Sharing perspectives on mathematical methods: A dialogic investigation

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Bakhtin's work on perspectives forms the basis for a dialogic investigation into perspectives on the mathematical methods used by secondary school students and teachers. I aim to find out more about why people use the mathematical methods they do, both when teaching and when working individually. As part of this PhD project, students from Years 7 (aged 11) to Year 13 (aged 18) in a UK comprehensive school completed questions designed to encourage a range of different methods to be used. The artefacts will form prompts for teacher and student discussion groups which will focus on sharing perspectives on mathematical methods. These perspectives will form a framework for classroom observations to see teachers and students talking about methods in context. This report of discussions held during a BSRLM session highlights two points regarding methods that participants viewed as showing understanding and then two issues with the activity.

Key words: mathematical methods; perspective; dialogue; Bakhtin

Context

The new National Curriculum for Mathematics was published in 2013/14 (DfE, 2013 and 2014) and has been interpreted by the National Centre for Excellence in the Teaching of Mathematics (NCETM) as the chance for students to develop 'mastery' of mathematical concepts (NCETM, 2014, p.1). However, there is a contrast between the curriculum's aim that "pupils develop conceptual understanding" (NCETM, 2014, p.2) and some of the specific requirements of the curriculum that advocate the use of certain methods or dictate that some facts must be learnt by heart.

For example, the Year 6 programme of study states that students should use "the formal written method of long division" (DfE, 2013, p.39) with the justification that this method demonstrates a deeper conceptual understanding of the underlying mathematics than the 'bus stop' method – a traditional method for short division. However, primary school students are also required to memorise their times tables up to 12 x 12 (DfE, 2013, p.25) and GCSE Foundation level students are required to memorise the exact value of trigonometric ratios (DfE, 2014, p.10) which seems to emphasise rote memorisation rather than conceptual understanding.

This confusion between teaching for deeper conceptual understanding and teaching students to have a good recall of certain key facts or algorithms has a particular impact on low achieving students for whom there is an ongoing debate around how best to help their progress in mathematics. Kroesbergen and Van Luit (2005) raise the importance of 'automaticity' and the teaching of standard algorithms for lower achieving students, whereas programmes such as Realistic Mathematics Education (RME) emphasise validating students' instinctive mathematics and using their ideas to build more 'sophisticated' methods with deeper conceptual understanding (Van den Heuvel-Panhuizen & Drijvers, 2014).

Why does long division demonstrate a deeper conceptual understanding of division than the ‘bus stop’ method or even a tallying method of sharing out? Surely, some students will still have to do long division by rote memorisation, does this demonstrate the intended deeper conceptual understanding? How can we say if a method demonstrates mastery? In order to address these questions I first want to see what it is that makes us choose the methods we use – both as teachers and as students, and what these methods look like. Do we vary the methods we use for different situations? Why? What do we think it is that others look for in our methods?

Project outline

I am in the early stages of a three-phase project that will focus on generating texts or ‘voices’ from different sources to discuss mathematical methods. I am forming teacher groups to discuss what methods teachers use both when they are answering questions themselves and what they would use to demonstrate to a class. I am then going to ask student groups what methods they use when solving problems themselves, what they use when someone else will see it, and what they would use in an examination. These initial ideas will lead to a wider discussion around mathematical methods. I am then going to carry out a series of classroom observations to see methods being used in context. I aim to bring together these ‘voices’ and identify tensions from the dialogue generated.

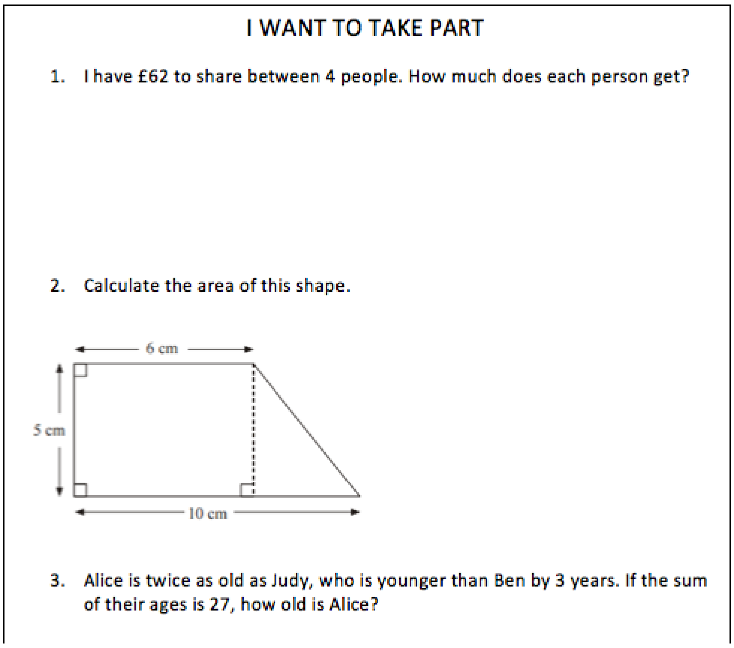
Approach

The idea of ‘voice’, as I am using it, has a particular Bakhtinian meaning as “the speaking personality, the speaking consciousness. A voice always has a will or desire behind it, its own timbre and overtones” (Holquist in Bakhtin, 1981, p.434). I am looking to collect a variety of voices through my study. Every person contributing will contribute a different voice and come to the investigation from a different perspective. I am interested in the differences and tensions between what these voices have to offer. I intend to carry out a dialogic investigation in that I am not aiming to arrive at a consensus between all of the different voices that will be coming through in my research. Indeed, it would be simplistic of me to assume that so many different contributors would all reach a point of agreement. Instead, I intend to “recognise that difference is productive” (Reed, 2005, p. 88) and not only a starting point for creating agreement. This dialogic investigation takes a hermeneutic approach which “seeks understanding rather than explanation; acknowledges the situated location of interpretation; recognises the role of language and historicity in interpretation; views inquiry as conversation; and is comfortable with ambiguity” (Kinsella, 2006, p.2). My involvement is much deeper than just the fact that I will be sharing ideas in group discussions and is dependent on the relationships that I have built with colleagues, students and parents and reflects “the active role played by the knower” as part of the hermeneutic approach (Gardiner, 1999, cited in Kinsella, 2006, p.5).

Generating Artefacts

I have gathered a series of artefacts from the students I teach. Approximately 200 students took part in this stage, ranging from Year 7 (11 years old) to Year 13 (18 years old). They were given a document divided into two sections. One contained problems for students who were happy to take part in my study (Figure 1). The other half had similar problems for those who wanted to opt out. After they had completed their work, I cut the pages in half and discarded the half that contained the work of those who did not want to take part. All students were asked to complete one half of the sheet as I did not want to incentivise taking part or not. Students at all levels were asked the same three questions, designed so as not to guide students towards using a particular method. These artefacts will be used in a teacher group session to begin discussion about student methods and again in the student sessions to gather the students' perspectives on the methods their peers have used.

Figure 1: Blank question sheet


I WANT TO TAKE PART

1. I have £62 to share between 4 people. How much does each person get?

2. Calculate the area of this shape.

3. Alice is twice as old as Judy, who is younger than Ben by 3 years. If the sum of their ages is 27, how old is Alice?

Aim of the session

Participants formed groups and considered randomly selected examples of the artefacts. They were asked to discuss what was the same and what was different in the artefacts they were given. Each of the groups' examples came from a student in a different year group. Once the groups had some time to discuss the examples, they were brought back together for a wider discussion where groups fed back aspects of the articles they had chosen to spend time discussing. The purpose of the discussion was to collect the participants' voices about what the students had done. This allowed me to collect perspectives of participants from outside the school where I am doing my research.

I begin by writing about two points regarding methods that participants highlighted as showing understanding or a lack of it. I then highlight two issues raised by the participants with regard to the activity.

Discussion

Context of questions

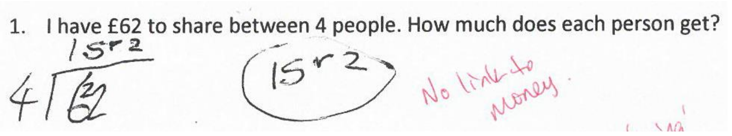


Figure 2: Student work

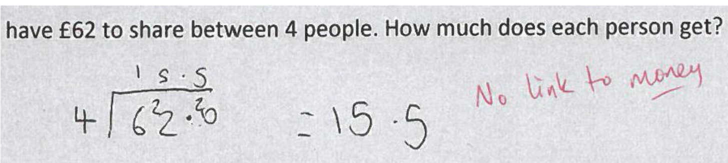


Figure 3: Student work

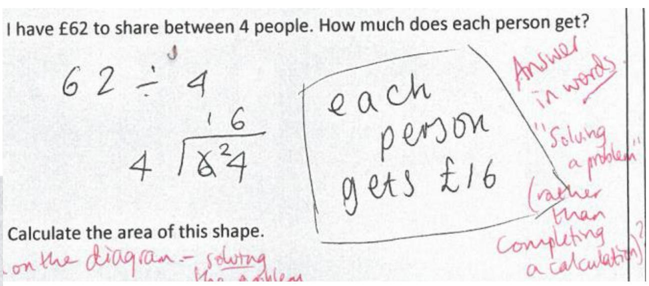


Figure 4: Student work

Question 1 brought up the issue of context in a problem and the ways in which the students did, or did not maintain this link. The group working with these examples noted that two of the students had carried out an algorithm and then stopped with a numeric solution (Figures 2 and 3), whereas in the other two examples (Figures 4 and 9), the students had linked their solutions back to the original problem of sharing money. The question was asked; ‘after the problem has been represented as part of, say, the ‘bus stop’ method, what happens?’ The groups suggested that two of the students seemed to ‘throw away’ the problem once it had been reduced to an algorithm. Does the lack of a link back to the original problem show less understanding? Or could it be considered that the students have not answered the question?

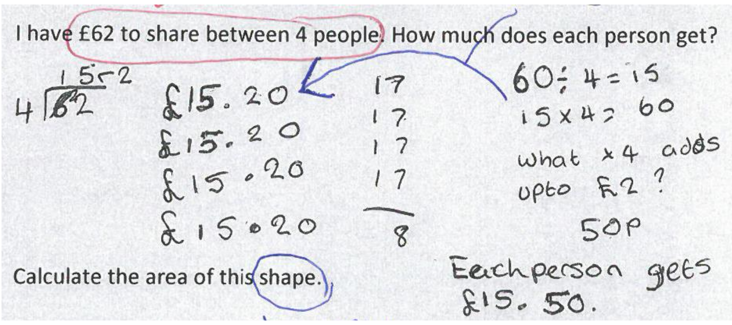


Figure 5: Student work

With these questions in mind, our discussion considered Figure 5. This student began by using the ‘bus stop’ algorithm for division, but misinterpreted their remainder as representing 20p – not the only student to do so. However, the student has corrected their mistake and reinforced that the remainder 2 is the same as £2 divided by 4. Their solution illustrates the difficulty some students had interpreting their answer to the ‘bus stop’ algorithm. However, this student has overcome this by linking their solution back to the problem.

Methods for question 3

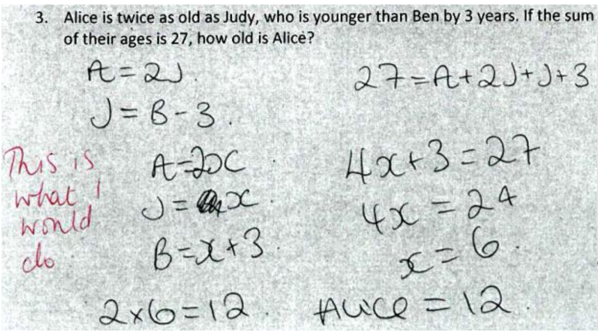


Figure 6: Student work

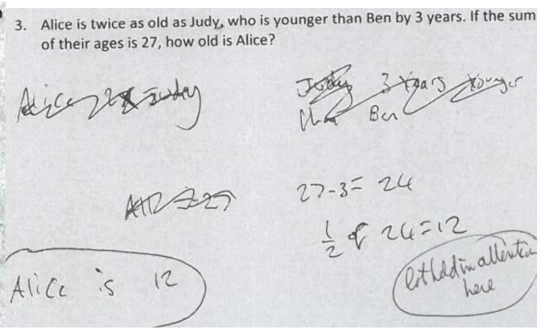


Figure 7: Student work

Analysis of solutions to question 3 interested several groups and highlighted the variety of solutions offered by students, which participants felt demonstrated varying levels of understanding of the problem. One group noted that a ‘standard’ algebraic method was familiar and is what they would have done given the same problem (Figure 6). Another group discussed the method given in Figure 7, pointing out that the student obviously held a lot in attention while working on the problem to use such a sparse method successfully.

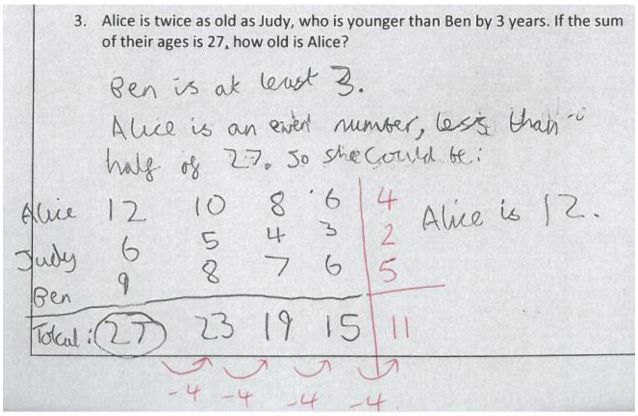


Figure 8: Student work

Figure 8 was given as an example demonstrating good understanding whilst not using a ‘standard’ algebraic method. The written reasoning at the beginning was highlighted particularly as showing the student’s understanding of the initial problem. The group interpreted the table as showing that the student had completed the first column and found the correct solution and that the remaining columns were the student checking their answer. However, other groups pointed out that the student could have been completing the table row by row instead. This does not detract from what is a clear method that shows understanding of the processes involved without generalising it to an algebraic problem, however, it highlights a different issue – that of interpreting written work.

Issues

Interpretation of methods from writing

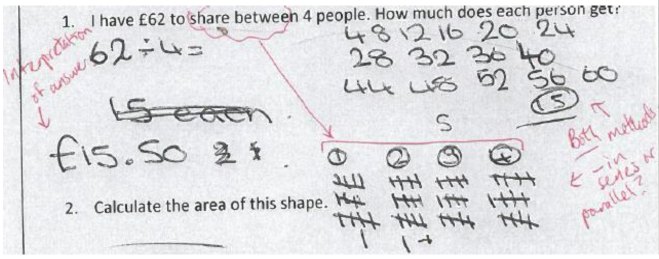


Figure 9: Student work

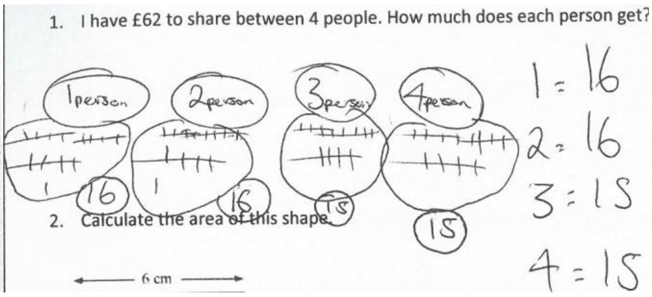


Figure 10: Student work

Along with the example from question 3, the groups differed in their interpretation of another method used by two students in solving problem 1. Figures 9 and 10 show two students who used tallying to help solve the division problem. For the first example, the group discussing it found it difficult to tell if the student had carried out the two stages “in series or in parallel” i.e., an initial counting-on method with the four times table to get £15 each and then a tallying method to check it, or if they had used their tally to somehow help the counting on. However, after discussion with the other groups it was suggested that the student had instead used the four times table to find ‘15’ as the largest number of fours in 62 and then used the tallying to find how many they had left to divide. This was suggested as being more sophisticated than Figure 10, where the student has used tallying alone to divide 62. That student has also not interpreted ‘divide’ from the question as ‘divide equally’. This example demonstrates the difficulties in interpreting understanding from a written method without the opportunity to discuss the method further with the students involved.

Design of question 2

The discussion from the groups was much less rich around question 2. One group highlighted that the dotted line used to divide the trapezium (see Figure 1) had guided students towards a method of splitting the shape into a rectangle and a triangle to find the area (or perimeter for some students). The line was intended to support some of the weaker students taking part, but it had led to less variety in the methods used by the students and therefore less for the groups to discuss.

Conclusion

The artefacts clearly prompted discussion about the methods being used by the students. There were some interesting examples of students' individual methods that worked to solve the problems as alternatives to 'standard' algorithms or methods. The groups involved in this short session found it particularly interesting to discuss what these methods demonstrated of the students' underlying understanding of the problems. We did, also, come across some examples of methods that were difficult to interpret purely from a written text. In these cases, it would be helpful to have further discussion with the student as to their intentions.

I can see that it is going to be powerful if I start my teacher and student groups by asking participants to complete the problems for themselves before introducing examples of the methods of others. This will give me the chance to discuss their methods and to get more insight into what their written texts represent to them.

The ongoing question of interpretation of texts was mirrored for me in writing this paper as I was trying to interpret the writing on the artefacts that had been left by the session participants. Despite my attempts to circulate between the groups, I could not be present for all of the conversations. This meant I was attempting to interpret another layer of written text on top of the original artefacts. I am going to find it particularly interesting to tape record the teacher and student group sessions to get more detail of the discussions.

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