Student involvement in a workplace inquiry activity: solution of the solar panel problem

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This study explores the effects of two teaching interventions that focus on workplace and inquiry learning in problem solving. This research is part of the European Project MASCIL and refers to the solution of the Solar panel problem, which was assigned to 11 heterogeneous groups of high school students in Greece (year ten). In particular, by using audio and video recordings and qualitative content analysis, we discuss the ways in which collaborative inquiry learning and authentic workplace can be used to bring out and enhance the students' mathematical argumentation. The results of the experimental teachings show that the workplace context and the inquiry activity favored the involvement of students in solving the problem. It is important to note the negotiations to cover the surface with a maximum number of photovoltaic panels that can be placed on the roof of a house and the students' difficulties in trigonometry and three-dimensional space.

Keywords: inquiry activity; authentic workplace situations; trigonometric ratios; students' involvement

Theoretical framework

The term "inquiry-based learning" generally refers to a more student-centered way of learning and teaching, in which students are encouraged to ask questions, direct their own activity while exploring mathematical situations and develop their strategies of finding solutions (Maaß & Artigue, 2013; Borasi, 1992). With this type of teaching, students are asked to work with methods similar to those used by mathematicians and scientists in general. Inquiry approaches in teaching and learning mathematics have been developed in many countries. Investigation and inquiry approaches have a tradition in the United Kingdom (Cockcroft, 1982; Jaworski, 1994; Blair, 2008). In the National Curriculum of 2014, one of the three aims is to ensure students can "reason mathematically by following a line of enquiry" (DfE, 2014). Moreover, the current inquiry learning in mathematics and science in recent years has wide influence in Europe, where large programs are planned and implemented, such as the European project PRIMAS (http://www.primasproject.eu) and the Fibonacci project (http://www.fibonacci-project.eu) (Maaß & Artigue, 2013). The European Project Mascil belongs to this category.

The inquiry is a completely different approach to learning, where both students and teachers leave their traditional roles. Students ask questions, explore ideas in collaboration, engage, plan and monitor their activity, make conjectures, explain their reasoning, prove their results, extend and communicate actively (Artigue & Blomhøj, 2013). Teachers encourage students to explore and connect concepts and procedures, combine different forms of reasoning and develop conceptual understanding, engage students' initiative on challenging mathematical tasks and support independence and responsibility in their own learning.

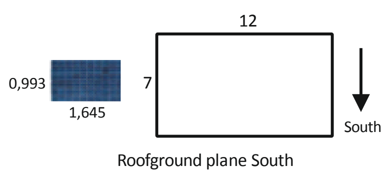
Most researchers agree that one of the basic difficulties of mathematics teaching is the weak connection with real life. According to (Gravemeijer, Cobb, Bowers & Whitenack, 2000), the cognitive difficulties of students are due to the gap which exists between daily life and formalistic mathematics. He believes that the gap can be bridged, because the daily experience and abstract mathematics are not completely different entities. He proposes a progressive mathematization process, in which formalistic mathematics is built as a natural extension of the empirical reality of students (Gravemeijer et al., 2000).

A combination of inquiry learning activities and authentic workplace facilitates challenging problems assigned to students. The use of the activities, which are related to the original work, allows the development of pedagogical strategies that will bring the student closer to abstract mathematics. Students, pretending to be professionals, are invited to combine their knowledge in order to cope with the inquiry activities. Furthermore, students can recognize the limitations and difficulties, which many workers encounter in authentic workplace situations using mathematics (Wake, 2014).

Students are invited to explore and model real problems, which are derived from the authentic world of work. The authentic workplace problems arise from common real life situations and its solution is used in real life workplace. As part of inquiry learning, students engage in activities that encourage them to investigate problematic situations. The workplace in this research was the basic context in which students worked. In the future, students will face authentic problems and the way in which they use their knowledge to solve them is of particular interest. Therefore, the research of innovation depends on the authenticity of the activity and the different orientation on existing experience of students within the school (Triantafillou & Potari, 2014).

Methodology and research questions

During the academic year 2014-2015, two two-hour experimental teachings were organized in a Lyceum in Athens, Greece. In this paper we will present an application entitled "solar panel problem", which was given to 15-year old students. The two experimental teachings were organized according to a teamwork method. Every class was divided into heterogeneous groups of 4-5 students and there were 11 groups examined in total. Enough time was given to the students to think and write the solution of the problem. For the purposes of this research, teaching was recorded and videotaped. Data analysis is qualitative (Collins, Joseph, & Bielaczyc, 2004). The problem has the following formulation:



The solar panels of a photovoltaic installation system will be placed on the roof of a house, in horizontal rows one next to each other. The dimensions of solar panels are 1.645m x 0.993m, while the dimensions of the roof of the house 12m x 7m. Each row usually has a distance 50 cm from the others, in order to create enough space for the technician to be able to repair the system in case of damage or break down. For maximum efficacy in the production of electric current the panels should have south exposure, while their inclination angle is suggested to be 30^0 from the horizontal. According to existing regulations, the panels should be 1m from the perimeter line for safety reasons. Find the maximum number of panels that can be placed on the roof.

The worksheet and a detailed description of the problem are available from the mascil-project website (http://www.mascil-project.eu/classroom-material). Students in both classrooms have been taught trigonometric ratios, but their familiarity with applications in geometric solids (e.g., projections) was rather limited. Moreover, workplace inquiry activities are not so common in the Greek Mathematics Curriculum.

In this paper, we examine the possibilities and difficulties of 15-year old high school students' involvement in an authentic problem of the world of work. The research questions are the following:

(a) Did the solar panel problem promote student inquiry?

(b) Did the workplace help the involvement of students in solving the problem?

Presentation of results

In the students' mathematical activity, we can distinguish three cycles of inquiry: a) dividing the roof area by the area of a panel, b) translation of the problem in three-dimensional space using trigonometric ratios, and c) considering alternative ways to the installation of panels on the roof.

a) In the first cycle of inquiry, students divided the area of the roof by the area of a panel and found an incorrect solution because they ignored the inclination. They calculated the area of the roof as $12 \times 7 = 84$ m^2 and the area of a panel as $1.645 \times 0.993 = 1.633$ m^2. They found by division the result $84 \div 1.633 = 51.439$ and concluded that we can place 51 panels on the roof. Then, they corrected their solution, taking into account the distance of the panels from the edges of the roof. Seven groups of students implemented this method with some variations. Here is an episode:

> S1 ... So in the whole roof we can place 51 panels. 51 fit.
>
> S2 We can't place so many panels on the roof.
>
> S3 We didn't take into account the distance of 1 metre around from the perimeter of the roof.
>
> S1 Then instead of 12m we will use 10m and instead of 7m, 5m. OK?
>
> S3 Now it will be: $10 \times 5 = 50$ m^2, so $50 \div 1.633 = 30.618$, i.e. 30 panels...

For these students it was difficult to use the trigonometry knowledge of right triangles. They also had difficulties in understanding three-dimensional space and the concept of projection. Their narrowness and their inflexibility are associated with persistence in the two-dimensional space and an inability to change their own point of view. However, the failure of students to discover a mathematical solution does not mean the absence of inquiry. Subsequently, students during the class discussion revised their misconceptions.

b) In the second cycle of inquiry, students translated the problem in three-dimensional space and used projections and trigonometric ratios. Four groups of students followed this strategy. Here is another episode:

> S4 ... We shouldn't lie down the panels on the roof. They must be placed with some inclination.
>
> S5 With the inclination, each panel will cover less area on the roof...
>
> S4 ... We find … 35 panels.

Using trigonometry of the right triangle they found: $1.645 \times \cos 30^0 = 1.645 \times 0.866 = 1.425$ m. The projected area of the panel on the roof is: $1.425 \times 0.993 = 1.415$ m^2. The number of panels is: $50 \div 1.415 = 35.336$. If we project the small side we have: $0.993 \times$

$\cos 30^0 = 0.993 \times 0.866 = 0{,}860$m. The projected area of the panel on the roof is still the same: $1.645 \times 0.860 = 1.415$ m^2. So the number of panels is again 35.336.

One group, used trigonometry and the following dialogue took place about respecting the constraints of the problem:

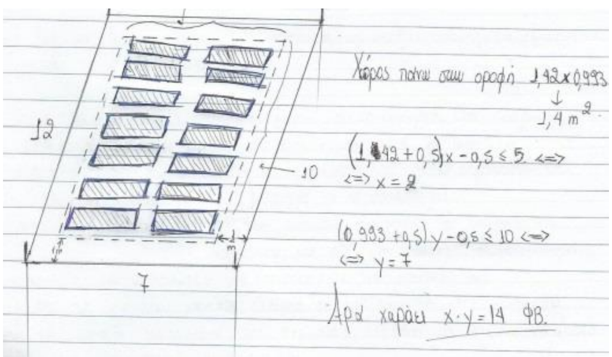
> S6: Cosine and dimensions… The dimensions ... cosine30^0 $\frac{\sqrt{3}}{2}$ [...]
>
> S6: […] Can't we find the area of that and answer how many panels can be placed?
>
> S7: Yes, but it says that there is a distance of 50 cm between each panel.
>
> S6: Let's put it in our solution. Can't we measure it and implement it?
>
> S7: So you should do it one by one. You have to put one panel, leave 50 cm, put another, etc.
>
> S6: I think it comes out 3 × 5. Three here and five there. So, 15 panels fit.
>
> S7: But we want to find the maximum, we can't put them approximately.
>
> S8: I think the distance between the rows should always be 50 cm.

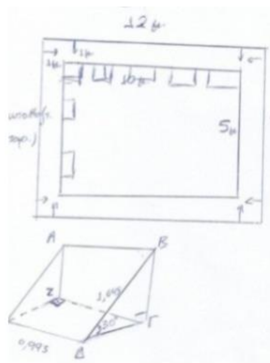
In the previous episodes, students decided to calculate the dimensions of the projection of photovoltaic panels using trigonometric ratios. In addition, workplace restrictions emerged, as students observed that they must be consistent and accurate in their planning.

c) In the third cycle of inquiry, students looked for alternative ways of placing the panels on the roof. We can observe a shift from areas to the lengths of the roof and of the panels. In this context, three teams found alternative ways.

> S9 [...] I'm not sure that on the terrace we can fit 35 panels maximum.
>
> S10 What do you mean?
>
> S9 It's impossible to place 35 panels. They can't all fit; but less than 35 fit. The surface at the edges of the roof isn't enough to put whole panels. We don't have smaller panels. We have to draw them in order to see what will happen in reality.
>
> S11. I agree! We should make the design to see. I think… The area does not matter, but how many times does the length and the width of a panel fit in the two dimensions of the roof?
>
> S9 That's it! Let's go. Design!

During the inquiry activity most student groups undertook the role of designing the placement of photovoltaic panels. The result of their work looks similar to the corresponding products that are produced in authentic workplace situations. In the following schemes two different drawings are shown.





During the whole class discussion, students communicated on the basis of their findings, assessed mathematics achievements of other groups and formulated new arguments. Teachers listened carefully to the ideas expressed by the students and recognized their different ways of thinking. On the one hand, they helped the students to reach the correct answers and on the other hand, they tried to understand their way of thinking. After the presentation of results, negotiations about the best solution followed.

> S12: We found the dimensions of photovoltaic panels as the other groups did and the available space to place the panels is 10 metres by 5 metres. We put the small dimension of the panel along this side (10 m) and we didn't leave space between them, only between the rows here. [...] So to find how many will come along I divided 10 by 0.99 and the result is 10 panels in length. To find the width, I took the other dimension and I made an equation. Let us assume that the distance 1.42 fits x times in the small dimension of the roof that is 5 cm. Then the distance 0.5 can be placed one time less. This should be less than or equal to 5. So eventually we can accommodate two panels. Therefore 10 times 2 equal 20 panels.

In the previous episode there is an alternative way adopted by the group of students. The number of panels in the small dimension is x = 2 and resulted from the solution of the inequality:

$$(1.42 + 0.5)\,x - 0.5 \leq 5 \Leftrightarrow \ldots \Leftrightarrow\ x \leq 2.864$$

It should be noted that students had not considered all the cases to choose which leads to maximizing the number of panels. At the end, the teachers made a summary of the different strategies and presented the best solution to the problem.

Discussion and conclusion

The classroom environment organized by the teachers during the solar panel problem of the Mascil Project, offered rich learning opportunities to students. The objectives of the inquiry activity were widely achieved as children linked mathematics they knew with the workplace context and everyday life, they examined possible outcomes, they cooperated constructively in finding the solution and participated actively in the learning process (Goos, 2004; Artigue & Blomhøj, 2013).

Concerning the first research question, as shown by the presentation of previous results, the students thought about the restrictions of the problem; they explored the solutions and gave their answers for the maximum installation of photovoltaic panels. Although a few teams found the solution (projection of the panels on the roof, use of cosine, effective covering) many creative ideas were exposed by students, which were discussed in the whole class, by using further exchange of arguments and learning enrichment. The right solution is connected with the students' abilities to represent the problem and develop appropriate strategies. Students have negotiated about the number of photovoltaic cells to cover the surface and designed multiple ways of placement. The mathematical dimensions explored by students refer to concepts that were taught as projections in three-dimensional space, trigonometric ratios, areas and arithmetic operations. The open formulation of the problem, which brought out more than one solution method, favored the inquiry activity of students. The inquiry process motivated the students, enhancing the learning of mathematics, which is the aim of the project Mascil.

Concerning the second research question, we conclude that the solar panel problem was an excellent opportunity for active involvement of students in an authentic mathematical activity. Our observations show that initially the connection to the workplace situations surprised high school students and teachers. However, during the process of the activity, the context of the authentic workplace favored the involvement of the whole class to explore the problematic situation and helped students to reflect creatively and cooperatively on the concepts associated with solving the problem. The workplace was a powerful learning motivation for students, which positively affected the finding of the mathematical solution and the negotiation of mathematical meanings in the whole class (Triantafillou & Potari, 2014). Our observations show that the connection to the workplace context was consistent. The context of the problem with the interdisciplinary connections, gave feedback to the reflective dialogue of most students' pragmatic arguments about energy saving and environmental protection in general.

Finally, the inquiry activity of students helped to reflect the formalism and the highlighting of human and social dimension of mathematics. Thus, we can conclude that coupling inquiry learning with authentic workplace situation is a challenge to improve the teaching of mathematics in high school.

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References

Artigue, M., & Blomhøj, M. (2013). Conceptualising inquiry-based education in mathematics. *ZDM*, *45(6)*, 797-810.

Blair, A. (2008). Inquiry teaching. *Mathematics Teaching Incorporating Micromath, 211*, 8-11.

Borasi, R. (1992). *Learning Mathematics through Inquiry*. Portsmouth: Heinemann.

Cockcroft, W. (1982). *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*. London: HMSO.

Collins, A., Joseph, D. & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *Journal of the Learning Sciences, 13*(1), 15–42.

DfE (2014). National curriculum in England: framework. London: Department for Education.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258-9.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, modeling and instructional design. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms* (pp. 225-274). Mahwah, NJ: Lawrence Erlbaum Associates.

Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. London: Falmer Press.

Maaß, K. & Artigue, M. (2013). Implementation of inquiry-based learning in day-to-day teaching: a synthesis. *ZDM*, *45*, 779–795.

Triantafillou, C., & Potari, D. (2014). Revisiting the place value concept in the workplace context: the issue of transfer development. *Educational Studies in Mathematics*, *86*(3), 337–358.

Wake, G. (2014). Making sense of and with mathematics: The interface between academic mathematics and mathematics in practice. *Educational Studies in Mathematics, 86(2)*, 271–290.