Evolution and comparison of mathematical abilities in China's mathematical curriculum standard for elementary schools

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The teaching of mathematics in China's elementary schools is based on Mathematical Curriculum Standard, which is a programmatic document issued by the national education authorities. In the standard, specific requirements have been provided for the mathematical abilities of elementary school students, mainly including such abilities as the operational ability, spatial imagination, logical thinking and problem-solving ability. In this paper, on the one hand, we have roughly compared and summarized the different requirements for mathematical abilities in the Primary Mathematics Syllabus and the Mathematical Curriculum Standard issued by the national education authorities in the past years and explained what mathematical abilities are and why we should develop students' mathematical skills; on the other hand, we have introduced how to train students' mathematical abilities through specific mathematical examples.

Keywords: Mathematical Curriculum Standard; operational ability; spatial imagination ability; ability of logical thinking; problem-solving ability

Introduction

With the rapid development of the modern information technology, mathematics will be more widely used in various aspects of our social production and daily life. In the 21st century, society has more and more diversified demands for talents. The Compulsory Education Mathematical Curriculum Standard (2011) proposed: "The mathematical literacy is the basic literacy that every citizen should have in the modern society" and "Mathematical education should serve all students and everyone can get good education in mathematics and different people can get diversified development in mathematics." (Ministry of Education of the People's Republic of China, 2011, p.1) This requires that mathematical education should not only transfer mathematical knowledge or train skills, but more importantly cultivate mathematical abilities, and only in this way can we really improve students' mathematical literacy and promote the all-round development of students in mathematics. In this paper, we'll discuss three problems as follows: What are primary mathematical abilities? Why should we cultivate mathematical abilities? How can we train students in these mathematical abilities?

Connotation of mathematical abilities in primary mathematics syllabus and mathematical curriculum standard

As a teaching instructional document issued by the national education authorities, the

Mathematical Curriculum Standard (hereinafter referred to as the 'Standard') has clarified the importance and guiding ideology of mathematical teaching and provided

Table 1: Comparison of the requirements for mathematical abilities of elementary school students in each syllabus

Name	Operational ability	Space imagination ability	Thinking ability	Problem-solving ability
Syllabus (1952)	It requires complete correctness and accuracy in written and oral operations.	No explicit statement	Cultivate and develop children's logical thinking	Enable children to acquire the skills for practical use of the learned knowledge
Syllabus (1956)	Able to complete correct and rapid operations of integers	Develop students' sense of space	No explicit statement	Able to use the learned knowledge, skills and techniques to solve arithmetic problems
Syllabus (1963)	Cultivate the ability to correctly and rapidly complete the four arithmetic operations	Cultivate students' preliminary sense of space	Cultivate students' preliminary ability of logical thinking	Able to solve practical questions correctly
Syllabus (1978)	Able to correctly and rapidly complete the four major operations of integers, decimals and fractions	Cultivate students' preliminary sense of space	Cultivate students' preliminary ability of logical thinking	Able to use the learned knowledge to solve the simple practical problems in daily life and production
Syllabus (1986)	Able to correctly and rapidly complete the four major operations of integers, decimals and fractions	Cultivate students' preliminary sense of space	Cultivate students' preliminary ability of logical reasoning	Able to use the learned knowledge to solve the simple practical problems in daily life and production
Syllabus (1992)	Able to complete the four major operations of integers, decimals and fractions	Cultivate students' preliminary sense of space	Cultivate students' preliminary ability of logical thinking	Able to use the learned knowledge to solve the simple practical problems
Syllabus (2000)	Able to complete the four major operations of integers, decimals and fractions	Cultivate students' preliminary sense of space	Cultivate students' preliminary ability of logical thinking	Able to explore and solve the simple practical problems
Curriculum standard (2001)	Able to express a number in different ways; able to understand the quantitative relation and change rule indicated in symbols; able to complete transformation between symbols	Establish students' preliminary sense of space	Develop students' imaginational thinking	Solve the problems in daily life and other subjects for developing the practical ability
Curriculum standard (2011)	Establish the sense of numbers and symbol consciousness for initial formation of the operation ability	Establish the space concept for initial formation of the geometric intuition	Develop the imaginational and abstract thinking	Able to find out, put forward, analyze and solve problems

the objective requirements, teaching contents and teaching instructions of mathematical teaching as well as the teaching requirements and contents for each grade, so it is the standard to master the teaching schedule and check the teaching

quality. In view of the authority and guidance of the Standard in teaching, let's study the requirements in the elementary Mathematical Curriculum Standard for students' mathematical abilities.

From the concrete analysis of the purposes or objectives and related contents in China's nine official syllabuses or curriculum standards, we have found that no syllabus or standard has expressly stated the concept of elementary school students' 'mathematical abilities' or further clearly put forward the contents of mathematical abilities for elementary school students, but we have concluded the changing trend in thought regarding mathematical abilities in such syllabuses or curriculum standards by means of the research on the changes of the objectives for elementary school mathematical teaching or curriculum standards.

In China's educational circles, mathematical abilities have been more consistently divided into mathematical operational ability, space concept (or space imagination ability), ability of logical thinking and problem-solving ability (namely, the 'three major abilities' + the ability to solve the actual problems).

Table 1 has shown us the requirements for elementary school students' operational ability, space imagination ability, ability of logical thinking and problem-solving ability in three syllabuses, from which comparison we can see the changing trends. Here follows the analysis of the requirements for elementary school students' mathematical abilities in the *Standard* from the perspective of the four major abilities.

(1) Operational ability

The second column in Table 1 states the operational ability. We can see that the requirements are correctness and rapidness for operational ability in the Syllabuses before 2000. The teaching objectives require the 'correct and rapid four operations of integers, decimals and fractions' in terms of the operational ability. Besides the necessary requirement for 'correctness' in operation, the requirement of 'rapidness' may be easily misinterpreted as the sign of the strength and weakness of operational ability, but such a way of training has affected students' interest in mathematics and may let students think mathematics is boring. As shown in Figure 1, an operation card for elementary Grade 2-3 students (usually 7-9 years old) requires that the students should correctly work out the given questions within the prescribed time. Some would argue that it is not significant to mechanically train the correctness and rapidness of operation in the information age. In the curriculum standards (2001 and 2011), operational ability has been redefined by further requiring number sense and symbolic consciousness, namely, to cultivate students' sense of numbers and symbols. To this end, we have designed such exercises as follows to enhance students' symbolic consciousness. As shown in Figure 2, it requires that the students should be able to list the formula through figures or symbols in order to cultivate their sense of symbols and their consciousness to change abstract things into numbers. In mathematical education, we should train students to complete the transformation from daily language to mathematical language and then to symbolic language, so that the students could establish their symbolic consciousness, accurately express their mathematical thought and avoid the complexity, lengthiness or vagueness of daily language. Such a learning process, together with specific situations, can let students understand the needs for generating mathematical symbols and realize that only symbols can be used to clearly and conveniently express the quantity and the change rules in such specific situations.

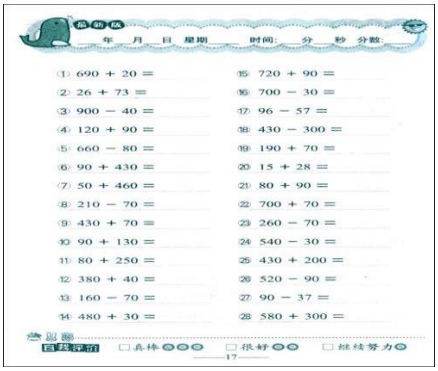

Figure 1: Operation card for elementary students

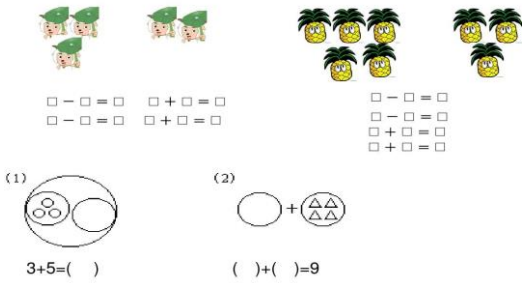

Figure 2: Exercise for training of operational ability

(2) Spatial imagination ability

From the third column of Table 1, we can see that the original requirement is relatively low for spatial thinking, only to cultivate students' initial space concept. Students should be able to identify the basic graphics (such as circle, rectangular and trapezoid) and remember the formulas for the areas of the circle and rectangular, etc. In 2011, however, the curriculum standard has heightened the requirement for the ability of spatial thinking. Such requirements have been specified as follows: 'able to abstract geometric figures based on object characteristics'; 'visualize the actual objects described in the geometrical diagrams'; 'imagine the object orientation and their positional relations'; 'describe the movement and changes of graphics'; 'draw the graphs according to the language descriptions and form the geometric intuition', namely, use graphics for problem descriptions and analysis (Wang, Q., 1991).

For example, we use the exercises in Figure 3 to train students' spatial imagination ability.

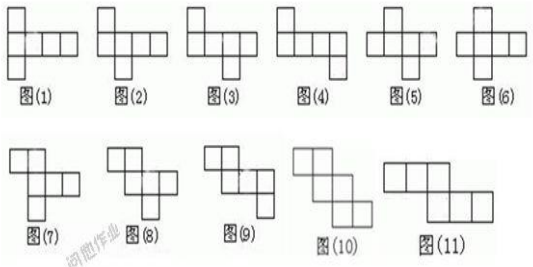

Figure 3: Training of spatial imagination

The question in Figure 3 is as follows: Which of the images can be folded into a cube? This kind of exercise can allow students to form visual images in their imagination and thus train their abilities for spatial imagination and geometric intuition. Moreover, students can complete folding on their own, train their geometrical intuitions in practice and simultaneously verify their ideas through practice.

The teaching can be designed as follows:

1. Firstly, let students guess the results by imagination and write down such results. The students can manage it individually or in groups.
2. Make physical objects, so that students could practically observe the results.
3. Verify their results in contrast to their previous records.

(3) Ability of logical thinking

From the fourth column of Table 1, we can see that, in regard to the ability of thinking, the syllabuses before 2001 just required cultivation of students' preliminary ability of logical thinking, but now the requirements have put emphasis on the ability of logical thinking and also on cultivation of students' abilities in imaginal and

abstract thinking. For example, for students in the second phase of studying (Grade 3-6), we can use such examples as follows to train students' abilities of imaginal and abstract thinking.

Example: As shown in Figure 4, some buttons are scattered on the table. Please classify such buttons. Let's think: How to determine the criteria of classification? According to such criteria, how many categories can we divide such buttons into?

Then get down to specific operations and record the results by means of words, drawings or forms (Wang, Q., 1991).

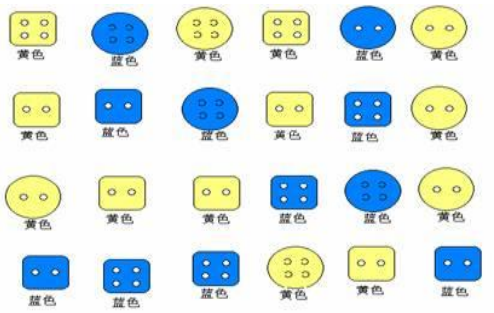

Figure 4: Training of logical thinking

The teaching can be designed as follows:

1. The teacher can put forward questions, guiding students to discuss the standards for classification and enlighten them to think as follows: Firstly, focus on one indicator as the classification standard, for example, their colors. On this basis, further focus on two indicators as the classification standard, such as their colors and shapes; finally, focus on their colors, shapes and number of holes. In this way, we can avoid confusion.
2. Group the students according to the classification standards determined after discussion and guide them to complete the practical operations and finalize the counting in cooperation; let all groups present their results.
3. The teacher can organize the students to report the statistical results, guide them to make remarks and help them organize their thoughts.

(4) Ability to solve practical problems

Mathematical education is directly to target the training of students' ability to solve problems or solve practical problems. From the last column of Table 1, it can be seen that the curriculum standards before 2001 only require students' ability to solve practical problems as the curriculum objectives, but now it is required that the students should be able to find out, put forward, analyze and solve problems. The practical question given to students before 2001 always has one unique answer subject to a fixed pattern for solution. Let's see such an example as follows for the students in the second phase of studying:

In a room, the four-legged chairs and three-legged stools add up to 16 chairs, and if they have 60 legs in all, then how many chairs and stools respectively? (Wang, Q., 1991)

We can guide students to try exploring the law of solving the problem, so that they could feel it is an effective way to explore mathematical problems.

The teaching can be designed as follows:

1. List as follows:

Number of chairs each	Number of stools each	Total number of legs each
16	*0*	*4*×16=64
15	*1*	*4*×15+3×1=63
14	*2*	*4*×14+3×2=62
...	...	...

Continue and you can see that exactly 60 legs are available when there are 12 chairs and 4 stools respectively.

2. From the table above, the students can be enlightened to think: Once one chair is reduced, a stool will be added and the total number of legs should be reduced by 4-3 = 1. When the total number of legs is 60, the number of chairs that need to be reduced should be 64-60 = 4, then the number of chairs should be 16-4 = 12 and then the number of stools should be 0 + 4 = 4. Finally, let's verify it as follows: 12×4+3×4=60, which is true. Of course, you can also think about it from the number changes of stools: Once one stool is reduced, a chair will be added and the total number of legs should be increased by 4-3 = 1.

3. For those students with more competence, the teacher can encourage them to discuss the problem of the "chickens and rabbits in the same cage" or further use letters to replace the numbers of chairs and stools for achieving a model to work out the total number of legs and initially forming the thought of mathematical modeling.

Conclusion

We have summarized the progressive course of the understanding and requirements for mathematical abilities from four major aspects of mathematical syllabuses and curriculum standards, explained the connotation of mathematical abilities and explained why it is necessary to train mathematical abilities and how to develop such abilities. The new curriculum standard points out: "The elementary mathematical education should make students becoming stronger in knowledge and intelligence" and "The intelligence development and ability cultivation should be targeted throughout the teaching of each grade."(Ministry of Education of the People's Republic of China, 2011,p.3) It is a basic task of modern teaching to develop students' mathematical abilities, which should be some ideas and methods established through mathematical learning as well as such abilities to deal with and solve problems by means of mathematical thinking. The core of mathematical abilities is a foundation of mathematical ideas and methods achieved based on such specific mathematical knowledge and skills as numeracy, calculation, measurement and statistics, featuring sufficient comprehensiveness, perfect integrity and enough durability.

Acknowledgements

The authors thank the faculty of education and communities of the University of Wales Trinity Saint David for their invitation and hospitality. This research is supported by the leading scientific research project of Teachers' College of Beijing Union University under grant numbers 12213991724010201.The first author also thanks for the encouragement of the corresponding author (Jian Zhang).

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