical tasks in the classroom

How tasks are designed and how teachers relate to those tasks matters because of the prevalence of tasks in mathematics classrooms. Sierpinska (2004) called on the mathematics education researchers to refocus attention on mathematical tasks, followed within a decade by an ICMI study on task design in mathematics education (Margolinas, 2013), introducing tasks as "mediating tools for teaching and learning mathematics" (Watson et al., 2013, p.12). 'Task' here is as in the ICMI study, namely

"anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something" (Margolinas, 2013, p.12). Here I consider 'curriculum' and 'task' as synonymous other than in scale; teachers use and relate to curriculum as to task, true inasmuch as tasks constitute the curriculum.

Curriculum might comprise tasks, but how tasks relate to learning is a matter of debate. Doyle (1988) wrote influentially on academic tasks in the mathematics classroom, "based on the premise that the tasks teachers assign determines [sic] how students come to understand a curriculum domain" (p.167). 25 years on Ainley and Margolinas (2013) identified this as a tacit assumption which persisted despite research findings disputing such a simple relationship.

One study revealing complexity was from Stein, Grover and Henningsen (1996), who built on Doyle's finding that "a central theme in academic-task research is the extent to which tasks can change their character once unleashed in real classroom settings" (p.460). In their model tasks are described as existing at three distinct levels: as represented in instructional materials; as set up by the teacher; as implemented by students. An apparently singular task can take many guises, because students' "understanding of the objectives of the task can transform the task, [and] teachers also can wittingly (or unwittingly) change the nature of tasks by stressing less- or more-challenging aspects" (p.460). Tasks with higher cognitive demand often resulted in a decline, the task as 'implemented' was lower than as 'set up'. Stein et al. focused on the task as set up by a teacher and as implemented by students, rather than on the relationship between the task as represented in the curriculum and as set up by the teacher, maybe because 39% of the tasks used were created by teachers, rather than published curricular materials. This prevalence of unpublished curricular materials may be less common now; Thompson and Watson (2013) recognised that "most teachers use textbooks and/or online packages of materials as their total or main source of tasks" (p.281).

## Participatory relationship between teachers and tasks

Stein et al.'s (1996) analysis of tasks took a particular stance regarding the role of the teacher, implicitly assuming that teachers were striving to meet certain expectations in line with a curriculum reform effort. Remillard (2005) identified and challenged this stance in her examination of key concepts in research on teachers' use of mathematics curricula, detailing an alternate view which considers

> the teacher as interpreter of the written curriculum. This outlook holds to an interpretive view of text and assumes that fidelity between classroom action and written words in a teacher's guide is impossible, that teachers bring their own beliefs and experiences to their encounters with curriculum to create their own meanings, and that by using curriculum materials teachers interpret the intentions of the authors (pp.219-220).

In this view teachers inevitably interpret curricular tasks, just as jazz musicians play many different versions of a single published musical score (Brown, 2009). Remillard described this as a two-way participatory relationship, in which teachers are themselves changed as they modify tasks, a process also included as a feature of Gueudet and Trouche's (2009) notion of documentational genesis.

## Teacher learning

Rather than a reason for curriculum reformers' despair, this inevitable lack of teacher fidelity and associated teacher change offers the opportunity for tasks to be educative,

the site of teacher learning. Clarke and Hollingsworth (2002) proposed an Interconnected Model of Professional Growth to account for changes in the domains of practice, salient student outcomes and teacher knowledge and beliefs (principally in that order), with mediating processes of enaction and reflection. These processes may be supported by curriculum guidance, professional development, collaborative working or teachers' normal practice, but teachers in the West do not always value or otherwise cannot take the time to seek guidance nor reflect on tasks and activities (e.g. Sherin & Drake, 2009), contrasting especially with teachers from high-performing jurisdictions such as Japan (Fernandez & Cannon, 2005).

### *Teachers' perception of mathematical tasks*

In the common absence or lack of reference to clear and explicit guidance on the intended use of the task, teachers' decisions on task use can rely heavily on initial perception of the task. Kahnemann (2011) described decision-making as characterised by two distinct systems: System 1 is automatic, instinctive and inflexible; System 2 is more considered, rational and wilful, and can moderate System 1 judgements. Although very efficient, System 1 can give rise to irrational decisions, causing us uncontrollably to oversimplify. A given example is that "the statement that 'the odds of survival one month after surgery are 90%' is more reassuring than the equivalent statement that 'mortality within one month of surgery is 10%' " (p.88). With some statistical nous and application (System 2) we can assert that the two statements are quantitavely identical, yet our gut reaction (System 1) to each remains different.

Because perception here is internal, there is methodological difficulty to make it the subject of investigation; perception is automatic, and not necessarily consciously known by the perceiver. Investigating perception, researchers rely on external characteristics such as self-reports or actions subsequent to the perception, to be interpreted with reference to the teacher's experience, belief and understanding.

### *Teacher Decision-making*

The work of the Teacher Model Group (Schoenfeld, 1998) served as the theoretical framework and basis of the thematic analysis in this study. For over three decades this group has developed a model of teaching-in-context, about which mathematics educator Alan Schoenfeld has written extensively (e.g. 1998, 2010), for whom "the model describes, at a level of mechanism, the ways in which the teacher's goals, beliefs, and knowledge interact, resulting in the teacher's moment-to-moment decision-making and actions" (1998, p.1).

The group's persuasive research methodology was to parse video-recorded episodes of teaching, interrogating these data to inform research questions about the interaction of teachers' goals, beliefs, and knowledge. This model allowed for analysis of what came before the lesson, in terms of the teacher's previous decision making, and what was anticipated. This group's literature gave an account before any teaching took place, describing teachers' thinking as a created lesson image, a putative and idealised version of the pedagogical activity which the teacher intended.

## The study

14 teachers were given the realistic-context Taxi Cabs task, developed by Bowland Maths (Bowland Maths, 2010), and planned to use the task as if with 14 year-olds. Teachers had an hour to plan, one week before and one week after observing a

'lessonette', a 30-minute vignette of a lesson – intended to contrast rather than to guide – after which the group immediately discussed what had just been observed.

Four teachers were selected for semi-structured interview to explore the meaning and motivation of any changes made in their written scripts, and one further question asking if they thought their approach was in line with the task designers' intentions. Three teachers' responses were reported.

## Results

Katy, Diane and Mark responded to the study in varying and distinctive ways.

Katy initially submitted a detailed lesson plan, and used the task herself before the lessonette. She was enthusiastic about the task, and her pre-lessonette script exactly anticipated a conceptual issue which arose in the lessonette. Her post-lessonette changes referred to both the lessonette and her own lesson. The teacher in the observed lessonette had been mindful of intervening too soon when students struggled, which Katy saw as a higher tolerance for discomfort than her own. Her own students had 'lost' the meaningful context when engaged in analysis, which caused Katy to set a new overarching goal for the lesson using the task, that students deepen their understanding by relating their mathematical work to everyday life, a deeply held conviction of her own which had not been clear in her pre-lessonette script. She felt that her approach was in line with the task designers' intentions, with which she agreed, but that she had taken it further than the published version.

Diane made significant changes to her script after the lessonette, and had used the task with her own class by the time of interview. Her initial plan clearly identified a list of topics to solve the problem, her own System 1 (Kahnemann, 2010) interpretation that the task required her to find "the proper maths [that] students are supposed to learn here." In post-lessonette discussion she commented on the task from students' perspective, which also characterised the changes in her intended lesson. Revisions included removal of reminders for "scaffolding", now emphatically "not required". The revised script was much less directive, with more choice because "students should think: when I go to the real world, what are the best options for me?" Diane felt that her revised approach was not at all what the task designers had intended. She challenged the intentions of the task, which she had and continued to perceive as intended to be a means of introducing some decontextualised topics.

Mark made very few changes, and those he did make were designed to prepare students early for the more challenging aspects he had observed students struggle with in the lessonette, which he, like Katy, had identified as key concepts in the problem. Mark didn't teach his own lesson using the task. His approach to planning the lesson was to complete the task himself, then to design steps to lead students through his solution. He identified a mathematical goal of "methodical thought / set out work" in his initial script, and was satisfied not to revise this, as "I don't see the point of reinventing the wheel." When asked if his approach was in line with the task designers' intentions, his response was that "this has been written by somebody a lot more able than me at maths", a deferential response revealing his perception of the task as representative of an authority which he didn't seek to question or challenge.

## Discussion

Presented with the same physical material and expectations, these teachers perceived the same task in distinct ways, and responded quite differently in their revisions, expressed most clearly in terms of their goals for student learning.

Remillard (2005) called for further research on individual teachers' "perception of and stance toward curriculum materials and the teacher's professional identity as it relates to the use of curriculum resources" (p.237), and considering the teachers in these terms is revealing. Katy's initial perception was to be encouraged by the task, which aligned with her pre-existing beliefs, and went on to assume some notional authority associated with the task; her stance was as an advocate of the task. Diane was immediately compliant, then had the confidence to subvert (in her view) the intended aims; without rejecting it, her stance was to challenge the task. Mark perceived the task as a contractual obligation, where an apparently superior task designer had done their bit, and he fulfilled his side of the bargain by designing his version of a lesson around the task; of the three, his stance was the most compliant.

As well as seeking to describe teachers' perception of mathematical tasks, this teacher education study intended to elicit change, offering opportunities for initial perception – a System 1 process (Kahnemann, 2011) – to be held up for inspection and questioned, a System 2 process. All three teachers had multiple opportunities to reflect on the task and to develop their relationship with it, but it is notable that the two who used the task in their own teaching had more to say by the end of the study. Clarke and Hollingsworth (2002) described both reflection and enaction as critical mediating processes in teachers' professional growth, and in this study the enactment of both the jointly observed lessonette and their own teaching appeared to be instrumental in the changes Katy and Diane made, in both cases leading to a new appreciation of salient student outcomes, also consistent with the Clarke and Hollingsworth model (2002). Mark did not use the task in his own teaching, so did not undergo that process of enaction and subsequent reflection, maybe explaining his relative lack of change. In these three cases correlation alone might not imply causation, but interview responses suggest an impact of enaction and reflection. Relative to Mark, Katy and Diane often described critical moments in their thinking in terms of students' experiences with the mathematics, an empathetic dimension which Jaworski described as a sensitivity to students (Jaworski, 1992). This propensity could account for the close observations made by Katy and Diane throughout the study, and raises the question of whether observation alone can be considered as enaction, or at least sufficiently enactive to effect change in a domain, say of salient outcomes?

Although this teacher education intervention may be argued to have effected a change in two of these teachers, any improvements are too late for the students who have already had the lesson. A Japanese approach called kyozai kenkyuu (Fujii, 2016), part of a cycle of a professional development practice called lesson study, may be one way to address this problem. Fujii (2016) described how a group of teachers researched teaching materials to make pedagogical design decisions. By anticipating students' responses and making modifications to realise mathematical goals as seen from the students' perspective, in some detail the teachers hypothetically considered the mathematical flow of an intended lesson, which enabled them to make key improvements before the lesson was actually taught.

Teachers' perception of mathematical tasks is through the prism of their beliefs. It may be possible to deploy cognitive psychology, as in advertising or political messaging, to productively manipulate or even coerce teachers' perception of tasks, but seeking to control beliefs is not an ethical proposition. The potential for teachers to deeply enact and reflect on student learning within formal, structured collaborative practices such as lesson study offers the prospect of mathematics teachers acknowledging and benefitting from the broad range of perceptions which are fundamental to how they teach and grow professionally.

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