The weakest link of Polya's stages through integral problem solving process: what to check

Özkan Ergene[1] and Ali Delice[2]

[1]*Sakarya University, Turkey;* [2]*Marmara University, Turkey*

In this research, the processes university students used to solve integral volume problems were analyzed using Polya's problem solving stages, especially focused on the 'look back' stage. An integral volume test and a semi-structured interview form were administered to the participants. Qualitative data were analysed by descriptive analysis and content analysis. Interestingly, at the end of their integral volume problems' solutions, even though almost all the students performed the 'look back' stage, almost a quarter of the students modified or corrected some parts of their solutions. Discussions reveals that of Polya's stages of problem solving, the 'look back' stage was performed with less care than other stages. Consequently, the participants' movements in the 'look back' stage occurred in three phases, one is from the beginning of solution to the end of the solution or vice versa or among the stages of the solution irregularly.

Keywords: integral; problem solving; Polya's stages; 'look back'

Introduction

Many researchers reveal the necessity of finding the answer of the question "What is the problem?" before examining the problem solving skills (Kilpatrick, 2010; Lester, 1983). The problem which has been described differently by mathematics educators, is defined by Van de Walle (2007) as a work for which individuals have desire and need in order to find a solution without having an organized preparation. The problem defined by Lester (1983) as a situation where the individuals with desire and necessity feel obliged to find a solution even if there is not an available and accessible solution for the individual or the group. In consideration of the definitions presented by researchers, the problem may be considered as "the situation where the individuals with desire or need make an endeavor by using their knowledge and skills to generate solutions for the problems that can arise in the daily life or the obstacles that are supposed to be overcome" (Ergene, 2014). It has been emphasized for years that the problem solving process has a meaning which involves multiple and opposing views (Schoenfeld, 1992).

In mathematics education, problem solving can be seen as a process where mathematical skills are implemented to given situations. In this context, individuals take actions by understanding the problem with respect to given and intended situations in the light of their paradigms during problem solving. Several variables such as skills, process, demographic features, instructor and learning environment may have an effect on problem solving processes (Ergene, 2014). In the consideration of these effects, there are several classifications for examining the problem solving processes in order to determine misunderstandings, absence of the necessary stages and mistakes (Polya, 1957; Gott & Murphy, 1987; Adair, 2010). Polya's problem solving stages involving understanding the problem, making a plan, carrying out a plan and 'looking back' have been actively used for more than a half century in order

to examine the problem solving processes. Within this time, one of the most significant points is that the 'look back' stage, the last stage of Polya's problem solving stages, was usually not studied compared to the first three stages by researchers (Lee, 2015; Jacobbe, 2007).

Regarding the definitions of problem and problem solving process, students have difficulties in understanding and describing especially limit, derivative and integral during teaching and learning mathematics. Many researchers have revealed the difficulty in understanding the concept of integral which is one of the analysis courses (Ergene, 2014; Delice & Sevimli, 2011; Rasslan & Tall, 2002; Ferrini-Mundi J. & Graham, 1994; Orton, 1983). The integral can be examined in two parts. The first one is the indefinite integral that aims to find the primitive form of a function; the second one is the definite integral that involves the practical and geometric solutions such as area-volume calculations (Finney, Thomas, Demana & Waits, 1994). The subject of "Finding the volume of solids of revolution", one of the implementations of definite integral, takes part in the curriculum conducted within the scope of the Bologna Process whose aim is to bring common standards to higher education and academic subjects in Europe. In the solving process of the integral volume problems, geometric solving methods (Delice & Ergene, 2015a) are used as well as the methods of disc, washer and shell (Finney et al., 1994). Visual and analytic solving preferences are made by using spatial skills such as drawing and rotating along with the current solving methods (Delice & Ergene, 2015b). Examining the solution processes of integral volume problems can be important to determine factors such as individuals' choice of method, skills and the usage of multiple representations.

In this study, the solution processes of the integral volume problems were examined with respect to Polya's stages and the 'look back' stage which are encountered rarely in mathematics education research focusing on the changes students make through integral volume problems solving process after the solution is completed The research questions investigated are the following:

1. How do university students use Polya's problem solving stages in the solution of integral volume problems?
2. What have the university students done in the 'look back' stage of the solution processes of integral volume problems?

Methodology

Design and Participants

Since this research aims to examine deeply the processes university students use to solve integral volume problems in the context of Polya's problem solving stages, a qualitative research method was adopted. The design of the research is determined as a case study. Participants of research consist of 142 university students selected from four different faculties of two different state universities by using purposeful sampling technic of random sampling (Table 1). Besides, semi structured interview (SSI) was conducted with 3 or 4 students randomly selected from each faculty.

University	Faculty	Department

A	Faculty of Education	Secondary Mathematics Education
B	Faculty of Education	Primary Mathematics Education
B	Faculty of Science and Letters	Department of Mathematics
B	Engineering Faculty	Departments of Environmental Engineering, Civil Engineering and , Mechanical Engineering which take service course

Table 1. Participants

Data Collection Tools

In the research, Integral volume test (IVT) is used for the purpose of investigating integral volume problems solving process. IVT consists of 7 problems, some of questions selected from Thomas Calculus and the others created by researchers. Expert opinion and trial practice were applied for the content validity. Besides, SSI form have used in order to determine the reasons lying behind students answers given to IVT and in order to analyze those made in the 'look back' stage of problem solving processes.

Data Analysis

Qualitative data were analyzed in two phase. The answers given to the IVT were primarily analyzed with descriptive statistics by using Polya's problem solving stages. Afterwards, each stage was analyzed with content analysis. The answers given to SSI form were analyzed as question-based and were used supporting findings for IVT.

Findings

Polya's Problem Solving Stages

The answers that university students gave to the IVT have been analyzed with Polya's problem solving stages. After the deeply analyze, it has been observed at some stages, performance differences in the stages between the university students. Therefore the researchers have divided stages and they have created an integral volume solving process steps table (Table 2). During the creating process it has been asked two mathematic educations experts who had studies in the integral field, for their opinion. When the realizing percentage of university students at problem solving stages (Table 2) is been examined, it can be seen that the "understand the plan" stage the most realized stage is and more than more to the 'look back' stage the realizing rates decrease. Besides, it has been observed that the Figure-math effect which is taking part at the Carry Out stage is the least realized step. Also at the 'look back' stage it has been noticed that nearly all students have realized the check-out step.

Focusing on 'look back'

At the end of IVT, the working group was questioned with "Have/How you check out your solutions after solving problems?" questions. Nearly the whole participants (94.6%) has answered this question with "yes" and have stated that they control the solving process and what they do at the solving process. When the answers at the IVT are analyzed it has been come across with crossing out, getting into box (get the answer in a checkbox), erasing, emphasizing, controlling which can be indicators for the look back stage (Figure 1).

Stage	**Step**	**Explanation**	**Percentages**

Understand a Plan	Start a solution	Forming a start after reading the problem, in order to find the requests.	100
	Figure appears	Explanation or creation of figure-graphics which are in the IVT	80.7
	Mathematics appears	Making sense or creating mathematical statements relating to the problem	100
Devise a Plan	Determination Method	Choosing the method which mediate the volume of the solids of revolution (Disc, Washer and Shell methods)	72.4
	Determination Formula	Choosing the algebraic statements and formulas which will mediate the solving	68.4
	Drawing a figure	Choosing the area during the solution and considering the axis of revolution and determining the axis focused borders	60.5
Carry Out the Plan	Calculate	Having the required procedures via the determined formulas or algebraic statements	71.4
	Figure-Math effect	Determining the revolutions solids which is given or drawn and the mathematical transfer	48.4
	Result	To find a numerical or verbal statement after doing the procedures	68.3
Look Back	Check-out	Analyzing the solving process again, approve or repeat the things have done.	94.6
	Erasing-Crossing out	Crossing out, getting into box or emphasizing the founded results after being sure of the trueness.	52.4
	Others	Rewriting or scratching a part of statements which take place in the solving process	24.4

Table 2. The explanations and implementation percentage of Polya's problem solving stages

More than the half of university students (52.4%) has stated that the process is finished with various marks around the numerical results. Also a small percentage of university students (24.4%) used erasing or scratching stages.

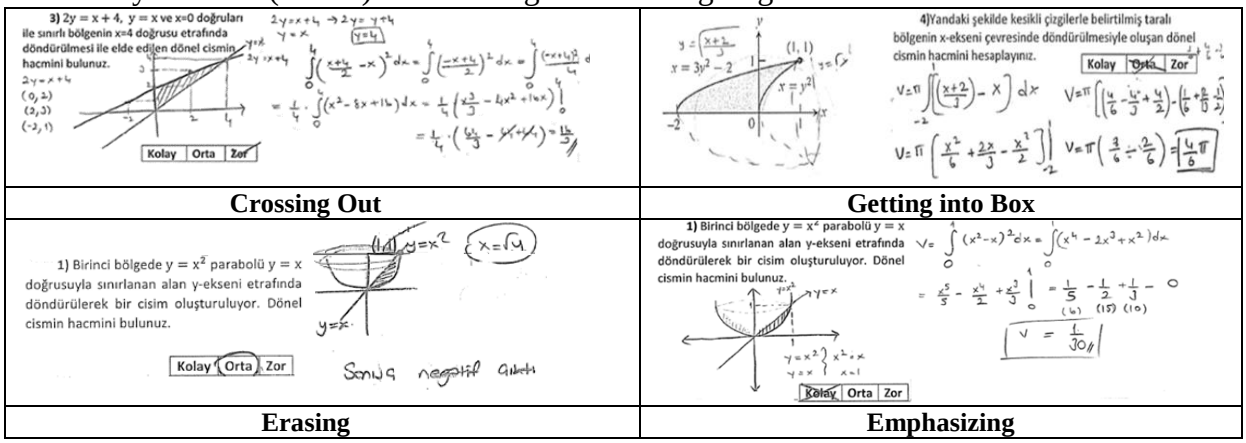

Figure1. Students' answers to IVT test; the indicator of 'look back' stage

SSI Form's Findings

In SSI, the university students were asked questions below.

- How did you found IVT? Is it hard or easy? Why?
- When getting the result, have you think for checking?
- How did you check? Is it useful for you?

After the interview, the students were evaluated according to their departments they are associated with, in different ways of the IVT difficulty level which includes the shape, solving method, integration processes as easy (24%) – average (42%) – hard (34%). Even if nearly all of the students (92.6%) have stated that they have more

times analyzed their solutions, when they were asked what they did, they said that they analyzed formula, method or result focused (S5 and S10).

> S5: All questions which I solved, I start to check from end to first step carefully, this method is so useful.
>
> S10: I have check but only I have used my eyes and I have started from solving methods like disc, shell or washer.

All of the students have stated that they found the evaluation useful. Besides, they have started to control the steps in points like from beginning of solution to end of the solution (S3) or vice versa (S5). Also they start the controlled from the method (S10) or from the formula (S7) which is needed for the solution.

> S3: I start to check from reading a questions and I investigated all steps in my solutions.
>
> S7: I'm not busy in all steps because some steps are only memorizing, so I have examined integral calculate, formula or drawing area.

Also the S14 students who did not control the solving process, has stated that "there is no reason why he didn't underline the result, just because it is a habit."

Discussion and Conclusion

At this research which analyzes the integral volume problems of university students according to Polya's problem solving stages, it has been observed that problem solving processes are cyclic process with transitions between stages. Integral volume problem solving process steps (Table 2) can be handled as functions which are photographing the process.

The things done with the aim of evaluation and controlling of the integral volume problem solving process are inadequate because they don't fulfil the necessary situations. In addition to this there are differences between university students in terms of evaluation and controlling of problem solving processes. Even when the university students say that they are controlling the solving process, the findings of this research shows that those made in the 'look back' stage are not enough. Many students appear to believe that they finish their mission after solving a problem Lee, 2015; Jacobbe, 2007), in reality they miss an important and instructive phase of the work (Polya, 1973). At this situation, it is seen that, Polya's stages in integral volume problems the 'look back' stage is less care than the others as in the studies of Lee (2015).

After the participants get the result, they performed controlling the process from the beginning of solution to the end of the solution or vice versa. Some of the participants performed controlling merely; by the method they determined, shape they draw, mathematical operations they have done through the problem solving process. These situations has shown that university students have actualized in evaluation phase that we can call a "down-upper" in the 'look back' stage during the process of solution of the integral volume problems with respect to the direction and "mixed method" with respect to the context. Consequently, it appeared that the participants' movements in the 'look back' stages occurred in three phases one is from the beginning of solution to the end of the solution or vice versa or among the stages of the solution irregularly.

References

Adair, J. (2010). Decision and problem solving stragies. 2nd Ed. The Sunday Times. British Library Cataloguing-in-Publication Data

Delice, A. & Ergene, Ö. (2015a). İntegral Hacim Problemleri Çözüm Süreçlerinin Bireysel İlişkiler Bağlamında İncelenmesi; Disk, Pul ve Kabuk Yöntemleri. Sakarya University Journal of Education, 5/1. p. 37-54

Delice, A. & Ergene, Ö. (2015b). Investigation of Drawings and Rotation Skills in Integral Volume Problems Solving Process within Communities of Practices. Journal of Theory and Practice in Education. 11(4) pp.1288-1309

Delice, A., & Sevimli, E. (2011). İntegral kavramının öğretiminde konu sıralamasının kavram imgeleri bağlamında incelenmesi; belirli ve belirsiz integraller. Pamukkale Üniversitesi. Eğitim Fakültesi Dergisi. 30(1), 51-62

Ergene, Ö. (2014). İntegral hacim problemleri çözüm sürecindeki bireysel ilişkilerin uygulama topluluğu bağlamında incelenmesi (Unpublished master dissertation). Marmara Üniversitesi. İstanbul

Ferrini-Mundi, J. & Graham, K. (1994) Research in calculus learning: understanding of limits, derivatives and ıntegrals. In J. J. Kaput & E. Dubinsky (eds.), Research Issues in Undergraduate Mathematics Learning, MAA (Notes 33, pp.31-45). Washington DC: MAA.

Finney, R., Thomas, G., Demana, F. & Waits, B. (1994), Calculus, 2th Edition Addison- Wesley Publishing Company.

Gott, R., & Murphy, P. (1987). *APU Science Reports for Teachers9: Students Aged 13 to 15* – National STEM Centre. Department of Education and Science.

Jacobbe, T. (2007). Using Polya to overcome translation difficulties. Mathematics Teacher, 101(5), 390–393.

Kilpatrick, J. (2010). Research on problem solving in mathematics. *School Science and Mathematics.* 78(3), 189-192

Lee, S. (2015). Students' Use of "Look Back" Strategies in Multiple Solution Methods. *International Journal of Sciences and Math Education.* DOI 10.1007/s10763-014-9599-9

Lester, F. K., 1983, *Trends and Issues in Mathematical Problem Solving Research*, in Lesh & Landau, eds. (1983), page. 229-261.

Polya, G. (1957). *How to solve it?* (2 nd Ed.). Princeton, N.J.: Princeton University Press

Rasslan, S. & Tall, D. (2002). Definitions and images for the definite integral concept. In Cockburn A., Nardi, E. (Eds.). *Proceedings of the 26th PME*, 4, 89-96.

Orton, A. (1983). Student's Understanding of integration. *Educational Studies in Mathematics*, 14(1), 1-18.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). New York: MacMillan.

Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th Ed.). Boston, MA: Pearson /Allyn and National Council of Teachers of Mathematics, (2000). Principles and standards for school mathematics (Reston, VA: National Council of Teachers of Mathematics