Words and contexts: Teaching mathematics vocabulary

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Research into vocabulary learning emphasises the need to teach vocabulary in authentic contexts and through making connections to students' prior knowledge or experiences. In mathematics, we have vocabulary that is also widely used in everyday situations or in other school subjects with a similar meaning but we also have words that are used in everyday situations that have very different meanings. Finally, there are also words that students are only likely to meet in their mathematics lessons. In this paper, we explore the discussions from the workshop at the June conference about what might be meant by an authentic context in mathematics for these different types of words, and how these contexts may, or may not, support students in making connections.

Keywords: vocabulary; contexts; tasks.

Introduction

Words can change how we see the world. They add precision and clarity, and they are tools for mathematical reasoning: for convincing oneself and for persuading others. Mathematics makes use of a wide variety of words, some of which share their use and meaning in our everyday lives, some of which are highly decontextualised and we are unlikely to encounter them outside the mathematics community or classroom, some of which fall somewhere between these two extremes. In this paper, we explore how focusing on these different types of words and the meaningful contexts in which they might be used supports teachers in providing students with opportunities for learning and using the language of mathematics.

We draw a distinction between static knowledge and dynamic knowing of mathematical vocabulary. Learning mathematics is far more than coming to know specialised vocabulary. Rather knowing how and when to use these words and phrases is fundamental to being able to communicate mathematical ideas and concepts. Furthermore, we take the stance that the primary purpose of mathematics teaching practices should be the learning of mathematics. Therefore, these definitions of learning and teaching lead us to emphasise practices that integrate the need for and use of mathematical language over those that treat vocabulary learning as something discrete and separate from the main event of learning mathematics. Indeed, such separation may be seen as an obstacle to effective vocabulary learning as Siebert and Draper (2008) have shown how such practices can perpetuate negative attitudes and resistance towards mathematical literacy.

The ability to communicate and use mathematical language, both the technical vocabulary involved and the ways in which we use language within mathematics, gives students access to meanings and powerful reasoning tools. Veel notes how this ability can be divisive, "provid[ing] some students with the access to the technical meaning potential of mathematics while simultaneously denying access to others" (Veel, 1999, p.206). We are therefore concerned with practices that afford all students access to the vocabulary that supports their mathematical thinking rather than viewing the technical register as the realm of the previously higher attaining.

In the workshop at conference, we explored the relationships between words and contexts within two key foci: using contexts to generate a need for words, and using experiences and definitions as contexts for learning new words. Before discussing examples from our research of inclusive teaching practices that prompted these discussions, we draw on language learning literature in order to consider what it means to learn a word and how this applies in the complex case of learning mathematical words.

What it means to learn a word

There are different aspects to what it means to know a mathematical word. Students may know the definitions of words for example, but not know how to use these words within sentences in a meaningful way (Herbel-Eisenmann & Otten, 2011). On one level, Bravo, Cervetti, Hiebert, and Pearson (2008) talk about knowing a word in terms of the degree of control a student has over a word. This ranges from low control where a student can decode the term, to passive control where a student can provide a definition or synonym, to active control where a student can use the word meaningfully in oral and written communication. The challenge is thus how to move students through these degrees of control so they go from recognising mathematical words to being able to use them in mathematically meaningful ways.

On another level, to 'know' a word means more than knowing the word's definition and when to use it. Students also need to know how that word functions similarly in different contexts, and indeed know in what ways the meaning may change in different contexts. For example, the word *simplify* works in both similar and different ways when working with algebraic expressions, fractions or probabilities.

Finally, the nature of mathematics also has important implications for what it means to know a mathematical word. As mathematicians, we seek to expand the example space to which a term can be applied beyond the context in which it was first experienced. For example, knowing the word *parallel* potentially means more than being able to give its definition in terms of straight lines in the plane, and recognising two lines as parallel. It also means more than being able to describe two sides of a quadrilateral as parallel, a common context in which students will meet parallel line segments. It means knowing how to use the word parallel in a wider range of contexts, such as potentially what is parallel to a curve in the plane or what does parallel mean in three dimensions. Whilst students might not directly meet these wider contexts the meaning they attach to the word needs to be flexible and rigorous enough for them to generalise to these unfamiliar contexts.

How we learn words

Research into generic vocabulary learning largely focuses on the relationship with learning to read or learning an additional language. This research emphasises the need to study words in their natural habitat; within sentences with a communicative purpose or within contexts where the need for the word arises (Moschkovich, 2015). There is an emphasis on: enabling students to make connections with what they already know; experiencing words in the range of contexts in which they might arise; and encountering words in context when reading. We consider each of these below and evaluate them by considering the case of learning mathematics vocabulary.

Many researchers have shown that effective word learning integrates new words in a network of other words and ideas (e.g. Bravo et al., 2008; Stahl & Stahl, 2004) but this can be more challenging with some mathematical terminology that students encounter for the first time in a mathematics classroom alongside a new concept. For example, the National Numeracy Strategy (DfEE, 2000) recommended that “you need to plan the introduction of new words in a suitable context, for example, with relevant real objects, mathematics apparatus, pictures and/or diagrams" (p.2). However, as Lemke (1990) states “Concepts are just thematic items…we never use them one at a time; their usefulness comes from their connections to one another.” (p.91). In the circumstances where the words are abstract and new, this would imply that the meaningful contexts are those which connect and contrast with existing mathematical words or concepts with which the students are more familiar.

The development of meanings associated with words is generally considered to be incremental as students experience a word several times in a variety of contexts (Nagy, Anderson, & Herman, 1987). Yet in mathematics students may experience a word several times in a lesson, and possibly across a series of lessons, but there is typically little 'recycling' of words in the ways described in the generic vocabulary learning literature, and furthermore in many circumstances the students may not meet the word again until the topic is revisited months or even years later. This may be a particular issue for learning words that students are only likely to encounter in the mathematics classroom. Nagy, Anderson and Herman further argue that learning word meanings from context during reading is more effective than any other type of vocabulary instruction. However, there are several aspects of word learning that influence comprehension, and in mathematics many words are encountered through hearing them within a context rather than reading them. Where words are encountered through reading, the context is usually shaped by the written text, such as the sentence the word appears within, the paragraph that sentence appears in and so on. The context in mathematics often includes visual representations and/or oral explanations and the teacher has more control over the nature and content of this context than they may have over the situations in which students might encounter mathematical words in written text (such as in textbooks or examination papers). This links to Tall and Vinner’s (1981) notions of concept image and concept definition where the contexts both broaden and deepen the concept image associated with the word, as well as refine the concept definition as students become more aware of how the word works in different contexts. However, within a mathematics classroom we tend to use language more precisely than we do in everyday contexts and we use technical vocabulary in fewer contexts. Sometimes the contexts within which students encounter a particular word, e.g. mutually exclusive, may be limited in nature and variation which can make it easier to misuse words or lead to broader concept definitions that those generally accepted by mathematicians. There is also often very little emphasis placed on the high-frequency words and structures that provide a basis for communication across mathematical topics, such as solve, show, and calculate.

Categorising mathematical words

A clear distinction can be drawn between technical words, such as *hypotenuse* or *congruent*, that students are unlikely to encounter outside of a mathematics classroom and words ‘borrowed’ from the English language that have a technical mathematical meaning, such as *similar* or *face* (Pimm, 1987, p.78). But further distinctions can be made between those ‘borrowed’ words that have a related meaning to that within mathematics (such as *event*, *enlargement* or *altitude*), those where the technical meaning might be derived from breaking down the word (such as *bisector*) and those words where the meaning is totally different (such as *product*, *function* and *identity*). There are also some borrowed words that students might first encounter in their technical mathematical sense before they encounter their wider meaning (such as *column* or *cumulative*). And technical words that become used beyond mathematics that may be first encountered outside of mathematics (such as *pyramid*) or subsequently (such as *common denominator*). There are borrowed words that might have multiple meanings beyond mathematics (such as *opposite*), and even technical words that might have multiple meanings within the mathematics classroom (such as *sum*).

Furthermore it is not just single words that are borrowed and used differently within mathematics. Word combinations can also have specific meanings in mathematics. These combinations might not be used beyond mathematics, and their mathematical meaning may be unrelated to the meanings attached to the individual words. For example consider possible meanings of the words *absolute* and *value* and contrast these with the meaning of *absolute value* within mathematics.

Given this background, it is perhaps unsurprising that many students and researchers liken the technical vocabulary of mathematics to a foreign language (Kotsopoulos, 2007). But more than this, each mathematical topic often has its own particular vocabulary almost like a set of regional dialects. This has important implications for learning vocabulary because curricula designed around topics potentially constrain even further the opportunities for 'recycling' that were discussed in the previous section. There is the risk that a student becomes a tourist, dipping instrumentally into the language of each region.

The role of the context in which a word is encountered was highlighted as significant in the language learning literature discussed in the previous section. In contrast to many other curriculum areas, for mathematics some contexts can be highly abstract. Whilst some mathematical words refer to objects than can be visualised or physically handled, many refer to abstract concepts, relationships or processes. It is those words that students first encounter in the mathematics classroom (and possibly only encounter in the mathematics classroom) that refer to these abstract concepts, relationships or processes in which we became particularly interested during the course of a wider study. From here, we shall refer to these as *technical words*. We posed the question:

How do we create meaningful contexts that enable our students to build an understanding of the meaning of technical words so that they can ultimately use them themselves in meaningful ways?

The study

The wider project involved working collaboratively with two mathematics departments in comprehensive secondary schools in the UK to explore ways of developing students' mathematical talk during lessons. Both schools had a whole school priority of developing students' literacy which had resulted in a range of generic suggestions for practice focusing on the teaching and learning of vocabulary. These included referring to a word's etymology, having keywords written on the board, and making connections to the experiences the students already have with the words. In this paper, we explore two meetings that focused on introducing or working on new vocabulary. The first example discussed the issues around generating a need for the technical word being introduced and the second example discussed the relationship and order of experiencing and defining the technical word.

Generating a need for words

The discussion around generating a need for the technical word arose from a video clip that one teacher shared of her class working on factors, multiples and prime numbers. The clip focused on a task, the design of which required students to use mathematical words. The class were split into two teams and a team was awarded a point when they gave a mathematical reason for their answer or included mathematical language in their answer. The mathematical purpose of this task was for students to identify that a group of numbers were all multiples of five. Throughout the clip both the teacher and the students used the phrase 'in the times table' comfortably and hence the need for the technical term *multiple* did not arise from the need to explain the mathematical properties. Instead the task design itself developed a need to use the technical term.

In contrast one group explored generating a need for a technical word through grouping mathematical objects, initially transformations of shapes and then algebraic equations and expressions. Here the need for the technical word arose from the need to give a collection of objects a label: translations, rotations, reflections or equations and expressions. This label is somewhat arbitrary but is connected to specific defining characteristics that distinguish objects within the collection from objects in a different collection.

Experiences and definitions

Following the example offered above around generating a label for equations and expressions, in a subsequent meeting a video was shared that illustrated that experience of each type of object alone was not enough for students to develop a meaning for the terms equation and expression. After a sequence of lessons simplifying expressions and solving equations, two of the teachers, one from each school, found that not all their students were able to define what was meant by equation or expression in a mathematically acceptable way. For the first teacher this was prompted by a student asking "if you have an expression and then you work it out, does it become an equation?" and for the second teacher it was from the responses to "can someone tell me how they know that that was an expression, not an identity or an equation" following a task asking the students to sort objects into these categories. To be clear, in both situations the students had a range of experiences working with both expressions and equations; they had simplified expressions and solved equations in previous lessons. Yet in the discussions that followed each of these questions the students were not clear as to what were the defining characteristics of an expression or an equation. Similarly in the meeting with one of the group of teachers the question arose as to whether the use of '=' is sufficient to describe an equation.

Concluding comments

There is an emphasis from the research on the need to teach vocabulary in authentic contexts and through making connections to students' prior knowledge or experiences. We have explored a range of types of vocabulary in mathematics: those that are also widely used in everyday situations or in other school subjects with a similar meaning; those which are used in everyday situations that have very different meanings; those where students are only likely to meet them in their mathematics lessons. In this paper, we have described our explorations with teachers regarding what might be meant by an authentic context in mathematics for these different types of words, and how these contexts may, or may not, support students in making connections.

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