

Replication of a British study in Brazil: how do teachers describe mathematical tasks?

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British researchers recently used factor analysis to investigate how mathematics teachers qualify classroom tasks. Based on 360 answers to an online survey, they identified seven factors underlying a comprehensive set of adjectives and expressions commonly used to describe mathematical tasks. This part of their study was replicated by the authors of this paper with 415 Brazilian mathematics teachers and the results presented many similarities and some interesting differences when compared to the British context. For instance, our analysis also suggested seven factors, but only four were similar to factors identified in the original study. In this paper, we will discuss the main results of both studies, compare them and present some possible explanations for the differences.

Keywords: mathematical tasks; factor analysis; replication; Brazil

Initial motivation

There is a huge variety of adjectives being used by teachers, policy makers and researchers to describe mathematical tasks, such as rich, open-ended and exploratory, and little agreement in terms of their meanings (Foster & Inglis, 2017). The term rich, for instance, is widely used in English official documents related to the national curriculum and little or no definition, explanation of its meaning or illustrative example is given. If one recognizes that the nature of the tasks used in classrooms is important for the learning outcomes (Shimizu, Kaur, Huang, & Clarke, 2010), it is obvious that a shared understanding of what is meant by these adjectives, when they are used to qualify tasks, is relevant.

To approach this issue, Foster and Inglis (2017) decided to investigate how teachers use the adjectives instead of trying to define them. Their approach relies on the statistical technique called factor analysis, a tool that seeks to group a large number of variables (the adjectives) into a smaller set of factors while at the same time accounting for as much of the original variance as possible. It works by searching the matrix of correlations between the original variables for subsets of these variables that are all strongly intercorrelated. If such a subset exists, then they can be thought of as representing a single underlying construct, called factor. Details regarding the use of the statistical technique utilized will be omitted in this paper due to space restrictions, but it followed the suggestions made by Bandalos and Finney (2010).

Foster and Inglis (2017) created a comprehensive list consisting of 84 adjectives used to describe mathematical tasks, extracted from a wide range of relevant sources.

A link to an online questionnaire was distributed via emails, online discussion forums, professional journals and social media. The questionnaire asked the teachers

to think about a mathematical task that he/she had used recently or saw another teacher using with students, then rate in a 5-points Likert scale how accurately each adjective described the chosen task.

Results of the British study

In total, 360 British teachers completed the questionnaire and 92% of them were teaching 11-18 year old students. An exploratory factor analysis suggested 7 factors. This means that the 84 adjectives could be grouped into 7 underlying factors, for instance, the words ‘difficult’ and ‘complicated’ were group under the factor named Demand, while the word ‘procedural’ loaded high on a different factor, named Routine.

The factors were named trying to encompass the general meaning of as many highest-loading adjectives as possible. Thus, the seven factors were: Engagement, Demand, Routineness, Strangeness, Inquiry, Context and Interactivity. The table below shows the adjectives with loadings higher than 0.5 on each factor.

Engagement	Demand	Routineness	Strangeness	Inquiry	Context	Interactivity
enjoyable	difficult	routine	strange	open	real-life	hands-on
fun	complicated	repetitive		inquiry-based	realistic	cooperative
pleasing	demanding	procedural		deep	context-based	collaborative
appealing	perplexing	formal		exploratory	applied	practical
attention-grabbing	easy ^a	mechanical		investigative		
motivating	challenging	rule-based		rich		
stimulating	simple ^a			thought-provoking		
memorable	problematic			closed ^a		
boring ^a	puzzling			analytical		
interesting						
absorbing						
exciting						
inspiring						
dull ^a						
engaging						

Table 1: Adjectives with loadings higher than 0.5 on each one of the 7 factors. ^a means negative loading.

The Brazilian study

We replicated this study with Brazilian teachers (Barichello & Guimarães, in press). First, we generated a new list of adjectives in Portuguese based on: the British list, our judgement about the relevance of each adjective and our knowledge of other adjectives that could be included. As a result, our list had 88 adjectives. A similar online questionnaire was set up, using a 5-points Likert scale for each adjective, and disseminated through emails, social media and other networks focused on mathematics teachers.

Results

We obtained answers from 415 teachers, from which 77% taught 11-18 year old students. Following the same procedure adopted by Foster and Inglis (2017), an exploratory factor analysis suggested 8 factors. However, inspection showed that one of them did not have the same characteristics as the others, making it impossible to interpret in the same way. Then, we conducted an analysis similar to the one used by Inglis and Aberdeen (2015) and concluded that this factor was not an actual factor, but a proxy to “not applicable” and the adjectives with high loading on it were removed from the data set (10 in total). All the removed adjectives had a low mean response rate, meaning that teachers, in general, found all those adjectives not appropriated to describe the task they had chosen.

A new factor analysis was run and the same tests suggested 7 factors, all very similar to the remaining 7 factors from the first analysis. The factors were named (in Portuguese with literal translation indicated in brackets) as follows: Efetividade (Effectiveness), Exigência (Demand), Rotina (Routineness), Contextualização (Contextualization), Abstração (Abstraction), Inovação (Innovation) and Interatividade (Interactivity). The table below shows the adjectives with loadings higher than 0.5 on each factor.

Efetividade	Rotina	Demanda	Abstração	Contextualização	Inovação	Interação
útil	comum	demorada	abstrata	contextualizada	incomum	colaborativa
agradável	mecânica	complicada	formal	vida real	chocante	cooperativa
séria	rotineira	trabalhosa		interdisciplinar	surpreendente	em grupo
acessível	repetitiva	difícil		resolução de problema	diferente	manipulativa
atrativa	previsível	rápida ^a		aplicada	inesquecível	experimental
significativa	técnica			realista		exploratória
pode ser extendida	fechada			concreta		
prende atenção	fácil					
prazerosa	devagar					
interessante	simples					
estimulante						
rica						
motivadora						

Table 2: Adjectives with loadings higher than 0.5 on each factor. ^a means negative loading.

Comparison between studies

The first similarity between the studies is the final number of factors, 7. It is important to clarify that this was not forced upon the Brazilian data, it was the result of a series of tests designed to inform the decision of how many factors should be considered for a given dataset.

However, what is mostly striking is not the number of factors, but the similarities and differences between the factors identified in each study.

Similar factors

A quick check on the names of the factors in English and in Portuguese evidences some pairs of factors that are very similar to each other: Demand and Exigência, Routineness and Rotina, Interactivity and Interação, Context and Contextualização. The similarities also hold if the adjectives with the highest loadings on each factor are compared.

There is not much to add to this analysis, except that this coherence between results in two different countries, with different participants, suggests that the method employed is reliable, especially if you take into account the internationalization of tendencies in mathematics teaching and in academic research in mathematics education.

The other factors

There are three factors in each study that were not common to both.

We can say that the British factor Strangeness and the Brazilian factor Inovação (Innovation) are not related to each other. While only the word strange loaded high in the British factor, the words incomum (unusual), chocante (shocking) surpreendente (surprising), diferente (different) and inesquecível (memorable) loaded high on the Brazilian factor. This set of words suggests an activity that was new, innovative, not necessarily strange.

The other two pairs, Engagement and Efetividade (Effectiveness), Inquiry and Abstração (Abstraction), still bear some similarities.

Regarding the pair Engagement and Efetividade (Effectiveness), even though several high-loading adjectives are the same, in the British study the adjectives all seem to relate to how students experience the task, such as enjoyable, fun and pleasing. The Brazilian factor contains these adjectives but also a second group that seem to be more related to teachers' judgement on students learning, such as útil (useful), acessível (accessible) and significativa (meaningful). These 3 adjectives did not load high in any of the British factors.

Regarding the last pair, Inquiry and Abstração (Abstraction), at first sight they seem non-related. There are no matches among the adjectives that loaded higher than 0.5 in each of them. However, if the list is extended in order to include the next few adjectives, some commonalities start to appear.

Inquiry
Open
Inquiry-based
Deep
Exploratory
Investigative
Rich
Thought-provoking
Closed
Analytical
Conceptual
...
Abstract

Abstração	(Abstraction)
Abstrata	(Abstract)
Formal	(Formal)
Conceitual	(Conceptual)
Análítica	(Analytical)
Investigativa	(Inquiry-based)
Profunda	(Deep)

Tables 3 and 4: Adjectives with loadings higher than 0.3 on the factor Inquiry on the left and Abstração (Abstraction) on the right.

These tables show that, although weak, it seems that British and Brazilian mathematics teachers see a relationship between inquiry and abstract tasks. This relation is not salient in any of the studies if considered in isolation, but the comparison highlights it.

Source of the differences

The differences discussed before could have arisen from random noise in the data or from confounded variables. However, based on our experience as teachers, teacher educators and researchers in Brazil, we can point out one element that could explain the differences.

In Brazil, textbooks are widely adopted by teachers. The federal government provides them for free for every student in the country based on a list of pre-approved textbooks (the list contains about eight textbooks for each educational stage).

Bearing that in mind, the presence of the factor Inovação (Innovation) in the Brazilian results could be explained by the fact that the translation we used for the word task (atividade) in the online questionnaire may have influenced teachers to think about something not from textbooks and, consequently, unusual for their daily practice.

The absence of Inquiry as a factor in the Brazilian data could also be explained by the lack of open ended, investigative type of tasks in the textbooks.

Finally, the occurrence of Abstração (Abstraction) as a factor in the Brazilian data could be due to the presence of proofs and questions such as “Show that...” in the textbooks, making abstract and formal mathematics readily available in a lesson.

Final considerations

The similarities suggest that participants in both countries share a common meaning for the adjectives in the lists, but it is still uncertain if this meaning is shared among policy developers and researchers. Actually, the Brazilian data suggested that teachers see problem solving tasks as mostly related to real life and not pure mathematics,

while official documents present problem solving as a broader teaching strategy that should encompass tasks inspired by real life situations as well as by pure mathematical motivations.

Moreover, the differences might reflect cultural variations, but they can also result from lack of clarity regarding the meaning of some adjectives, since the list in Portuguese was highly inspired by the list in English. Our next step would be to conduct a confirmatory factor analysis, possibly with a more diverse sample of teachers (considering different regions in Brazil, for instance) and a replication of the second study reported in Foster and Inglis (2017).

Reference

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