Calculation strategies for year 5 children: 10 years on

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For 10 years we have collected year 5 children's calculation question responses. We examine the range of strategies used and the success of each strategy. This paper continues this research with a sixth data set. Like the previous research we have collated the proportions of children who are successful and explore the strategies employed. This study also includes examination of responses from a small group of children from the same class to consider how successful they are across the four calculation strategies. At a time when there is much debate about the teaching of standard algorithms, we explore if and when children are using these and whether the children in this study are demonstrating conceptual development, with any of the strategies employed.

Key words: calculations; written strategies; mathematics; primary; year 5; algorithms

Introduction

Being able to solve calculation problems accurately, effectively and with conceptual understanding remains one of the on-going issues in primary mathematics education (Raveh, Koichu, Peled & Zaslavsky, 2016). There is much debate amongst the education profession as to whether it is important and necessary for children to be able to simply complete calculation questions so they gain the correct answer even if they do not understand the mathematical thinking involved. As a result much time appears to be spent on teaching standard algorithms (which many perceive to be the most efficient and effective way to reach the correct answer), despite a great deal of research showing that many children have difficulties in using them and understanding the mathematics involved (Raveh et al., 2016, Kilpatrick, Swafford & Findell, 2001).

This research continues to focus on how successful Year 5 children are at completing calculations. In addition to collating the results for each individual operation and examining both the more and less successful strategies used by children, we have also extended this particular study to consider how successful across all four calculation strategies children were. One reason for this is as a result of the latest English National Curriculum for mathematics (DfE, 2013), which places importance on the mathematical skills of fluency, reasoning and problem solving. While these skills have been present in previous incarnations of the mathematics curriculum (although labelled differently) it would seem that there has been a shift in teachers' views to move back to more formal, standard algorithms. Reasons for this vary but the latest curriculum includes specific reference to formal, standard algorithms in particular year groups, which appears to have persuaded many more teachers to teach these to all children, whether they are ready for them or not. Our intention to look at how cohorts of children responded to all four calculation strategies will provide some indication of whether they have the skills to use these methods successfully and consistently. Our previous studies have focused on the individual calculations and explored the proportions of children gaining a correct answer, and the methods employed. However, there appears to be a disconnectedness in knowledge with some of the teachers we spoke to about how the teaching of a formal algorithm for one calculation can affect holistic mathematical understanding. For example, it may be appropriate, quick and successful to add digits in columns for addition questions, but then this transference of knowledge in using digits is applied incorrectly when subtracting or multiplying, because children are not thinking mathematically.

While there are many studies that examine the understanding and success of individual calculation strategies (e.g. Thompson, 2012) there are fewer that explore the teaching of formal algorithms as a holistic and connected way of thinking mathematically. One recent study (Raveh et al, 2016) did explore and offer an integrative framework of knowledge for teaching the formal algorithms and concluded that there must also be an emphasis on conceptual understanding and connected knowledge when using formal algorithms. Our study explores the methods used by children in all four calculation strategies and considers whether they show a level of this conceptual understanding and connected knowledge, whichever methods they are using.

Methodology, context and results

To maintain consistency with our previous research, data was collected from questions completed by 887 year 5 children from 17 of the original 22 schools. Children's responses were analysed using the same categories (e.g. Borthwick and Harcourt-Heath, 2014) to allow comparison between data sets if required (as demonstrated in Borthwick and Harcourt-Heath, 2014). The four tables that follow show the correct and incorrect responses for each separate calculation strategy.

Addition: 96% correct / 4% incorrect

546 + 423	Number Correct	Number Incorrect	Percentage Correct	Percentage Incorrect
Not attempted		1		
Standard algorithm	667	19	97%	3%
Number Line	33	7	83%	17%
Partitioning	55	4	93%	7%
Expanded vertical	75	4	95%	5%
Answer only	8	2	80%	20%
Other	11	1	92%	8%
Totals	849	38		

Table 1: Results from 887 children for the addition question

As our previous research has shown addition remains the most successful strategy in children gaining the correct response. However, an interesting development in the 2016 study shows that the proportion of children recording an answer only has decreased from 20% in 2008 to just 1% in 2016, suggesting that children either perceive the need to make some jottings or they actually need to in order to support their thinking, despite the question involving no bridging.

Subtraction: 79% correct / 21% incorrect

317 – 180	**Number Correct**	**Number Incorrect**	**Percentage Correct**	**Percentage Incorrect**
Not attempted		**5**		
Standard Algorithm – decomposition	**403**	**90**	**82%**	**18%**
Standard Algorithm – equal addition	**1**	**1**	**50%**	**50%**
Number Line	**271**	**30**	**90%**	**10%**
Negative Number	**2**	**8**	**20%**	**80%**
Counting Up	**3**	**2**	**60%**	**40%**
Counting Back	**5**	**0**	**100%**	**0%**
Answer only	**1**	**7**	**13%**	**87%**
Other	**13**	**45**	**22%**	**78%**
Totals	**699**	**188**		

Table 2: Results from 887 children for the subtraction question

Since the 2014 data set, the proportions of children gaining a correct response has increased by 4% for the subtraction question, yet one in five children still fail to gain the correct answer. Using a number line and counting back are the most successful strategies, as although the standard algorithm is the most selected method, there is a greater risk of getting an incorrect answer.

Multiplication: 57% correct / 43% incorrect

56 x 24	**Number Correct**	**Number Incorrect**	**Percentage Correct**	**Percentage Incorrect**
Not attempted		**13**		
Standard Algorithm	**94**	**123**	**43%**	**57%**
Grid Method	**334**	**141**	**70%**	**30%**
Expanded Vertical	**74**	**21**	**78%**	**22%**
Two partial products	**0**	**40**		
Answer Only	**2**	**7**	**22%**	**78%**
Other	**4**	**34**	**11%**	**89%**
Totals	**508**	**379**		

Table 3: Results from 887 children for the multiplication question

For the first time since 2006 our data now shows multiplication to be the least successful strategy. While the expanded vertical method has the highest rate of success in terms of the proportions of children achieving a correct response, the grid method is selected by more than half of the children within the study. As with subtraction, those children who selected the standard algorithm were more likely to achieve an incorrect than a correct response.

Division: 61% correct / 39% incorrect

222 ÷ 3	**Number Correct**	**Number Incorrect**	**Percentage Correct**	**Percentage Incorrect**
Not attempted		**29**		
Standard Algorithm	**295**	**123**	**71%**	**29%**
Chunking Down	**47**	**22**	**68%**	**32%**
Chunking Up	**146**	**18**	**89%**	**11%**
Number Line	**151**	**78**	**66%**	**34%**
Answer Only	**2**	**5**	**29%**	**71%**
Other	**2**	**69**	**3%**	**97%**
Totals	**543**	**344**		

Table 4: Results from 887 children for the division question

Since we began this analysis of calculation strategies in 2006 the proportion of children gaining a correct response to the division question has increased from 21% to 61%. More children are now gaining the correct answer although the strategies selected by year 5 children remains varied. The number of children now choosing the standard algorithm has increased, and although it is now the most selected method for division, chunking up demonstrates a higher rate of success within this study.

Discussion and conclusions

Our 2016 study shows that while the proportions of children gaining a correct response to the four calculation strategies continues to increase since this research focus began (Borthwick and Harcourt-Heath, 2007), there are still too many children, in our opinion, unable to employ a method which supports them in reaching the correct answer. In an attempt to understand why, we decided to alter our parameters of analysis to consider how successful children were in answering all four calculation questions. Our previous studies have only reported on the success of each individual calculation strategy. However, researchers such as Swan (2007), Ma (1999) and Hill, Ball and Schilling (2008) suggest that recognising connections between areas of mathematics is a necessary and effective strategy in children's learning, which we agree with, yet our previous studies have not focused on this. If a child can answer two questions correctly, why not all four?

One of the points we noted when analysing the 887 responses was that children would often use an informal strategy for one or two questions, but then a formal algorithm for the others. While we would support children having a variety of strategies to choose from, often the mistakes children were making with informal strategies were mental calculation errors, for example, adding up numbers incorrectly (e.g. 546 + 423 = 968) or multiplying partial products inaccurately (e.g. 50 x 20 = 100). However, we observed different mistakes when children used the standard algorithms. Often the answers seemed to suggest a lack of understanding of the mathematical concept. For example, children would write a larger answer for the subtraction question than the original number being operated on (e.g. 317 – 180 = 497).

However, there were many examples in our study where children had employed informal strategies for all four questions, but only achieved two correct answers. With perhaps a perceived increase in pressure to teach the standard algorithms (e.g. the current English mathematics curriculum, DfE, 2013) we must first analyse the errors in the incorrect responses before deciding to replace these with the standard algorithms. As the tables above highlight, our data shows that for many children, the standard algorithm offers more risk of obtaining an incorrect response. We have space in this article to include examples from one year 5 child for each question.

Case study: one year 5 child

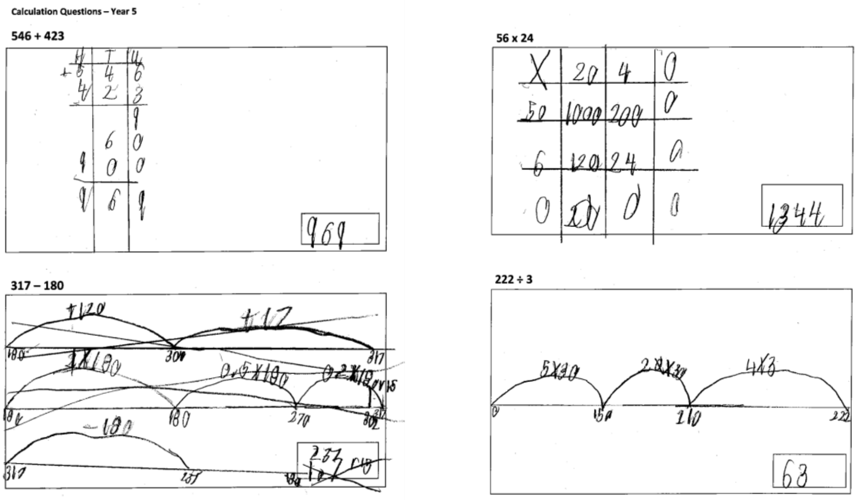

Figure 1: Four questions completed by Child A

Figure 1 shows that Child A was able to gain a correct response for the addition and multiplication questions but not for the subtraction and division questions. It is interesting to note that for all four questions the child has opted to use non-standard methods and, while not achieving a correct response for two out of the four questions, does appear to demonstrate some conceptual understanding (which many researchers, including ourselves, support, e.g. Rowland and Turner, 2007, Ball, Hill and Bass, 2005). With the subtraction question it appears that the child understands that one concept of subtraction is to find the difference between the two numbers, and although the answer is recorded incorrectly, the first number line does result in the correct answer. The division question uses the method of grouping the divisor and is correct. However, the child has added up the wrong chunks (e.g. 5 x 30 should have been 50 x 3). While the product is accurate, the divisor has been lost within the calculation, altering the final outcome.

It would seem that this child does have a network of knowledge which is connected. There is evidence of an understanding of place value across all four calculation strategies and also a sense that the child understands what is happening within each question. Yet with only a 50% rate of success some teachers may feel they need to replace the informal methods used by this child for subtraction and division calculations with the standard algorithm. Evidence from this study would suggest that this is not the case.

References

Andrew, P. & Sayers, J. (2015). Identifying Opportunities for Grade One Children to Acquire Foundational Number Sense: Developing a Framework for Cross Cultural Analyses. *Early Childhood Education Journal*, 43 (4) 257-267.

Ball, D. L., Hill, H. C. & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, 2005, 15-46.

Borthwick, A. & Harcourt-Heath, M. (2007). Calculation strategies used by Year 5 children. *Proceedings of the British Society for Research into Learning Mathematics.* 27 (1), 12-17.

Borthwick, A. & Harcourt-Heath, M. (2014). Calculating: How have Year 5 children's strategies changed over time? *Proceedings of the British Society for Research into Learning Mathematics.* 34 (1), 25-30.

Department for Education (2013). *The National Curriculum in England: Key Stages 1 and 2 Framework.* London: DfE.

Hill, H. C., Ball, D. L. & Schilling, S. G. (2008). Unpacking pedagogical content knowledge. Conceptualizing and measuring teachers' topic-specific knowledge. *Journal for Research in Mathematics Education*, 39 (4), 372-400.

Kilpatrick, J., Swafford, J. & Findell, B. (2001). *Adding it up: helping children learn mathematics*. Washington, DC: National Academy Press.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Ofsted (2011). *Good practice in primary mathematics: evidence from 20 successful schools*. London: Ofsted.

Raveh, I., Koichu, B., Peled, I. & Zaslavsky, O. (2016). Four (algorithms) in one (bag): an integrative framework of knowledge for teaching the standard algorithms of the basic arithmetic operations. *Research in Mathematics Education*, vol.18, No. 1, 43-60.

Richards, A. (2014). Working towards a 'child friendly' written progression for division. *Mathematics Teaching*, 240, 15-18.

Rowland, T. & Turner, F. (2007). Developing and using the 'knowledge quartet': A framework for the observation of mathematics teaching. *The Mathematics Educator*, 10 (1), 107-124.

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education*, 10, 217-237.

Thompson, I. (2012). To Chunk or Not to Chunk? *Mathematics Teaching*, 227, 45-48.