Raising attainment of middle-lower attainment GCSE students

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Year 11 students throughout England are currently attending ''intervention'' classes designed to raise their mathematics attainment ahead of their GCSE examinations, using methods of instruction that seem to have proven unsuccessful the first time they were taught concepts, and then again, unsuccessfully, in subsequent lessons. This paper reports on a study of one class of lower to middle attaining Year 11 GCSE students who have been taught algebraic concepts using multiple representations and using teaching designed to allow them to reason from key known facts. Qualitative data from lesson observation, student and teacher interviews and students' work is analysed to begin to construct a narrative interpretation of this small-scale classroom enquiry. This analysis demonstrates some promising outcomes in terms of pupils' perceptions of learning mathematics and their use of iconic representations of concepts.

Keywords: GCSE attainment; multiple representations; mathematical reasoning

Context

Issues with pupils' algebraic reasoning in UK schools are well documented, not least in the ICCAMs project (Hodgen, Küchemann, Brown, & Coe, 2009). Failure of the majority of UK learners to master mathematical concepts is equally well documented from Ofsted's Made to Measure (2012) report, to OECDs comparisons in PISA (2010) to many studies that suggest that the dominant pedagogy in UK classrooms privileges instrumental over relational understanding (Skemp, 1976). Linda Darling-Hammond looks at international comparisons to conclude that some regions have a curriculum that is a mile wide and an inch deep (2006) so that pupils are continually being taught the same concepts using the same learning models year after year, frequently justified by folk beliefs that are not necessarily supported by evidence of pupils' learning. This seems to describe the English school in this study. Within this context, this small scale enquiry explores the impact of enactive and iconic representations of concepts (Bruner, 1996) upon lower to middle attaining GCSE students' understanding of linear relationships.

The co-constructed classroom intervention was designed to provide GCSE pupils with experiences that support their knowledge of variable, linear expressions and linear equations. I designed the intervention in partnership with the class teacher and head of mathematics of the study school. Through this, the teachers expressed a desire to improve their pupils' attainment and contributed knowledge of the context for learning in their school, whereas I offered research-informed models that contributed to the teachers' pedagogical mathematical knowledge (Shulman, 1987). The mixed methods study includes quantitative data from GCSE assessments and qualitative data from observations, interviews and an interpretive analysis of pupils' responses to GCSE questions. This paper will focus on qualitative data from the first of two schools involved in the study.

The Study

Two schools 'in challenging circumstances' agreed to participate in the GCSE pilot project designed to raise attainment for lower to middle attaining pupils by using multiple representations of algebraic concepts and by using learning models designed to allow them to reason from key known facts (Watson, 2009). The intervention was designed in three stages: stage 1, linear expressions and linear equations; stage 2, sequences with linear general terms; stage 3, graphing linear relationships. The intervention class of 20 pupils was set 5 of 6 and the control group was the set above and below the intervention group. Learning models were agreed between me and the class teacher, and were largely informed by research that supports a meaningful concept of variable by allowing pupils to draw reasoned connections that are informed by the structure of mathematical problems (Watson, 2009: Andrews & Sayers 2012).

Within the study a pedagogical model similar to the connectionist teacher orientation (Askew, 2002) was adopted. In practice, the Year 11 pupils were provided with physical or visual problems chosen to stimulate connections with algebraic concepts. In turn, the teacher was able to respond to pupils' reactions in order to guide them to make connections with their experience and the symbolic representation of the concept as well as provide stimuli to conflict misconceptions.

In the first stage of the study, pupils were introduced to iconic and enactive representations of numeric expressions using arrays and images constructed from counters. Pupils matched iconic and symbolic representations as shown below:

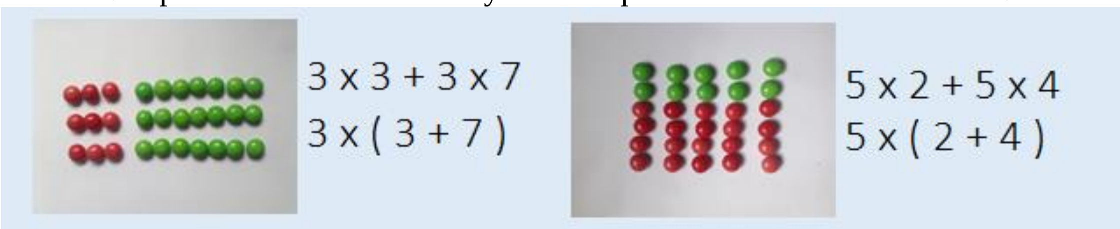

Figure 1: Iconic and symbolic representations of expressions

In each case, the expression had a value of 30 to encourage the pupils to focus on the structure of the expression rather than seek an answer from calculation. The class teacher and I believed that the existing culture of learning mathematics was dominated by repeated practice of manipulating symbolic algebraic representations. In order to prepare them for using other representations of algebraic concepts we needed to provide pupils with experiences of using iconic representations of numeric expressions. Within the intervention, their first encounter of variable was modeled using a variable 'number of sweets in a cup' activity. This provided the context for interpreting linear equations as two equivalent linear expressions. This led to a quasi-balance model where pupils were encouraged to ensure that the number of sweets in two groups remained equivalent. Figure 2 shows the pupils' use of this model:

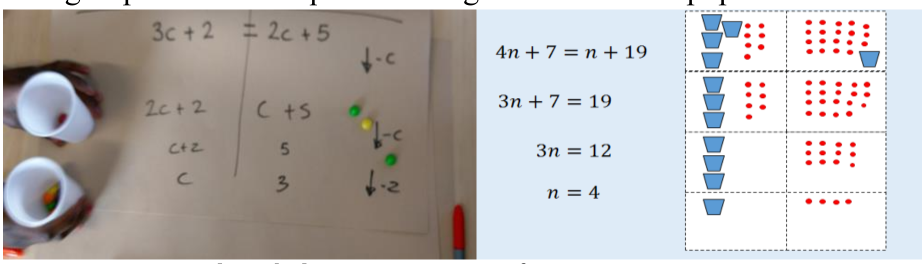

Figure 2: Iconic and symbolic representations of expressions

Pupils were guided to use enactive and symbolic models simultaneously, as illustrated above. The purpose of this paper is not to discuss the structure of the learning models in depth, but to report the pupils' responses to the models. However, parallels can be drawn between this and the balance model described by Andrews (2012). Clearly, the models are restricted to positive solutions and cannot be used authentically for negative values of *n*. However, it was our goal to provide pupils with an authentic experience of variable in the first part of the study, so that solutions for any value of *n* can be imagined once insight into balancing equivalent expressions had been realised.

A sample of the qualitative data from the project, presented using a narrative analysis (Clough, 2002) based on observations, assessments and interviews with one class of GCSE pupils, is described below. The cases of Alex and Ellie do not attempt to present generalisable outcomes of the study, but to illustrate the impact of the enactive and iconic representations upon the pupils' perceptions of learning algebra.

Alex and Ellie

Alex started Year 7 with mathematics attainment that was just above average for his year group. He was placed in set 2 for mathematics but moved to a middle-attainment set in Year 8 because the work was too hard. By Year 9, he was in set 4 and his attainment had slipped below national and local averages in mathematics. In Year 11, Alex appreciates the importance of achieving a GCSE Grade C. He claims that when he sees a GCSE question he assumes it is going to be tricky and that there is a lot of information to take in. He tries to tackle questions and attempts every question by repeatedly reading it and hoping that there is a fifty-fifty chance of getting it right. In mathematics lessons, Alex is co-operative and usually attentive, but sometimes appears withdrawn. He rarely volunteers answers in class discussions and avoids eye contact with his teacher when questions are posed. However, when pushed, he can explain his understanding of a problem to the class. He works co-operatively with the pupils that he sits with in lessons and they appear to laugh along with each other when they see 'stupid' mistakes.

At the start of the algebra project he did not think that the images used were going to help him to improve his understanding of algebra. By the end of the project, he seemed to have changed his perception:

> At first I was confused. But then it got easier when you do more. It's good. It gets stuck in your brain. When I realised I understood it, I wanted to carry on

Alex describes the disturbance that he and his peers felt when the culture of learning mathematics was altered. His expectations of what would happen in his mathematics classroom were being disturbed, which, as Bruner (1996) has described, causes anxiety that leads pupils to resist the change in culture. Through succeeding in tasks during the project (Coe, Aloisi, Higgins. & Major, 2014), Alex's perception was changing:

> It started to seem a lot easier. I remembered it due to its easiness.

He recalled the 'number of sweets in a cup' model that he had used in the classroom to try to make sense of a variable in a linear expression:

> I found the x bit really hard. Then I knew it was what's inside [the cup]. I can solve equations because I've got a better understanding. It's clearer. But you should have the cups in every lesson. Go over it on a weekly basis.

As this last comment suggests, Alex was reluctant to let go of the enactive model in most lessons, but gradually started to use an iconic representation by drawing images of the cups and sweets in his work. He seems to realise that he needs the iconic representation to allow him to understand more clearly. Alex's perception of the learning model was positive, he valued the model because he had learned through it and this success was motivating (Coe et al, 2014).

Alex was asked whether he thought that other Year 11 pupils would benefit from using images and practical models.

> They should give everything a chance. At the start I would have said no… Now… you should carry on. It's much easier. I couldn't see how adding pictures would be helpful, but I found it really is.

This view was supported by Alex's answers in the project assessments. In the pre-test he had demonstrated no understanding of how to solve the following equation:

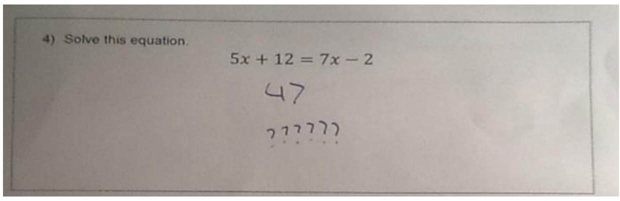

Figure 3: Alex's pre-test solution

The question marks suggest that Alex wanted to solve the problem, but lacked any starting point. The next image illustrates Alex's use of an iconic representation of the problem to successfully solve the equation in the post-test:

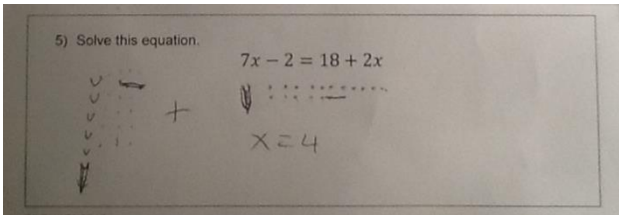

Figure 4: Alex's post-test solution

Whilst it is not clear from figure 4 how Alex has dealt with the –2 in the equation, it does appear that he has crossed out two cups in each part of the image to represent 'subtract 2*x* from both sides of the equation'. Alex's teacher reported that he successfully solved similar linear equations in lessons in the next term, again relying on a combination of iconic and symbolic representations. This supports Alex's own perception that his understanding of linear equations is becoming clearer.

There is no scope in this paper to illustrate every student's learning. However, Ellie's perceptions offer a contrast to Alex's story because she was more resistant to the intervention. In lessons, Ellie was very vocal in expressing her distrust of the learning models that she was being presented with. She explained why:

> It was stupid. It doesn't look right. I don't know if… [images]… helped but I didn't understand. On the test I can work it out- go up in times tables, calculate it.

This comment suggests that Ellie is actually using a repeated addition approach to solving equations that are typically presented in the format $ax+b=c$. Her answer in this post-test question illuminates this response and the error made in her calculation;

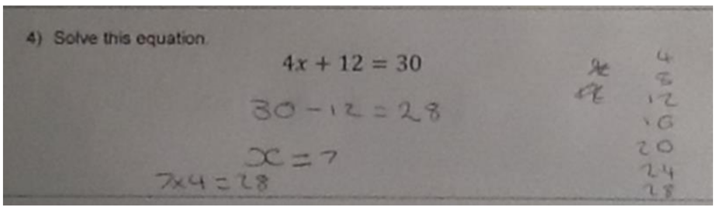

Figure 5: Ellie's pre-test solution

This suggests that the incorrect use of 28 is possibly an association error strengthened by her belief that the value of the $4x$ term must be a positive integer. Her perception that the enactive and iconic models are ineffective stems from her belief that she does not need them because she can solve equations already. She did not mention her inability to solve equations that equate two linear expressions, nor her unwillingness to attempt to solve linear equations that involve expressions with brackets. As Festinger's (1957) theory of cognitive dissonance illustrates, Ellie is very likely to resist new learning models, if the model that she currently uses is perceived as successful and if her existing knowledge has been hard-fought. Her comments in lessons suggest that both of these conditions are true. Furthermore, her nonchalant shrug, when asked to solve equations like $5x+12=7x-2$ supports the interpretation that her inability to do this fails to provide sufficient cognitive dissonance to motivate Ellie to seek new knowledge that could be applied to a broader range of linear equations.

Interestingly, it is the problem in Figure 6 that indicated any progress from Ellie's assessment in the pre- and post-test. Visibly, Ellie has annotated the diagram of the triangle in a manner that suggests that she has valued the enactive and iconic representations used in lessons even though her rejection of the cups model was resolute.

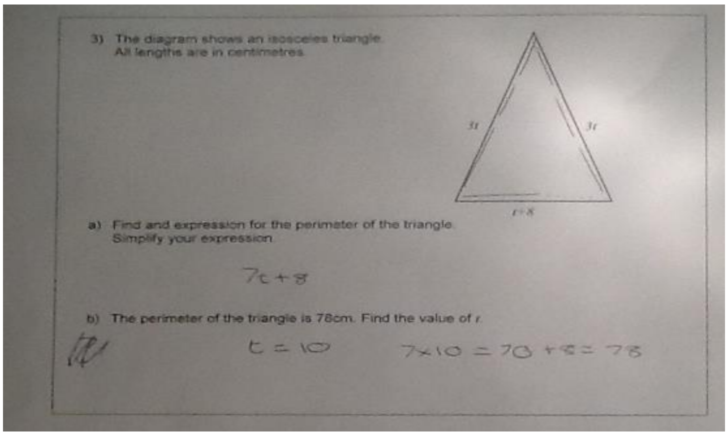

Figure 6: Ellie's post-test solution

Initial findings

By the end of the first stage of the intervention, all pupils said that they supported the use of the enactive and iconic models. All pupils had achieved some success in the post-test for stage 1, suggesting that their initial reservations changed once they had understood some of the concepts learned. Most pupils found verbal justification and reasoning challenging and did not enjoy this aspect of the intervention. This suggests that we changed the context for learning too much in stage 1. Pupils resisted forced pair work, but most adopted informal collaborations willingly after the first two lessons. Post-tests for the study group showed that more questions were attempted by most pupils, all pupils increased their attainment in at least one question, although some pupils confused equations and expressions in the post-test. A diagnostic interpretation of the post-test and solutions in lessons for the intervention class showed that pupils held onto iconic representations alongside semi-formal symbolic representations. Most of the solutions had increased 'meaning' in relation to the concepts being learned.

The assessment data also supports the value of the study, with the intervention class making far greater progress than the control groups in stage 1 of the study. However, the post-test was not taken seriously by the control group and so these outcomes cannot be used reliably at this stage.

I would not argue that these pupils achieved fluency in solving all linear relationships problems, but this small scale study shows that they have addressed some misconceptions about the use and the meaning of symbols in linear relationships and that they have started to attach some meaning to the solutions that they produce. For the pupils to gain a relational understanding, the interaction must be grounded in the experience of the learner and not the teacher's conception of what the learner's experience should be. The models used in this study are not intended to undermine the logic of abstractions that allow pupils to solve equations for any real number, but were designed to allow the symbolic abstractions that occur to be rooted in the logic of the pupils' experience. In using enactive and iconic representations, we needed to ensure that "authenticating a part of formal mathematics" (Hart, 1993, p.33 cited in Mason & Johnston-Wilder, 2004) was part of the pupils' experience once counters, cups and sticks are removed. As Alex and Ellie's cases suggest, these pupils are holding on to iconic representations, suggesting that they do not yet recognise meaning in the symbolic representations of the algebra problems that they encounter. However, for pupils who have been taught these concepts five or more times, without succeeding through a procedural understanding, I believe that their emerging insight through iconic representations cannot be a bad thing.

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