Describing the cycles of a modelling activity: The drug concentration in the human body

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This study explores the effects of two teaching experiments that focus on exponential modeling activity and collaborative inquiry learning in workplace problem solving. This research is part of the European Project MASCIL and refers to the solution of the drug concentration problem in the human body. Ten heterogeneous groups of year 11 students took part in this research. In this research audio and video recordings as well as qualitative content analysis are used. Results show that students, while solving the problem, developed mathematical abilities, which are divided in three modelling cycles: arithmetic, geometry and algebra. The constructions of mental images of students about the notions of monotony, rate of change and supremum (least upper bound) play a vital role. It is important to note the teachers' focus on mathematics rather than on realistic situation and the students' difficulties in the transition from recursive to the general type of geometric series.

Keywords: drug concentration; exponential function; modelling activity; authentic workplace problem solving; collaborative inquiry learning

Introduction

During the school year 2014-2015, under the European Programme Mascil, six two-hour experimental teachings which corresponded to different workplaces, were organized in classrooms of a Lyceum in Athens. Mascil (2013-2016) is a European professional development program aiming at disseminating inquiry teaching and learning in primary and secondary education through the connection of teaching mathematics and science in the workplace. In this study, we will outline a modelling activity entitled "the drug concentration by the human body" which was assigned to students of high school. In the current research, we focus on the following questions:

- Q1: What modelling cycles emerged in solving the problem of concentration by the human body, and how students developed corresponding competences?
- Q2: Did the authentic workspace encourage students' involvement in modelling the problem?

Theoretical framework

Modelling perspective has been important in mathematics education research for several decades. For researchers, modelling activities are closely linked with mathematics by solving realistic situations and open problems in the classroom (Sriraman & Lesh, 2006; Carrejo & Marshall, 2007). Mathematical modelling is the process of translating between the real world and mathematics in both directions (Blum & Borromeo Ferri, 2009). According to Blum and Niss (1991), the term "application" focuses on the direction of "mathematics to reality", whereas the term "modelling" focuses on the opposite direction from the "reality to mathematics". Modelling process starts from a reflection situation with usually indeterminate information that we face in real conditions. Among others, students' involvement in modelling activities provides opportunities for students to observe, communicate, describe, explain, reflect, and thus, build mathematical concepts based on meaning and inquiry (Kosyvas, 2016). Mathematical concepts, as they are more directly related to the real world, provide students with rich learning opportunities and encourage them to use a variety of ways in developing generalizations.

Most researchers agree that one of the main difficulties of teaching mathematics is the weakness of the subject of connection with real life. Cognitive difficulties of students are due to the gap which exists between the daily life and formalistic mathematics. Freudenthal argues that, in general, it is not fruitful to teach the abstract mathematics directly to students. In this case, the modelling of realistic situations is a pedagogical method to stimulate students' motivation to study mathematics by understanding (Gravemeijer, 1994; Dickinson et al., 2011).

Modelling activities which are related to an authentic workplace is suggested as a way through which students engage in meaningful mathematics and by working in real environments can develop strategies that will bring them closer to abstract mathematics and make connections between abstract mathematical models and physical phenomena (Wake 2015; Williams & Wake, 2007; Triantafillou, Bakogianni & Kosyvas, 2016). It is essential to introduce the modelling activity of authentic problems and collaborative inquiry in classrooms to bridge the distance between mathematics that students learn at school and the world of work. In the present study, under the light of the European Mascil Program we are going to investigate the modelling cycles applied by students while they solved an inquiry activity in an authentic workplace and the related benefits and difficulties.

Methodology

Two two-hour experimental teachings of year 11 students were organized in a Lyceum in Athens in Greece. Every class was divided into heterogeneous groups of 4-5 students and there were 10 groups examined in total. Enough time was given to students to think and write the solution of the problem. For the purposes of this research, teaching was recorded and videotaped. Data analysis is qualitative (Collins et al., 2004). A brief description of the modelling activity is the following:

A doctor presents the following details about the use of a specific drug:

- *An average of 25% of the drug leaves your body by secretion during a day.*
- *The drug is effective after a certain level is reached.*
- *Therefore, it takes a few days before the drug that you take every day is effective.*
- *Do not skip a day.*
- *It can be unwise to compensate a day when you forgot the drug with a double dose the next day.*

N.B. These details are a simplification of reality.

***Investigation**:*

- *Use calculations to investigate how the level of the drug changes when someone starts taking the drug in a daily dose of 1500 mg with for instance three times 500 mg.*
- *Are the consequences of skipping a day and/or of taking a double dose really so dramatic?*
- *Can each drug level be reached? Explain your answer.*

***Product**: Design a flyer for patients with answers to the above questions. Include graphs and/or tables to illustrate the progress of the drug level over several days.*

The worksheet and a detailed description of the problem are available from the mascil-project website (http://www.mascil-project.eu/classroom-material).

Presentation and discussion of results

According to our initial observations students encountered difficulties in understanding that every day they had to absorb the 0.75 of the total amount that was accumulated during the previous day, while others linked the problem to the linear model and designed a straight line. Misunderstandings were answered by other groups or with teachers' assistance.

During the mathematical activity students dealt with analyzing data and drawing graphics. In our study, we will distinguish three modeling cycles: the arithmetic, the geometric and algebraic. Examining and reconstructing the research material will shed light on the types of skills developed by the students in each cycle and their difficulties.

Arithmetic modeling cycle

In the first cycle the formulation of mathematical relations is done by numeric symbols. Most students used calculators, they found rates of drug concentration for each day and recorded them in a table or list.

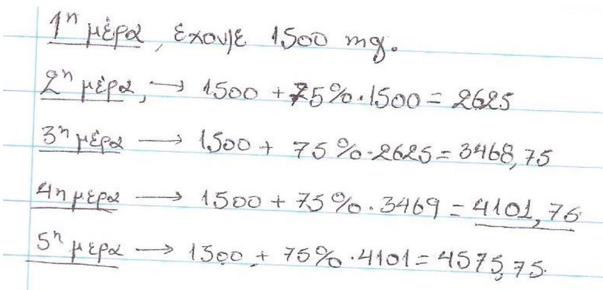

figure 1: a student's tabulation of results

The organization of the arithmetic results in the table shows a process characterized by analysis, systematic description and verification. The numerical knowledge allowed the students to identify the necessary information and to understand the problem. In addition, during the whole class discussion, students made further explanations and justifications. According to the question about consequences of skipping a day many students concluded, by using examples, that during the first few days the consequences of skipping a day may be dramatic on a patient's health.

Geometric modeling cycle

In the second cycle, students made a successful transition from arithmetic to geometric modeling. Most groups had used the previous numerical treatment of the problem and proceeded to the graph of drug concentration by the human body with respect to time.

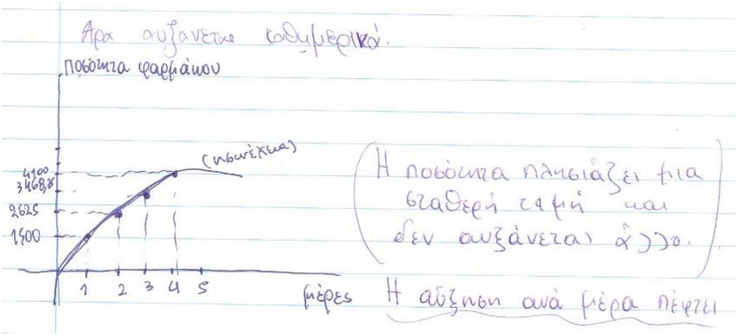

figure 2: graph of drug concentration by the human body with respect to time.

In figure 2 the student has written that “the amount of the drug increases every day”, “the amount approaches a constant value and does not increase anymore” and “the rate of change is reduced day by day”. According to the question about consequences of skipping a day a student said:

> S1: If we omit the dose of the third day, we will have: 3rd day: 2625 × 0.75= 1968.75. This is less than the drug quantity that there was the previous day, we have a large divergence… Therefore, as pharmacologists we conclude that this may create a serious problem to the patient.
>
> T: But will this go on? What happens, for example, the thirtieth day?
>
> S1: Then the difference is greater and more important ...

To answer the realistic question students did not examine the graph to observe that after a lot of days the rate of change decreases. Therefore, the omission of a dose is not important after many days. If we omit the thirtieth day dose, the human body has absorbed the needed quantity of drug. It was difficult for the students to use a deeper concept of limit and conclude that after many days of taking the medication there will not be perceptible effects on the quantity of drug in the human body.

In addition, six groups of students have concluded that the drug concentration in the patient’s body does not grow unlimitedly but is approaching 6000mg. These groups perceived intuitively, without formalities, mainly by using graph and table, that 6000 is an upper bound of concentration and some students concluded that is the lower least upper bound (supremum) of the function values.

Algebraic modeling cycle

The way in which a group has concluded that the supremum of drug concentration in the body is 6000mg is interesting. In attempting to justify this the group of students made the following argument that was formulated by a student:

> S2: The drug concentration in the patient’s body will stop increasing when the amount discharged in a day is equal to that taken.

In this group, they had found the recursion type $Q_{\nu} = 0,75 \cdot Q_{\nu-1} + 1500$, then setting $Q_{\nu} = 6000$ they found $Q_{\nu-1} = 6000$. So, they said, that it is not possible to be $Q_{\nu} = 6000$, thus the concentration is increasing continuously approaching 6000mg.

In the cycle of the algebraic modelling the conceptual system of students is more abstract than in the first and second. The awareness of the utility of symbolic representation provided a higher level of understanding the situation. Moving to the algebraic generalization the students expressed the assumption that the drug concentration in the human body is given by the recursive formula:

$$Q_\nu = 0,75 \cdot Q_{\nu-1} + 1500,\ \nu \geq 2.$$

Some students, despite their familiarity with algebra of exponents and the sum of n first terms of the geometric series they had difficulties to move from the recurrence relations of series to the general formula of the sequence, which is the quantity Q_ν (in mg) of the drug concentration in the body at the end of the nth day, depending on number n. This conclusion is consistent with other similar studies concerning the exponential function (White, 1985). Only one team succeeded in finding the sum of the first n terms of geometric series by giving the following general mathematical rule:

$$Q_\nu = 6000 \cdot \left(1 - 0,75^\nu\right),\ \nu \geq 1.$$

The majority of students had difficulties in finding the former type. The proof seemed complicated and laborious. The difficulty of students transferring their knowledge from the geometric series on the concrete situation was evident. The role of teachers, the problem, and the environment created in the classroom helped students improve their mathematical abilities. Teachers strove to shift from directive teaching strategies to student centered ones. It was obvious that inquiry was a new approach for them. Uncertainty was dominant in students' behaviours, answers and arguments as mathematicians pushed students to move from the realistic context of the problem to mathematical modeling. Despite their efforts, the teachers' practices followed a quite guided inquiry method.

Conclusions

As far as the first research question is concerned, as this is derived from the previous results presentation, this shows that students inquired the problem of "drug concentration" developing mathematical modeling skills, which are divided into three modeling cycles: arithmetic, geometric and algebraic with references in mathematical analysis branch. By using prior knowledge students analyzed the problem trying to find what mathematical concepts, theorems and algorithms, were useful for the solution. While one out of ten groups found the optimal algebraic solution (by using recursive relations for proving the general algebraic formula of the sequence) all groups succeeded in the first two levels (arithmetic, geometric). Some of them also used the geometric visualization approaching an intuitive understanding of some new analysis concepts such as increasing function, rate of change and least upper bound, giving them mathematical meanings. More particularly, by using effectively and successfully graphs and tables they illustrated the progress of the drug level over several days and they provided right answers concerning the consequences of skipping a day, only for the first few days. Despite the difficulties collaborative inquiry practices in the teaching of mathematics strengthened the critical competences of students, formed a continuous research environment in which students were the protagonists of the learning process, which is the main aim of the Mascil project (Kosyvas, 2016).

As far as the second research question is concerned, the drug concentration problem was an opportunity for active involvement of students in an authentic modeling activity. Our observations show that many students partly adopted the professional role of analyst in pharmacology or medicine, attempting to find out how drugs work in the body, they thought how a pharmacologist may use mathematics models to explore changes in bloodstream drug level under varying conditions, they examined the reality of the problem interpreting the results and evaluating the validity of the solution in the context of everyday life. The workplace was a beneficial and powerful learning motivation for students, which affected the finding of the solution and the negotiation of mathematical meanings in the whole class. The context of the problem gave feedback to the reflective dialogue of most students' pragmatic arguments about issues concerning the harmful or useful effects of medications in the human body and generally the importance of the medical science.

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References

Blum, W. & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, 1(1), 45-58.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects: State, trends and issues. *Educational Studies in Mathematics*, 22, 37-68.

Carrejo, D., & Marshall, J. (2007). What is mathematical modeling? Exploring prospective teachers' use of experiments to connect mathematics to the study of motion. *Mathematics Education Research Journal*, 19(1), 45-76.

Collins, A., Joseph, D. & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *Journal of the Learning Sciences*, 13(1), 15–42.

Dickinson, P., Hough, S., Searle, J., & Barmby, P. (2011). Evaluating the impact of a Realistic Mathematics Education project in secondary schools. *Proceedings of the British Society for Research into Learning Mathematics*, 31 (3).

Gravemeijer, K. (1994). *Developing realistic mathematics education*. Utrecht, The Netherlands: Freudenthal Institute.

Kosyvas, G. (2016). The students' involvement in a workplace inquiry activity: solution of the solar panel problem. In: Adams. G. (Ed.) *Proceedings of the British Society for Research into Learning Mathematics*, 36(1), 47-52.

Sriraman, B., & Lesh, R. (2006). Modeling conceptions revisited. *ZDM- The International Journal on Mathematics Education*, 38(3), 247-254.

Triantafillou, C., Bakogianni, D. & Kosyvas, G. (2016). Tensions in students' group work on modelling activities. In C. Csikos, A. Rausch & J. Szitanyi (eds.). *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 283-290, Szeged, Hungary: PME.

Wake, G. (2015). Preparing for workplace numeracy: A modelling perspective. ZDM *- The International Journal on Mathematics Education*, 47(4), 675-689.

White, J. (1985). Micromaths. *Teaching Mathematics Applications*, 4(3), 123-126.

Williams, J. S., & Wake, G. D. (2007). Metaphors and models in translation between college and workplace mathematics. *Educational Studies in Mathematics*, 64(3), 345–371.