Singapore bar models appear to be the answer, but what then was the question?

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The use of Singapore bar models has been a topic of great interest in recent years in response to the success of Singaporean students in international mathematics tests. However, results for English students have improved without the use of these diagrams. This research sought to identify which strategies were used by four Year 6 pupils who have not been taught how to use bar models to solve word problems to help ascertain the need for a new initiative.

Research was conducted in the summer of 2016 adopting a mixed methods approach to gather data which included artefact analysis, observation and interviews. The research findings suggested that pupils had a useful repertoire of problem solving strategies and could successfully solve a range of worded problems and create a range of diagrams for a variety of purposes to suit their needs for each problem.

Keywords: Singapore bar models; problem solving; strategy choice

Context and research approach

Each national curricula in England has reflected the conclusions of the Cockcroft report which stated that "mathematics at all levels should include opportunities for problem solving" (1982; paragraph 249) and that problem-solving ability lies "at the heart of mathematics" (p.73). The most recent review of the national curriculum also reflected considerable government interest in how mathematics is taught in East Asian territories such as Shanghai and Singapore (James, Oates, Pollard, & Wiliam, 2011).

Much research has been undertaken into mathematics teaching in Singapore and the jurisdiction's cohesive organising framework, clear focus and careful use of multiple models were found to be key (Ginsburg, Leinwand, Anstrom, & Pollock, 2005, Ruddock & Sainsbury, 2008). Diezmann (2000), and Mudaly (2012) identified the importance of the diagrams used in Singapore as they provide a visual representation of mental processes. Diagram use has many benefits: they convey the structures and relationships within a problem in a meaningful way (Winn, 1987); they help to communicate their solutions to others (Cai & Lester, 2005); they deepen pupils' mathematical understanding as the drawing encompasses three different modes of representation: text, pictorial and symbolic (Ng & Lee, 2009).

In Singapore, a particular variant of diagram is used: the bar model. Hoven & Garelick (2007) state that bar models are a particular example of the 'draw a picture' problem-solving strategy. The creation of the bar model diagram represents visually the information that the learner already knows and what is unknown and helps to identify how that information can be used to solve the problem. Another advantage of bar models is that their flexibility and versatility mean that pupils can use this one representation consistently as they will know what kind of picture to draw (Hoven & Garelick, 2007).

The research undertaken in this small research project sought to identify the strategies four Year 6 pupils used when presented with a set of word problems and to consider the following question: *What strategies do pupils use to solve word problems?* The research was undertaken in one session in an urban primary school in Hampshire. The pupils chosen were judged to be working in line with national expectations. The four children were seen at the same time, each was presented with a selection of questions to rank in order of difficulty (really easy, quite easy, quite hard, really hard). Pupils were then invited to solve some of the problems and explain their solutions. The session was video recorded and field notes were taken. The research adopted Plowright's (2012) mixed methods model and comprised observations (in the moment and in review via video recordings), semi-structured interviews (discussion of the solutions) and artefact analysis (the pupils' written notes and researcher field notes).

Pupils were given routine problems, the type of problem most commonly seen in mathematics tests. Kantowski (1977), defined them as problems where the appropriate application of an algorithm "will certainly lead to a solution" (p.163). There are many problem solving frameworks to solve routine problems which frequently reflect Polya's (1945) four principles: understand the problem; devise a plan; carry out the plan; review or extend. However, according to Hegarty, Mayer & Monk (1995) some pupils are not able to apply given rules appropriately or develop their own. Instead, unsuccessful problem solvers latch onto a key word, such as 'more than' which triggers in the mind of the child an addition calculation. This phenomenon is termed by Stigler, Lee & Stevenson (1990) as "compute first and think later"(p.15) and by Littlefield & Rieser (1993) as number grabbing. In contrast, successful problem solvers are able to translate the words into a visual representation of the situation presented in the problem (Edens & Potter, 2008).

However, Diezmann (2000) pointed out the usefulness of a diagram is determined by whether the linguistic and numeric information has been successfully decoded. Cheng (2004) found that children's ability to solve problems was affected by the position of the 'knowns' and 'unknowns' in a calculation. Children generally only found a question 'easy' if the 'unknown' was the result of a calculation. However, much of this research has been with young children (approximately 7 years of age) and has not been undertaken in England. It was therefore of interest to see how applicable their findings were to English Year 6 children in 2016.

Findings and analysis

The problems were categorized using a combination of Riley & Greeno's (1988) and Cheng's (2004) classifications on the complexity of word problems (table 1). The research group thought that most of the questions were 'easy' even if the categorization may suggest otherwise with only Child C suggesting any of the questions were hard. It is perhaps predictable that a group of children who have been thoroughly prepared for their end of key stage tests would find the questions relatively straightforward. What was of greater interest was how diagrams were used by different pupils for different purposes.

Question	Question Type	Cheng grading	Really Hard	Quite Hard	Quite Easy	Really Easy
1. Robert had 52 X-Box computer games. He sold 12 of the better ones, gave away [illegible] How many games has Robert got left?	Change, result unknown	Easiest				A, C, B
2. The coldest temperature recorded on the school thermometer this year was – 6 degrees Centigrade. The highest temperature was 25 degrees Centigrade. What is the difference between the two temperatures?	Change, result unknown Change, start unknown	Easiest Hardest			D	
3. Robert had some X-box games. He sold 15 of them, gave away 17 and threw away 3 that weren't very good. How many games did Robert start with?	Change, start unknown	Hardest		C	A, D,	
4. There is 425g of flour in a bowl. Some sugar is added, the mixture now weighs 517g, how much sugar was added?	Change, change unknown	Hardest			C, D	A,
5. Justin's birthday party was a bit crazy last year so this year he is only having seven people at his party. This is four fewer than last year. How many people were at the crazy party last year?	Change, start unknown	Hardest				D, C, B
6. Together Greg and Polly have 542 stickers. Polly has 439 stickers. How many does Greg have?	Combine, subset unknown	Hardest			C, B	
7. In Keenan's toy bin there are 24 red blocks. There are 13 more yellow blocks than red blocks. There are also 14 more blue blocks than red blocks. How many blocks are there in all?	Combine, result unknown	Easiest			A, B, D	C
8. Beverley and Sally collect stickers. Beverly has collected 235 stickers. Sally has 48 more stickers than Beverley. How many stickers does Sally have?	Combine, result unknown	Easiest			A, D, B	

Table 1: Pupils' categorisation of word problems

Use of a taught diagram

Only child D attempted to solve the temperature question which she thought was 'quite easy', although this is not reflected in her explanation of the question:

> LC: Ok, so, if you had to write that as a number sentence, what is it that you know already and what is it that you are trying to find out.
>
> Child D: ummmmm, I'm trying to find out what the difference is, ummmm, and the, the, the coldest temperature was minus 6 and the highest temperature was 25 so you've got to, ummmm, like find the difference
>
> LC: Ok, so it's find the difference, ok, so how would you find the difference between minus 6 and 25?
>
> Child D: subtract 6

Child D's comments reflect Ng et al findings that some unsuccessful problem solvers misunderstand one text element resulting in inappropriate solutions. Child D had correctly identified that the question involved negative numbers and that difference was a structure of subtraction. However, the inclusion of a negative number was problematic. She appears to have 'grabbed' the word 'difference' and translated this into 'subtract 6' rather than subtracting -6. What was of interest for this study was that her use of a diagram (an unstructured number line) (Figure 1) helped her to recognize the relationship between the numbers in the problem, understand the problem, devise a plan and carry out the plan.

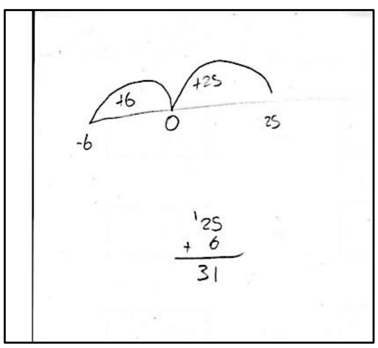

Figure 1: Child D's use of a non-structured number line

The diagram represented what was known (the values -6 and 25) and what was unknown (the difference between them). It also helped her to calculate the difference and explain her solution and check whether her answer was correct.
In this case Child D had appropriately applied a type of diagram that she had been taught how to use, to reach a successful solution. The use of a diagram was certainly significant as it helped her to recognize that her initial interpretation of the problem and selection of strategy was incorrect.

Use of a created diagram

Child A also produced a diagram. When solving Keenan's Toy Bin (question 7) Child A categorized the problem as 'Quite Easy' and his explanation indicated that he could recognize the relationship between the numbers and the context.

> Child A: because there was three colours and they've all, they're all, the yellow blocks and the blue blocks are higher than 24 so I do 24 times 3 because that was the amount of red blocks and then I times then I plus 13 which would be sss… eighty sss five then I plus 13 which will be 99
> LC: so why are you adding the 13 and the 14?
> Child A: because if I didn't it would be 72 and that's not the amount that there is because I took 13 and 14 off of the original numbers of yellow blocks and blue blocks to make it 24

Child A had understood the relationship between the numbers and the context of the toy bin. He did not need to draw a diagram to solve the problem however he produced a diagram to support his explanation of his solution. The diagram produced was idiosyncratic and created to reflect his interpretation of the problem and explanation. It mirrored the multiplicative thinking he had explained (Figure 2). He drew three columns (or bars) the first showing 24, the second showing 24 and 13 more, the second showing 24 and 14 more. Echoing Winn's (1987) findings, the diagram produced by Child A represents the relationships between the numbers within the problem and represents both the structure of the problem and its solution.

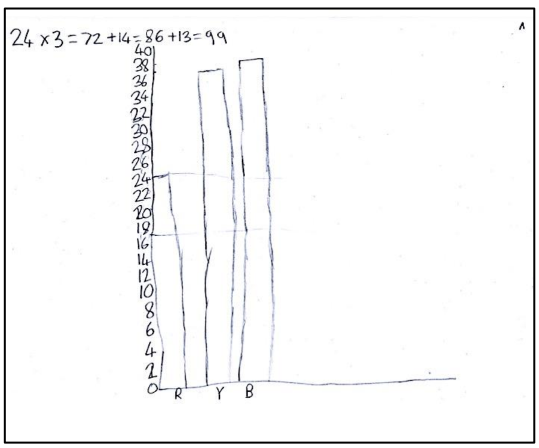

Figure 2: Child A's diagram to support his explanation of Keenan's toy bin question

His diagram and explanation suggested that Child A fully understood the question and his solution and had not applied only learned procedures but had developed his own diagrammatic representation to support his explanation of his solution.

Child A: so I worked out that the highest amount of the colour is 28 because 13 plus 14 is 18, so I went up to 40 because it's an even number so then I went up in 2s because there wasn't enough room for ones, and umm and did like bars going up to where the certain number is
LC: uh huh
Child A: so …
LC: so the red bar goes up to 24
Child A: and then the yellow bar goes up to thirty … seven and the blue bar goes up to thirty eight
LC: Can you tell me the words in this number problem that told you the yellow bar was going to go up to 37?
Child A: It wasn't really any words that said it, you just, you had to read between the lines and work out that 24 plus 13 is 37

Conclusions

The solutions provided by Child A and Child D support the view that diagrams can be powerful and effective tools as part of the problem solving process. This study also suggests that diagrams are used at different stages of the process for different purposes. Child D's diagram helped her understand, solve and review the problem, whilst for Child A the diagram was only useful for reviewing his solution. That each of the pupils successfully solved the problems provided and could draw upon, adapt and create appropriate diagrams could support a line of argument that bar models are not needed as the pupils already had a repertoire of strategies. However, even within this very small study questions emerge about the efficiency of the strategies used and that there was little consistency about the approaches used by the pupils. This could suggest that these pupils could benefit from learning how to deploy a consistent and flexible model such as the bar model in order to improve efficiency and increase equality of opportunity to succeed. This research suggests there is still much to learn about the strategies that pupils use when solving problems. Further research will involve comparing the solutions of pupils who have been taught to use bar models to solutions of pupils who have not been exposed to this model.

References

Cai, J., & Lester, F.K. (2005). Solution representations and pedagogical representations in Chinese and US classrooms. *The Journal of Mathematical Behavior*, *24*(3), 221-237.

Cheng, P. C. (2004). Why diagrams are (sometimes) six times easier than words: benefits beyond locational indexing. In A. Blackwell, K. Marriott, & A. Shimojima (Eds.) *Diagrammatic Representation and Inference* (pp.242-254). Berlin Heidelberg: Springer.

Cockcroft, W. H. (1982). Mathematics counts: Report of the committee of inquiry into the teaching of mathematics in schools. *London: Her Majesty Stationery Office*.

Diezmann, C. D. (2000).The difficulties students experience in generating diagrams for novel problems. In T. Nakahara, & M. Koyama, (Eds.) *Proceedings 25th Annual Conference of the International Group for the Psychology of Mathematics Education, (*pp.241-248), Hiroshima, Japan.

Edens, K., & Potter, E. (2008). How students "unpack" the structure of a word problem: Graphic representations and problem solving. *School Science and Mathematics, 108*(5), 84-196.

Ginsburg, A., Leinwand, S., Anstrom, T., & Pollock, E. (2005). *What the United States can learn from Singapore's world-class mathematics system (and what Singapore can learn from the United States).* Washington, DC: American Institutes for Research.

Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of educational psychology*, *87*(1), 18.

Hoven, J., & Garelick, B. (2007). Singapore math: Simple or complex? *Educational Leadership*, *65*(3), 28-31.

James, M., Oates, T., Pollard, A., & Wiliam, D. (2011). *The framework for the National Curriculum: A report by the expert panel for the National Curriculum review*. London: DfE

Kantowski, M.G. (1977). Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education*, *8*(3), 163-180

Littlefield, J., & Reiser, J.J. (1993). Semantic features of similarity and children's strategies for identifying relevant information in mathematical story problems. *Cognition and Instruction*, *11*, 133-188.

Mudaly, V. (2012). Diagrams in mathematics: To draw or not to draw? *Perspectives in Education*, *30*(2), 22-31

Ng, S.F., & Lee, K. (2005). How primary five pupils use the model method to solve word problems. *The Mathematics Educator*, *9*(1), 60-83.

Plowright, D. (2012). *Using mixed methods frameworks for an integrated methodology*. London: Sage

Polya, G. (1945). *How to solve it*. Princeton: Princeton University Press

Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction*, *5*(1), 49-101.

Ruddock, G., & Sainsbury, M. (2008). Comparison of the core primary curriculum in England to those of other high performing countries – Research report DCFS-RW048. London: Department for Children Schools and Families

Stigler, J.W., Lee, S-Y., & Stevenson, H.W. (1990). *Mathematical knowledge of Japanese, Chinese, and American elementary school children.* Reston, VA: National Council of Teachers of Mathematics.

Winn, B. (1987). Charts, graphs, and diagrams in educational materials. In D.M. Willows & H. Houghton (Eds.) *The psychology of illustration*, (pp.152-198). New York: Springer.