# 'When Mamta met Nancy and Emily to do some mathematics' – what intellectual and personal resources do primary student teachers draw on when doing and considering the teaching of mathematics?

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For a primary student teacher, developing secure content and pedagogical subject knowledge within mathematics is of paramount importance and sometimes a cause of anxiety. As a teacher educator I was keen to explore the intellectual and personal resources students on a BA (Hons) primary initial teacher education (ITE) programme draw upon, to enact their mathematical self ('I') and mathematical teacher identity. Two students participated and the domain of fractions was chosen due to its reputation as being difficult to learn and to teach. The study employed an interpretivist approach, fulfilling an intention to watch and observe rather than to 'intervene'. The four dimensions of Rowland's knowledge were used to review key literature and as the theoretical framework to interrogate findings. The students had very different relationships with mathematics. Both had strong levels of self-concept and self-efficacy in relation to teaching mathematics, with clear strategies arising from high levels of self-regulation.

**Keywords: primary teacher subject knowledge; knowledge quartet; teacher identity**

## Introduction

How student teachers (STs) fare when teaching mathematics is of importance to many stakeholders: the children they teach, their parents/carers, the schools they work in, the university that allows them to qualify and themselves. There is a moral purpose attached to the effective teaching of mathematics (or any subject); as a profession, we want all children to achieve. Regardless of the policy environment, it is essential that student teachers start their teaching career confident about teaching mathematics and committed to professional self-improvement as a reflective practitioner.

My aim was to explore both 'intellectual resources' such as mathematics subject knowledge, the ability to connect and relate mathematical ideas, and 'personal resources' such as confidence, resilience and self-efficacy as contributors to self-regulation (Jain & Dowson, 2009).

In considering the research questions, 'What intellectual or personal resources do Primary Education students draw on, when working on mathematics? How do they relate these to their own teaching of mathematics?', I document the resources drawn upon when two primary student teachers worked on mathematics (fractions) together and considered its teaching. Insights gained are referenced to the four dimensions of the knowledge quartet framework (Rowland, Turner, Thwaites & Huckstep, 2010).

## Research design

### *Locating a research paradigm*

To explore intellectual and personal resources students might draw on to do and consider the teaching of mathematics, I needed access to a genuine epistemic perspective of a student at this point of their journey on their ITE course. If, as Bjerke, Eriksen, Rodal, Smestad and Solomon (2013) assert, the STs' mathematical identities were built upon years of the "apprenticeship of observation" (Lortie, 1975, cited by Ball, 1988, p.2), it was necessary to get to the heart of this. I needed students immersed in doing mathematics out of my normal teaching context to explore 'how' participants engaged with the mathematics, related this to their teaching and to explore any emotions aroused. How did they approach mathematical tasks? What did they draw upon in terms of their own subject knowledge? Were distinctions made between different types (Shulman, 1986) and which other personal qualities and characteristics might be called upon? Through task-focused interviews, I would be able to truly watch, evaluate and use the interview as a conduit for conversation about their mathematician 'I' (Peshkin, 1988) and mathematics teacher identities. Thus, the study was located within the interpretivist paradigm and ethnographic features would allow some research that "gets to the bottom of things, dwells on complexity, and brings us very close to the phenomena we seek to illuminate". (Peshkin, 1993, p.28, cited in O'Donoghue, 2007, p.156).

### *Data collection methods*

I chose mixed methods. An initial questionnaire captured background information: qualifications; previous mathematical experiences and attitudes; self-concept and self-efficacy, expected to be key features of personal resources (Lee, 2009); comprising of self-ascribed levels of enjoyment, understanding, confidence and feelings of competency of own schooling and teaching (on placement) experiences. I conducted a group task-focused audio-recorded interview incorporating: work on rich mathematical (fractions) tasks; discussion arising from the questionnaire; reflection on tasks and discussion of a short video extract of a student teacher James, teaching fractions (Rowland et al., 2010). Three rich tasks were chosen to expose connections made, explore the existence of models in the personal resource base and illuminate beliefs about mathematics: **Rectangle tangle** (NRICH, 2016a), a large rectangle is divided into smaller rectangles and triangles where fractional parts must be determined. This appears non-threatening and allows for an intuitive way into task, gently developing in level of skill required to calculating with fractions. **Chocolate** (NRICH, 2016b), a classroom with chocolate on three tables and each child chooses where to sit to gain most chocolate. This task was chosen for its potential to draw out Lamon's (2012) observation of calculating fractional shares on the composite unit. **Fractions Made Faster (**NRICH, 2016c). The fraction wall was not offered to begin with, in order to observe innate strategies for comparing size of fractions; $1\frac{3}{4} \div \frac{1}{2}$ (Ball, 1988; Ma, 1999). Chosen to explore how the question would be interpreted.

## Findings and discussion

The four dimensions of the Knowledge Quartet Framework (KQF) (Rowland et al., 2010, pp. 35-37) were used as a theoretical framework to interrogate findings. The

KQF is based on extensive study of STs' teaching: **foundation** – elements relating to Common Content Knowledge (CCK), Subject Matter Knowledge (SMK) and beliefs; **transformation** - elements relating more directly to Pedagogical Content Knowledge (PCK), the teaching of or getting ideas across; **connection** – elements requiring connections made across a sequence of teaching episodes (my research focuses on connectionist (Askew, Rhodes, Brown, Wiliam & Johnson, 1997) and relational (Skemp, 1976) thinking) and **contingency** – elements relating to STs' thinking on their feet 'in the moment' of teaching as a response to unanticipated events in the mathematics lesson (Rowland et al., 2010, p. 32). My findings are discussed below using the four dimensions in pairs.

### *Foundation and connection*

The tasks proved to be a rich source for uncovering connected or relational thinking (Askew et al., 1997; Skemp, 1976). Nancy had recently re-taken GCSE mathematics to improve from grade C to A, something of importance to her high level of self-concept in doing mathematics. She appeared to have deeper understanding, particularly in proportional thinking, enabling her to work efficiently when calculating fractions. She saw the 'intertwinement' of many key concepts (Van den Heuvel-Panhuizen, 2008; Streefland, 1991). Strongly relational in her thinking, she saw patterns quickly, demonstrating a high self-efficacy. She was usually able to draw on a range of intellectual resources, i.e. make connections, switch between different representations to check accuracy and use formal methods.

This was in direct contrast to Emily's highly instrumental approach, indicative of a 'shaky foundation' (Lamon, 2012); she did not see relationships or make connections easily. Restricted by a mechanistic approach, possibly due to an impoverished intellectual resource base affected by personal instrumental learning experiences; she found it difficult to make connections. Although she holds GCSE grade B, she appeared unable to activate this mathematics knowledge; many times Nancy took the lead and acted as facilitator to support Emily's understanding. When comparing $\frac{5}{6}$ and $\frac{1}{3}$, though having just explored relationships of equivalent fractions, as modelled by Nancy, she reached an impasse, defaulting to the circular 'pizza' model, the only model she used. She also displayed a misconception during this task, assuming that her 'answer' must include a denominator contained within the question, a possible indication of instrumental learning.

One striking surprise was in the $1\frac{3}{4} \div \frac{1}{2}$ task, (Ball, 1988; Ma, 1999). Nancy displayed similar characteristics to participants in Ball's (1988) study, focusing on fractions rather than division, so gave a story for division by two (Ball, 1988, 1990). However, Emily was fixated on the division, firstly through fear and perhaps panic (Buxton, 1981) then genuine lack of understanding. Her use of language was more precise during this task, stating "$1\frac{3}{4}$ is divided **by** $\frac{1}{2}$" when looking initially at the question. She did not move to talking about dividing **into half,** signalling thinking of division by 2 until she had listened to Nancy's inaccurate story about dividing between two people. I directed Emily's attention to the relations on relations concept (Fosnot & Dolk, 2002).

**Mamta:** Has $1\frac{3}{4}$ got to be divided between two people?

**Emily:** Yes, because the 2 I'm looking at is 2 people.

**Mamta:** But it doesn't…. it doesn't say 2 there, that was what was stumping you at the beginning, what does it say?

**Emily:** It says $\frac{1}{2}$

**Mamta:** So…

Though initially distracted by Nancy's 'false' story, following this exchange, Emily appeared to begin to understand what the task required. (See Figure 1)

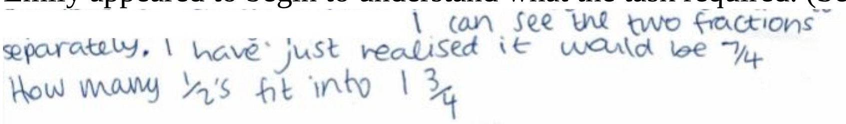


Figure 1 – Emily appears to understand

She moved from seeing $1\frac{3}{4}$ and $\frac{1}{2}$ as two separate fractions, struggling to understand how the ÷ symbol acts upon these two values, to beginning to understand the relationship between them (Fosnot & Dolk, 2002; Lamon, 2012). She made the connection that she was being asked how many $\frac{1}{2}$s fit into $1\frac{3}{4}$. However, when Nancy moved into decimal form in order to address the question (demonstrating her connectionist approach), Emily stated "I understand this is 0.5 now, but I don't understand how to divide by 0.5". This suggests she did not fully see the role of the fraction as a number here; did not connect that to divide by 0.5 means the same thing.

There were affective issues too, as Emily felt disheartened by her inability to see connections and relate with ease, even admonishing herself at times for leaving Nancy 'to do the work', for having to rely on her support to do the mathematics. This in turn made Nancy feel uncomfortable during the tasks and tension was evident as she tried to help while trying not to take over.

## *Transformation and contingency*

Although Nancy had a good awareness of the value of models and other visuals for teaching mathematics effectively, she said, **"**I have to do it formally, to - you know – check", immediately after using a visual representation (model) to check a previous answer when a formal algorithm failed her. A fascinating example of diametrically opposed beliefs of mathematics as a 'do-er' of mathematics and as a 'teacher' of mathematics. Generally, there was a weaker internalised schema for a 'range' of models and flexible application in the case of both STs. When investigating which fraction was higher in value in the chocolate on tables activity, rather than follow Streefland's trajectory (1991), in choosing an appropriate model to depict the fractional value of each, their strategy was to draw 'pizzas', reverting to the use of prototypical models (Ryan & Williams, 2007). This despite the fact that they had just used a bar model (drawn bars of chocolate) derived from a real-life context, a potentially more suitable choice. There was a sense of injustice, particularly from Emily when models such as the fraction wall were demonstrated. She expressed regret at the lack of personal opportunities to manipulate such resources to support her own mathematical thinking and understanding. Emily was aware a much wider range of models existed but these were not yet fully part of her consciousness for teaching.

Though the contingency dimension was less visible within this study, as no teaching took place, strong messages were conveyed regarding the pressure of the professional persona. This invaded the research space and weighed heavily in general; how they were perceived as teachers was deeply important to the students. The professional persona was also a real maxim for self-regulation (Dall'Alba & Sandberg, 2006), with Emily particularly recounting strategies for contingency including over-preparation of responses to questions to and from pupils, thorough checking of planning by the class teacher and similar strategies. This tension may relate to some extent to Feiman-Nemser and Remillard's (1996 cited by Dall'Alba & Sandberg 2006) suggestion of the survival stage of a journey into a profession.

Nancy looked for something else, a space between CCK and PCK; what might be seen as SMK. Nancy had a strong sense of self-concept (Jain & Dowson, 2009) due to her recent GCSE success, yet still struggled with self-efficacy around teaching mathematics, her ability to explain clearly, what she called "the next step on…"

## Concluding thoughts and implications for future practice

This research allowed access to insights which would have proved impossible in a normal teaching context. At times, I felt truly invited into the ST's lived reality, an honesty that could also prove disconcerting. Intellectual resources drawn upon included those expected: existing subject knowledge, the ability to make connections and relate key mathematical ideas. Personal resources employed when considering teaching mathematics were closely related to 'professional persona', typified by closely supported planning, micro-management, and over-preparation of lessons. Their generally weaker schema for the use of models meant the video clip was less revealing; it may be better suited to exploration with more experienced colleagues.

Implications for future work with STs includes careful thought about effective support for students around self-regulation and continued work on subject knowledge development. In particular, SMK arose as an important focus. It seems to straddle both KQF foundation and transformation dimensions, as distinct and specialised knowledge needed to enact the 'explainer' or 'teacher' me (Loewenberg Ball, Hoover Thames & Phelps, 2008). This requires a different type of thinking in moving from 'doing' mathematics to 'considering' teaching it, to mindfully consider concepts and connections. This can be a strange space, not simply concerned with skills, facts, knowledge, or resources and models employed in the physical act of teaching. Both students gave a sense of this space but Nancy had a particular awareness. She consciously felt something was missing, not quite connecting being able to do mathematics and being able to teach it. There were also areas for development in the PCK, as expected at this stage of her ITE course but the SMK seemed to be what she was navigating; almost constructing a bridge between her own CCK and developing PCK. Emily was less interested in this space and so seemed to jump directly between CCK and PCK, leading to a less connected and more instrumental approach.

My difficulty in neatly locating findings into any single dimension of the KQF served to reiterate the 'intertwined' (Van den Heuvel Panzuihen, 2008; Streefland, 1991) nature of mathematics learning and teaching. Multiple thinking is required to enact this wide ranging and sophisticated web of ideas. This may take all teachers many years to develop in a deep way, built upon many years of practice, of development and sophisticated understanding of and mediation between all aspects of subject knowledge. Furthermore, any teacher's ability to negotiate their own, at times complementary and conflicting identities, of mathematician 'I' and mathematics teacher 'I', will further serve to contribute to the mathematics teacher contained within.

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