

On learning number: The ‘-ty’ and ‘-teen’ confusion

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The aim of this paper is to draw out implications for the early learning of number from a detailed analysis of the work of one student, over an eight month period. The background to this work was our research interest in exploring the potential of approaching number without a focus on objects and cardinality, particularly for currently low-attaining students. We report on the difficulties that arose from a confusion of ‘-teen’ and ‘-ty’ numbers, for the student. We conclude that there is an argument for: (a) delaying work on ‘-teen’ numbers until students have worked on number structure more generally; (b) adopting a dual naming to regularise our naming system.

Keywords: early number; number naming; Gattegno chart; ordinality; cardinality

Introduction

We are ultimately interested in social justice and how we can support students who are in danger of being marginalised from the study of mathematics through their apparent repeated failure compared to their peers. In this article we want to question some assumptions of current early years number work in the UK and, in particular, practices that might, inadvertently, be disadvantaging some students. We raise questions through a close analysis of one student’s work, over time, on learning number. In the next section, we set our particular approach and our own assumptions around number learning. We then describe the methodology of the study before offering a selection of data and our analysis. We conclude by drawing out implications both for curriculum organisation and further study.

Learning number

There are (at least) two senses in which we can count. We can count objects (e.g., pointing as we speak) such that the last number we say tells us how many objects we have counted. This sense of counting is the basis of cardinality, which is the aspect of number to do with quantity and size. We can also simply count, as in a rhyme, and indeed many children enter school knowing the “number song” up to 10 or more without being able to reliably count and point simultaneously. The rhyme-like sense of counting is the basis of ordinality, which is the aspect of number to do with the order of the number sequence. The orthodoxy of primary school in the UK at the present time is that learning number must begin with the cardinal. Gelmen and Meck’s (1983) influential work in this area articulated an entirely cardinal conception of early number learning and we see this assumption persisting to the present day in national curricula (DfE, 2013) and depictions of learning trajectories or pathways (Cross, Woods & Schweingruber, 2009).

However, there is evidence that brings into question this cardinal focus. In particular, there are studies on primary aged children suggesting that skills in making judgments of ordinality correlate closely to overall mathematical attainment in children (Lyons, Price, Vaessen, Blomert & Ansari, 2014) and more closely than cardinality related skills. There is neuroscientific evidence (from studies of adults) that ordinal judgments of number symbols (e.g., ‘are the numbers 4, 7, 5 in order?’) draw on different patterns of brain activation than making cardinal judgments and, furthermore, that those ordinal-related patterns in the brain (unlike the cardinal ones) closely match brain patterns when engaged in more complex arithmetic tasks (Lyons and Beilock, 2011, 2013).

Coles and Sinclair (2017) draw, in part, on the evidence above combined with their own empirical data to suggest that place value is currently given far too much emphasis in the current curriculum. Place value, as it is depicted in curriculum documents (DfE, 2013) draws on a cardinal metaphor of learning number that is centrally about gaining an awareness of size. And yet, working in a more sophisticated manner with number (for example in performing algorithms) requires a suppression of awareness of size in order to work with number in a symbolic and game-like manner. Following Gattegno (1974) one way of characterising our position on learning number would be that, at the least, we want to ask the question: can we offer children, from the beginning, ways of working with number used by experts, rather than assuming they have to work in a more concrete manner that has to be left behind at a later date, in order to become successful at mathematics? And, judging by the estimated 25% of students in the UK who do not reach functional levels of numeracy by age 16 (OECD, 2016), we assume many students never leave behind the security of interpreting number in a cardinal manner, with counting as their ‘go to’ strategy for any arithmetic operation. In the work reported here, we have been exploring the potential for taking a more ordinal approach to learning number (not ignoring cardinality but offering a balance), particularly with those students who appear to be ‘falling behind’ their peers when offered a more exclusively cardinal focus in school. We are not the first to be suggesting cardinality is over-emphasised and this position is argued for strongly in Tahta (1998). Our interest in returning to questions around cardinality-ordinality has been, in part, the current neuroscience research in this area.

Methodology

The study was carried out in a primary school in the West of England serving an area of significant social deprivation. Learners were from a Year 2 class (6-7 year olds) and this paper focuses on one learner, Aidan (pseudonym), who was 6 years old and judged to be the ‘weakest’ in the year group in that school. Thirteen half-hour sessions were video recorded of one of the authors (Alf) working with Aidan individually. These sessions were carried out in a different room to the normal classroom. The sessions were recorded on a laptop webcam and Aidan could see on the screen the video as it was being recorded. He was consulted as to whether he was happy with what was in view, as were his parents, teacher and headteacher. We have software to translate the video into a ‘comic’ version for showing at conferences.

Analysis of the video comprised a number of stages. On the enactivist principle of equifinality (Coles, 2015), we began with the final video, as this is assumed to be the one in which patterns of action and interaction will be most marked. At this first stage both authors and a third collaborator, Nathalie Sinclair,

watched the last video on our own and met (virtually) to discuss patterns and what we had noticed. Through this process we identified common themes, including the mis-naming of numbers (particularly 20 and 12) and Aidan's shifts in attention. At the second stage of analysis, following an enactivist methodological principle (Reid and Mgombelo, 2015; Coles, 2015), we then looked (individually) through all the videos, with these themes in mind, noting passages of relevance to those themes. In the third stage, meeting via Skype, attending to the passages we had each noted through all the videos, further patterns became apparent. This resulted in the following elaborated themes being developed:

- The complexity of what Aidan was dealing with at the same time. There were different issues he was working on, such as: how to write the digits; how to say each digit; learning names for numbers bigger than 20, including hundreds; learning how to write bigger numbers; learning to say one more or less than a given number; learning to count in fives and tens.
- The non-linearity of Aidan's learning, where he would appear to be confident with something one session, unsure the next session and sure again another session later on.
- Where Aidan appeared to be attending at particular points in time.
- Particular numbers which were problematic for Aidan, such as 20, the *-teens*, the *-ty* names, and the digits 8 and 9.
- Aspects of learning where Aidan showed success and development.

A fourth stage of analysis then focused on one of the above themes in turn where detailed transcription of relevant passages was made and reflective comments about those incidents noted. This resulted in an awareness that 20 played a pivotal role comparing the relative success Aidan had with numbers greater than 20 and the difficulty he continued to have with many of the numbers less than 20. In this paper, we focus on the confusion between *-ty* and *-teen* numbers and draw on transcripts from the video recordings of sessions in order to exemplify the range of ways this confusion manifested itself.

Pedagogic approach

The sessions were built around the idea of working on relationships between numbers rather than the relationship between numbers and objects. As such the issue was not "How many?" but about developing a structure for number names and the order in which they may be said. The main visual tools used were the Gattegno Tens Chart and the App TouchCounts. This paper focuses on the use of the Tens Chart but sessions typically shifted between the two resources.

The Tens Chart used with Aidan was as in Figure 1. The structure of the chart is such that there are *digit* names which run down each column *one, two three, ... nine*. So, down the sixth column there is *six hundredths, six tenths, six, six-ty, six hundred, six thousand, six-ty thousand* with the word *six* appearing in all the names. Horizontally, row starting 100 has the word *hundred* appearing within all the names. Thus, the columns contain the *digit* words and the rows contain the *value* names. A number such as 463 would be shown by pointing to 400, then 60 and then 3.

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000

Figure 1: The Gattegno Tens Chart

The activities carried out using the chart with Aidan involved a mixture of: Alf pointing to the chart and asking the name; Alf saying the name and asking Aidan to point to the chart; counting in 1s, 5s and 10s with Aidan sometimes saying the names and sometimes pointing to the chart; Aidan saying one more or one less than a number pointed to; and writing numbers. The idea was to develop a sense of number through the relationships exhibited within the structure of the chart.

Confusing ‘-ty’ and ‘-teen’

Aidan had several difficulties with numbers either side of 20 when getting confused between the *-teen* and *-ty* endings. The difference between the word *fourteen* and *forty* can be subtle even when attending to it. At times Alf said during sessions that he was not sure whether Aidan had said *fourteen* or *forty*, for example, and asked him to repeat it. When analysing the videos, we often had to play a section repeatedly to be sure of which of the two Aidan had said. Some typical examples of when Aidan said the incorrect ending were:

- Saying “ninety” for 18 [session 11; 25th May]
- Alf asked “What does fourteen look like?” and Aidan pointed to 40 on the tens chart [session 12; 25th May 2016]
- Alf pointing to 10 & 6 on the chart and Aidan saying “sixty” [session 13; 12th July 2016].

The first example contained a combination of a continued confusion he had with saying “nine” for 8 and vice-versa along with the question of whether to say *-teen* or *-ty*. With this and the last example, Aidan said the incorrect word himself and so revealed his own uncertainty of when to use the *-ty* or *-teen* ending. However, the first example could be a case where he did not hear the ending clearly, as indeed we had when analysing the video in places. Another such possibility was when Alf asked “After nineteen what do we get?” to which Aidan replied “one hundred!” [session 4; 6th October 2015]. This could be a mis-hearing of *nineteen* as *ninety*, and him having worked previously on counting in tens, to then say “one hundred”. These and other occasions led to the *-teen* numbers being avoided for most of the time as they proved to be problematic for Aidan.

Higher numbers

The difficulty of differentiating between *-ty* and *-teen* meant that Aidan sometimes said the higher *-teen* numbers incorrectly, ending with *-ty* instead of *-teen*. Although we expect that each affected the way Aidan said the other, the key problem is that the *-teen* numbers are the irregular ones and as such cause problems with the more regular *-ty* names. Had there not been that irregularity in the language, this confusion would be less likely to occur. The influence of language was also noticed within the *-ty* names themselves. For example, Alf pointed to several numbers in the *-ty* row and asked Aidan to say those numbers [session 3; 29th September 2015]. Aidan was successful with all the regular sounding names (forty, sixty, eighty) but made errors with the irregular names. For example, when Alf said “fifty” Aidan pointed to 80; for “thirty he pointed to 70. However, when Alf re-worded these to a regular equivalent (“five-ty” and “three-ty”) Aidan was successful.

Over the sessions, Aidan gained increasing confidence with saying and working with numbers greater than 20, whilst the *-teen* numbers remained problematic. During session 11 [9th March 2016] Aidan was counting in 10s, pointing at the chart at the same time. From 10 to 90 he was tapping along the tens’ row. He then said “a hundred” and pointed to 100. He then said “a hundred and two/ten [unclear]” and pointed to the next number in the hundreds’ row, 200. Alf then showed him how to point to *a hundred and ten* (100 and then 10) and Aidan was able to continue to 190. He was then unsure how to proceed. Alf pointed to 200 and Aidan said “Two... two-ty... two hundred”. He then was able to continue again to 290. He needed reminding to continue with 300 but was then able to count to 490, pointing correctly as he did so. He showed increasing fluency with the counting and the pointing as he did so.

Another example of increased confidence with numbers greater than 20, whilst the *-teen* numbers remained problematic, was him successfully counting in 2s up to 98 [session 12; 25th May 2016]. A few minutes later he had written 450 successfully from hearing it said but, shortly after that, wrote 40 when prompted to write *fourteen*.

Implications

The UK National Curriculum (DfE, 2017) states that “children [should] count reliably with numbers from 1 to 20” (p. 12) in the Early Years Foundation Stage. For Year 1 it states that learners should “read and write numbers from 1 to 20 in numerals and words” (DfE, 2013, p. 6). There is a strong emphasis on learning the numbers from 1 to 20 before higher numbers. The numbers from 1 to 9 are necessary as they form the basis of the number system. However, the number names for 11-20 are all irregular, not only in the use of the name *-teen* but in the fact that the units word is said first before the *-teen* ending. This irregularity, along with the aural similarity of the *-ty* and *-teen* endings does not offer opportunities for learners to come to know the general structure of how number names are said and can affect the later learning of the *-ty* names. Ironically, the learning of the number names for numbers above 20 is easier as there is greater regularity and this regularity can help learners gain a sense of the place value structure of number through the number names themselves.

Aidan was gaining increasing confidence with larger numbers whilst still having difficulty with the numbers from 11-19. We put forward the argument that it is pedagogically more sensible to either use higher numbers before working on the *-teen* numbers. Alternatively, regular language for 11-19, such as *one-ty one*, *one-ty two*,

one-ty three, etc., would enable learners to notice the structure in the language and not become confused between 14 and 40.

To return to the social justice theme with which we began, we do not view Aidan's difficulties with number as a sign of lack of intelligence and indeed we consistently observed him able to engage in sophisticated work, e.g., linked to pattern. We are concerned to offer Aidan ways of accessing number work, making use of his evident powers to make abstractions, so that he does not end up cut off from opportunities in society and does not come to believe himself as undeserving of those opportunities partly as a result of his development of number sense not following 'normal' patterns.

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