Prospective mathematics teachers' views on pedagogical affordances of dynamic geometry systems for understanding geometry theorems

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This paper explores prospective mathematics teachers' views on pedagogical affordances of using technology for understanding geometry at upper secondary level. The study was situated within a four-year teacher preparation program in Turkey. Participants are eleven prospective mathematics teachers who worked in groups of twos and threes. Each group was asked to select and investigate two Geometry theorems one of which is an extra-curricular theorem. They prepared a report which reflected on pedagogical affordances of investigating geometry theorems and their proofs in Geogebra environment. Data obtained from the reports and Geogebra files were analysed using content analysis. Data analysis indicated that prospective teachers mostly reported on pedagogical affordances such as investigating various cases to make generalisations, providing feedback, self-discovery and permanent learning. Different themes emerged for extra-curricular theorems such as providing open-ended problems and creating investigative processes for students.

Keywords: prospective mathematics teachers; geometry theorems; Geogebra; DGS; pedagogical affordances

Introduction

The role of dynamic geometry systems (DGS) in geometry has been a focus of attention in mathematics education research for the last two decades. Research in this domain assumes that use of DGSs would support geometrical reasoning (Healy & Hoyles, 2001; Mariotti, 2001; Straesser, 2001). For example, Mariotti (2001) suggests that constructions in DGS's could help students in accessing the idea of a theorem and "moving from a generic idea of justification towards the idea of validating within a geometrical system" (p. 257). Similarly, Straesser (2001) emphasises that practical geometry in DGS environments could promote an understanding of deductive geometry. On the other hand, Komatsu (2016) questions the traditional approach which focuses on a process from empirical examination to proof construction. He investigates the opposite direction which focuses on proof construction followed by empirical examination. In each approach, the pedagogical affordance is constructing geometrical figures and exploring conjectures (Healy & Hoyles, 2001). In either case, however, "there is no general agreement about the contribution of dynamic geometry in the development of theoretical thinking, and especially in the construction of a meaning for proof" (Mariotti, 2001, p. 251). Furthermore, "measure" and "dragging" features of DGS's could be a limitation since they could result in the "further dilution of the role of proof in the high school geometry" (Chazan, 1993, p. 359).

The aim of the current study is not to discuss the effectiveness of different approaches in using DGS's. It should be mentioned that an evolution towards an understanding of deductive geometry and geometrical proofs is not simple and spontaneous (Mariotti, 2001). To achieve geometrical reasoning, teachers have a crucial role. It depends on the value of geometry tasks and the teachers' roles in enacting these tasks (Komatsu, 2016). This brings us to the importance of teachers' views and knowledge of pedagogical affordance and limitations of DGS's. Therefore, the current study explores prospective mathematics teachers' views on pedagogical affordances of using technology for understanding geometry at upper secondary level.

Contexts of the Study

The study is situated within a four-year teacher preparation program which awards its participants with a diploma for teaching mathematics at the upper secondary level. Participants of the study were eleven prospective mathematics teachers taking an elective course called 'Technology and Mathematics Teaching II'. The aim of the course was to develop an awareness of pedagogical affordances and limitations of using technology. The current study focuses only on the geometry component of the course. For this component, the instructor of the course (the author of this paper) constructed Morley's theorem using Geogebra. The theorem is not in the national curriculum and these kind of theorems will be called 'extra-curricular' theorems in this study. There are two reasons for working on extra-curricular theorems. First, mathematics teachers in Turkey are responsible for guiding students in 'National Competition of Research Projects in Mathematics among High School Students'. In recent years, upper secondary students who took part in this competition have been using Geogebra to explore and conjecture about theorems. The second reason is to give prospective teachers a chance to explore and conjecture about theorems that are new to them. New explorations might not be possible for theorems in school geometry which are already familiar to prospective teachers.

Morley's theorem states that the points of intersection of adjacent trisectors of any triangle form an equilateral triangle. Prospective teachers were asked to do further work which included: (a) justify the Morley theorem using Geogebra (b) modify the theorem and apply it to regular polygons such as square, regular pentagon and regular hexagon using Geogebra (c) explore whether there is a relationship between the inside figure and your original regular polygon? Prospective teachers worked individually in front of computers in a computer lab for two hours to explore Morley's Theorem.

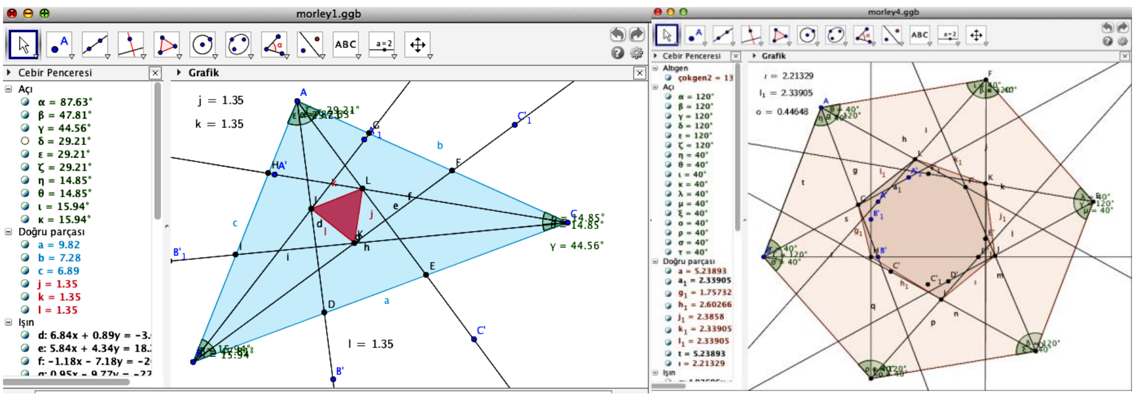


Figure 1. Construction of Morley's Theorem (left) and its modification for a regular hexagon (right)

Prospective teachers were asked to select and investigate two Geometry theorems one of which was an extra-curricular theorem. For the first theorem, they were asked to select a geometry theorem from the mathematics curriculum (grades 9-12), find the related educational goal as it is expressed in the curriculum and investigate the theorem and its proof using Geogebra. For the second theorem, participants were required to select an extra-curricular geometry theorem and investigate the theorem and its proof using Geogebra. For each theorem, they were asked to write a report on affordances of investigating the theorem and its proof in Geogebra environment. Each group presented their investigations to their peers in a computer lab.

Methodology

A qualitative study was conducted to investigate prospective mathematics teachers' views on pedagogical affordances of using technology for understanding geometry. Eleven prospective teachers participated in the study. They worked in groups in twos and threes for two weeks. There were a total of four groups. Data sources are prospective teachers' written reports on pedagogical affordances and Geogebra files they produced to construct the theorems. The data were analysed using content analysis. Themes were specified for pedagogical affordances reported by participants.

Findings

Four groups of prospective teachers selected and investigated a total of twelve theorems. In this section these theorems, their constructions using Geogebra and themes emerging from participants' reports regarding pedagogical affordances of Geogebra will be presented for each group.

The first group selected a theorem from grade 9. The theorem states that "all three altitudes of a triangle intersect at a point which is called orthocenter". They constructed the theorem as shown in Figure 2.

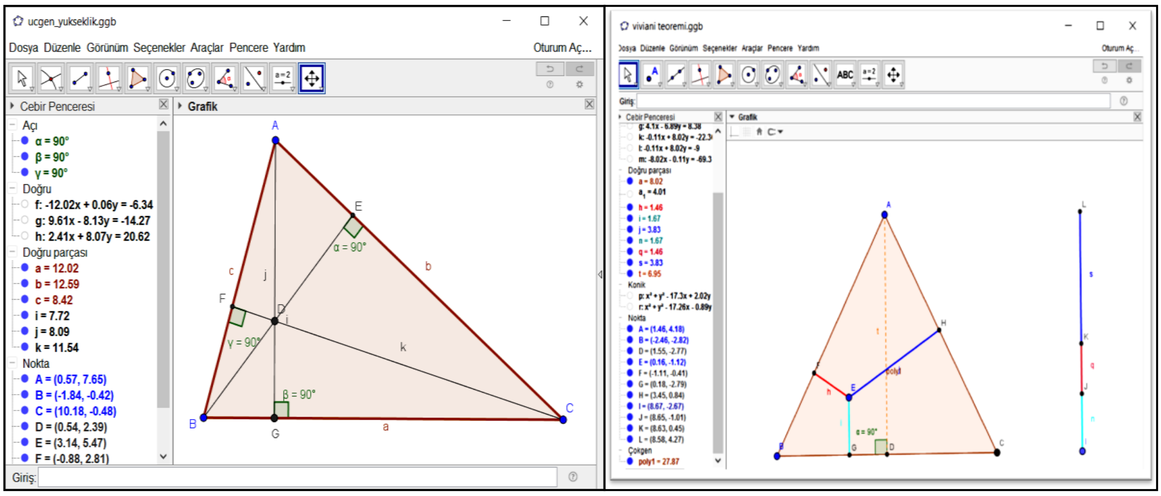
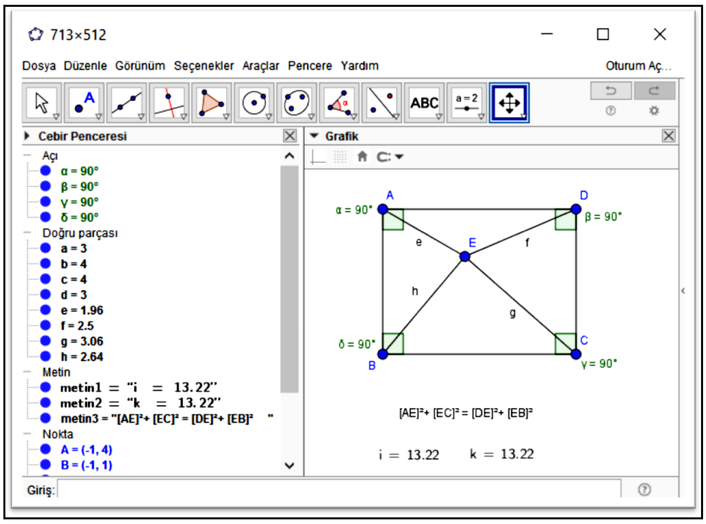


Figure 2. First group's theorems: one from grade 9 (left) and extra-curricular one (right)

The group reported on various pedagogical affordances. They stated that investigating the theorem using Geogebra is more convincing since it is more visual; promotes learning independently and permanent learning; and makes it possible to examine various triangles. They also wrote that different cases could be investigated using Geogebra.

The first group's second theorem was Viviani's Theorem (See the second construction on the right in Figure 2) which states that "the sum of the distances from any interior point to the sides of an equilateral triangle equals the length of the triangle's altitude". The group report mentioned various pedagogical affordances of using Geogebra. They stated that one could examine various cases and check the validity of the theorem for all cases; justify by dragging the point; and make new discoveries.

The second group's first theorem was from grade 9: "Let P be an interior point of a rectangle ABCD. If E is joined to each of the vertices of the rectangle then $|AE|^2 + |EC|^2 = |BE|^2 + |ED|^2$ (See Figure 3)".



The construction of the theorem using Geogebra is shown in Figure 3. The group report stated various pedagogical affordances of using Geogebra such as promoting permanent learning; seeing the effects of variables on the dependent variables (referring to free objects and dependent objects in Geogebra) and examining various cases.

Figure 3. Second group's first theorem from grade 9

The second group selected Leibnitz Theorem as an extra-curricular theorem: "Let P be a point in the plane of the triangle ABC and G be the centroid of the triangle. Then the equations in Figure 4 hold". Themes emerged from the analysis of second group's report concerning this theorem are as follows: getting students' attention; justification helps students understand the theorem; meaningful learning; being able to see the effects of variables on the dependent variables (referring to free objects and dependent objects in Geogebra); promoting new discoveries; monitoring students' learning through personal worksheets.

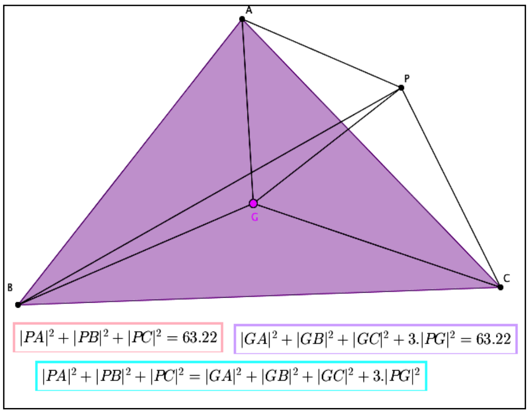


Figure 4. Second group's second theorem (extra-curricular)

The third group's first theorem was from grade 10: "Let P be a point in the interior of a parallelogram ABCD. Then A(ACP)+A(BPD)=A(APB)+A(BPD)". They constructed the theorem as shown in Figure 5.

The group reported various pedagogical affordances of using Geogebra. They mentioned that Geogebra could help students examine the theorem for all cases, get immediate feedback from the computer by observing various cases, learn independently, be more confident and creative and work at their own pace.

The third group's second theorem states that "Let M be the midpoint of a chord PQ of a circle, through which two other chords AB and CD are drawn; AD and BC intersect chord PQ at X and Y correspondingly. Then M is the midpoint of XY". They constructed the theorem in Geogebra environment as shown in Figure 5. In their report, they mentioned that it was difficult to explore the theorem using paper-and-pencil method but Geogebra was practical and explanatory and provided open-ended problems.

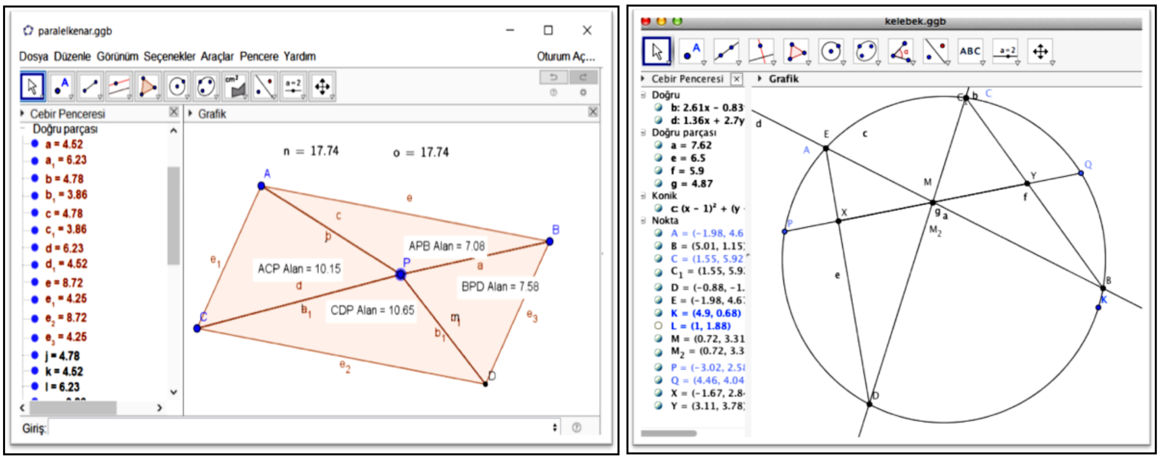


Figure 5. Third group's theorems: one from grade 10 (left) and extra-curricular one (right)

The fourth group's first theorem was from grade 10: "If A, B and C are points on a circle where the line AC is a diameter of the circle, then the angle ∠ABC is a right angle". They constructed the theorem using Geogebra as shown in Figure 6.

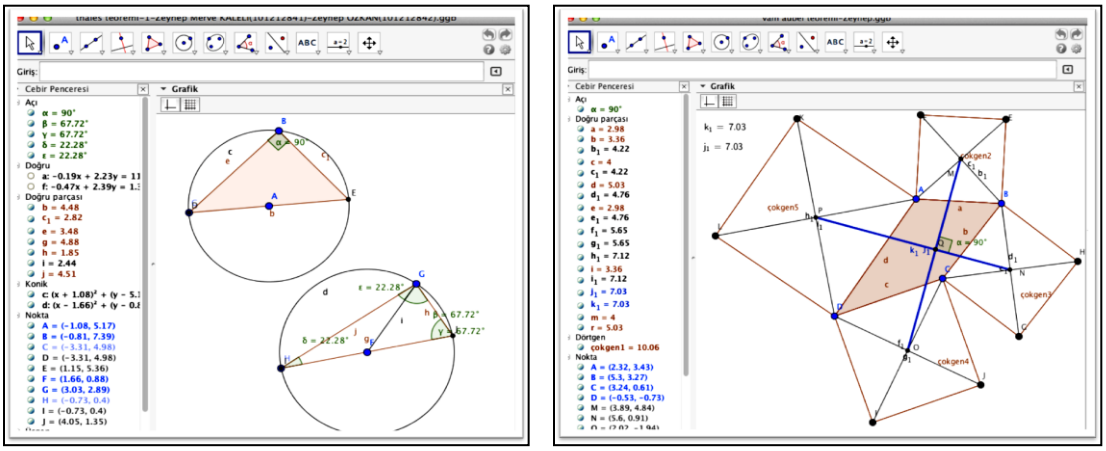


Figure 6. Fourth group's theorems: one from grade 10 (left) and extra-curricular one (right)

Themes emerged from the analysis of fourth group's report on the pedagogical affordances of using Geogebra for the theorem is as follows: avoiding long calculations, providing multiple representations, acknowledging the value of the role of technology in mathematics, dynamic constructions, effective participation, opportunity for after-class work, permanent learning and explanation on "what comes from where".

This group's second theorem was 'van Aubel's theorem': "Given an arbitrary planar quadrilateral, place a square outwardly on each side, and connect the centers of opposite squares. Then the two lines are of equal length and cross at a right angle (see Figure 6). Themes emerged from the analysis of fourth group's report on the pedagogical affordances of using Geogebra for the theorem above is as follows: promoting self-confidence using extra-curricular theorems; getting students' attentions; reasoning; analytical thinking; permanent learning by relating it to theorems in the curriculum.

Discussion and Conclusion

The data indicated that prospective mathematics teachers mostly reported on pedagogical affordances of using Geogebra such as examining various cases, permanent learning and learning independently. Although they reported that Geogebra could promote self-discovery, they used the software just to construct the theorem rather than exploring its proof. Most of the themes with regard to pedagogical affordances were general educational statements such as “permanent learning’’, “getting students’ attentions” and “critical thinking”. Therefore, these themes need further investigation which leads us to the following questions: “Are prospective mathematics teachers’ views influenced by voices of others? (e.g. teacher preparation program)” and “Can prospective mathematics teachers articulate their ideas on pedagogical affordances of using technology?”

Another important finding is that themes emerged for extra-curricular theorems were different from theorems in the curriculum such as providing open-ended problems and creating investigative processes for students. This finding indicated that extra-curricular theorem gave prospective teachers a chance to explore and conjecture about theorems that were new to them. Therefore, investigations of such theorems could be suggested for teacher educators to be used in teacher preparation programs. Further research is needed to explore effective ways to develop prospective mathematics teachers’ awareness and knowledge of pedagogical affordances and limitations of DGS’s for developing geometrical reasoning.

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