Examining prospective mathematics teachers' pedagogical content knowledge of limit using vignettes

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The aim of this study is to examine upper secondary prospective mathematics teachers' pedagogical content knowledge (PCK) with regard to students' difficulties with and misconceptions of limit. Participants are twenty-seven prospective teachers who were enrolled in a teacher preparation program at a state university in Turkey. Participants' PCK were explored using vignettes which illustrate students' difficulties with and misconceptions of limit. Scenarios in vignettes include students' answers and solutions. Data were analysed using content analysis. The findings will be discussed with regard to how participants (a) notice students' difficulties and misconceptions, (b) interpret their sources, and (c) decide on overcoming these difficulties.

Keywords: pedagogical content knowledge; limit; vignette; prospective mathematics teachers

Introduction

To provide effective mathematics teaching, mathematics teachers should have knowledge of the difficulties and misconceptions that their students might have. Training mathematics teachers who have the required knowledge becomes an important issue. Therefore, teacher preparation programmes should handle student difficulties and misconceptions. Knowledge of students' difficulties is a major component of pedagogical content knowledge (PCK), the knowledge required for teaching a subject (Shulman, 1986). The aim of this study is to examine prospective mathematics teachers' pedagogical content knowledge of limit with regard to this component.

The concept of limit was chosen as the subject matter due to several reasons. First of all, students have several difficulties and misconceptions with these concepts (Tall & Vinner, 1981; Bezuidenhout, 2001; Cornu, 2002; Roh, 2008). Second, the limit concept is prerequisite for other mathematical concepts such as continuity, derivative and integral (Cornu, 2002). Third, there is a gap in the literature concerning PCK of limit. This study focuses on two misconceptions with limit as listed below (Özmantar & Yeşildere, 2013):

- The limit is equal to the function value at a point, i.e. for a function represented algebraically, a limit can be found by a method of substitution.
- When a graph is given, the limit of a function can be found by finding the value of the function at that point.

Research questions

The research questions of the study are as follows:

- How do upper secondary prospective mathematics teachers notice the mistakes in students' responses in vignettes?

- How do upper secondary prospective mathematics teachers interpret the sources of students' difficulties with and misconceptions of limit?
- How do upper secondary prospective mathematics teachers intervene in student difficulties with and misconceptions of limit?

Methodology

This study uses a case survey method. Participants are twenty-seven (eighteen female, nine male) prospective mathematics teachers who were enrolled in a teacher preparation program at a state university in Istanbul, Turkey. They were in the third year and had already taken courses in mathematics, education and mathematics education. After graduating from the program, they will have earned a diploma for teaching mathematics at upper secondary level. All participants have completed calculus courses which covered limit, continuity, derivative and integral.

Prospective mathematics teachers were given a test on limit which included seven vignettes with scenarios. Scenarios in vignettes include students' answers and solutions (the first type), and teachers' responses to students' answers (the second type). Each scenario focused on one of the misconceptions as listed above. In this study, we will present only two of the first type of vignettes. The first type of vignettes includes a question asked by a teacher to the classroom and students' wrong answers to this question. Prospective teachers were expected (i) to notice the mistakes in students' responses (ii) to find the sources of these mistakes (iii) to respond to these mistakes. Five experts who are mathematics educators and mathematics teachers evaluated these vignettes. Revised vignettes were given to twenty-seven prospective mathematics teachers. Participants' answers to scenario questions were analysed using content analysis, and frequencies of categories are presented.

Findings

In this section, findings obtained from the first type scenario questions (a total of two vignettes) are presented.

Table 1: vignette 1 and related misconception

A mathematics teacher aimed to evaluate the knowledge of students about limit. So the teacher wrote "$\lim_{x \to 0} \frac{1}{x^3}$" at board and asked students to solve it (Akbulut & Işık, 2005). One of the students named Can said that "*I write 0 for x. The third power of 0 is 0. 1/0 is undefined, so there is no limit*". Determine whether Can's answer is correct or wrong? a) If it is correct, explain why? b) If it is wrong, explain the source of this mistake and explain how you will intervene?
The related misconception: "The limit is equal to the function value at a point, i.e. for a function represented algebraically, a limit can be found by a method of substitution."

Prospective teachers' answers to part (a) of vignette 1 were analysed, and frequencies of correct and wrong answers are presented in Table 2 below. 13 out of 27 prospective teachers realized that Can's answer was wrong, so they answered correctly. The other 14 participants have accepted that Can's answer was correct and it appears that participants themselves have the same misconception: *The limit is equal to the function value at a point, i.e. for a function represented algebraically, a limit can be found by a method of substitution*.

Table 2: findings from vignette 1 (noticing the mistakes)

Prospective Teachers' Answers (N=27)	**f**
Correct	13
Wrong	14
No Response	-
Total	27

Frequencies of categories for the sources reported by participants are presented in Table 3 below.

Table 3: findings obtained from vignette 1 (Interpreting the sources)

Sources reported by participants (N=27)	**f**
Correct Source	
Misconception (a limit can be found by a method of substitution)	8
Confusing definitions of limit and continuity	1
Inadequate Source	
Lack of understanding of definitions	2
Wrong Source	
Limited understanding of the notions of undefined and uncertain	3
No Response	13

Nine out of 27 prospective teachers mentioned a correct source for mistakes. Eight of them noticed that the students had the misconception: a limit can be found by a method of substitution. For example, one of the prospective mathematics teachers (PMT13) wrote: "The student (in vignette 1) thinks that finding the limit of a point is same as finding the value of the function at that point". Two participants just mentioned that students had "a lack of understanding of definitions" and these answers were considered as in the category of "inadequate source." Three prospective teachers stated that students had a "limited understanding of the notions of undefined and uncertain" which was considered as a "wrong source": "The student didn't understand the relationship between uncertain and undefined" (PMT14). Thirteen participants could not give any source for Can's answers.

Frequencies of categories of interventions suggested by participants are presented in Table 4 below. As can be seen from Table 4, a total of 16 participants suggested appropriate interventions for students' mistakes. Six prospective teachers considered emphasising the process of approaching while five participants found using graphic representation as a solution to students' difficulties. Five participants' answers such as "using different examples" and "revising the subject" were considered as "inadequate."

Table 4: findings obtained from vignette 1 (how to respond to students' answers)

Interventions suggested by participants	**f**
Correct	
Emphasis on the process of approaching	5
Using graphic representation	4
Emphasising the definition of the concept	3
Emphasising that the value of that point is not an issue	2
Using numeric representation	1
Using algebraic representation	1
Inadequate	
Using different examples	3
Revising the subject	2
Wrong	
Removing the undefined points	1
No Response	10

Participants also responded to another vignette as presented below.

Table 5: vignette 2 and related misconception

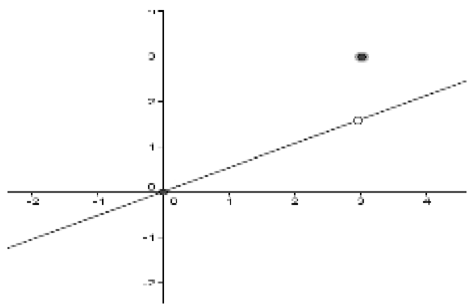
The teacher asks the class to find the limit for the function *g* given by the graphical representation as x → 3 (Jordaan, 2005)

The student (Ali) told us that "*The limit value is 3 because the value of function is 3 at x = 3*".

Determine whether Ali's answer is correct or wrong?

a) If it is correct, explain why?

b) If it is wrong, explain the source of this mistake and explain how you will intervene?



The related misconception: "When a graph is given, you can find the limit of a function by finding the value of the function at that point."

Frequencies of correct and wrong answers to part (a) of vignette 2 are presented in Table 6 below. 19 of the 27 prospective teachers realised that Ali's answer was wrong, so they answered correctly. The other seven participants accepted that Ali's answer was correct. It can be claimed that these prospective teachers themselves have the following misconception: *When a graph is given, you can find the limit of a function by finding the value of the function at that point.*

Table 6: findings obtained from vignette 2 (noticing the mistakes)

Prospective Teachers' Answers (N=27)	**f**
Correct	19
Wrong	7
No Response	1

19 out of 27 prospective teachers realised that the student's answer was wrong, so they answered correctly. The others have accepted that student's answer was correct and it appears that participants themselves have the same misconception.

Frequencies of categories for the sources reported by participants are presented in Table 7 below. Fourteen out of twenty-seven participants spotted a correct source for Ali's mistakes in vignette 2. Eleven of them mentioned that Ali had the misconception: "a limit can be found by a method of substitution" while three of them stated that Ali confused definition of limit with continuity. Nine participants could not give any source for Ali's answers.

Table 7: findings obtained from vignette 2 (interpreting the sources)

Sources reported by participants (N=27)	f
Correct Source	
Misconception (a limit can be found by a method of substitution)	11
Confusing definitions of limit and continuity	3
Inadequate Source	
Lack of understanding of the definition of the concept	1
Misconception	1
Wrong	
Lack of understanding of the process of approaching	1
Psychological reasons	1
No Response	9

Frequencies of categories of interventions suggested by participants are presented in Table 8 below.

Table 8: findings obtained from vignette 2 (how to respond to students' answers)

Interventions suggested by participants (N=27)	f
Correct	
Emphasising the process of approaching	9
Emphasising that the value of that point is not an issue	6
Reminding the limit definition	4
Inadequate	
Teaching limit using different methods	2
Using different examples	2
Wrong	
Linking limit and continuity in a wrong way	1
No Response	11

As can be seen from Table 8, a total of 19 participants suggested appropriate interventions for students' mistakes such as "emphasising the process of approaching." Four prospective teachers reported inadequate interventions which are not explanatory. Eleven participants did not suggest any intervention.

Discussion and Conclusion

The findings of this study showed that the majority of participants spotted the students' mistakes in vignettes. However, the percentages of participants who accepted students' answers as correct are still quite high. This finding indicated that prospective teachers themselves might have the same misconceptions and difficulties with regard to limit. Sources reported by participants and the way in which they address errors and misconceptions indicate a lack of pedagogical content knowledge. The majority of the participants had difficulties with interpreting mistakes, explaining the sources of these mistakes and developing approaches to overcome students' misconceptions.

Another important finding is that some of the participants tended to solve the given question in the vignettes rather than interpreting the sources of students' incorrect answers and finding ways to overcome their difficulties. In other words, participants focused on content knowledge rather than pedagogical content knowledge.

Considering the findings of this study, we can suggest that mathematics teacher preparation programs should focus on students' misconceptions and how to overcome these not only for the concept of limit but also for other concepts at upper-secondary level. Programs should develop different forms of knowledge as suggested by Shulman (1986): Propositional knowledge, case knowledge (using vignettes) and strategic knowledge (using videos).

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