Designing a clinical interview to assess algebraic reasoning skills

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The Irish Primary Mathematics Curriculum consists of five content strands, namely Number, Algebra, Data, Measure and Shape and Space (Government of Ireland, 1999). Children engage with material from all strands throughout their primary education. Thus the Irish education system fulfills the widespread recommendations in research of commencing algebra early (e.g. Kaput, 2008; Carpenter and Levi, 2000; Cai and Knuth, 2011). However, national and international studies of student attainment suggest that many children in Irish schools may not be developing robust skills in algebraic reasoning (Eivers et al., 2010; OECD, 2009; Eivers and Clerkin, 2011). In my research I plan to investigate to what extent children in Irish primary schools are developing skills in algebraic reasoning. In order to do so, an assessment instrument must be developed which will facilitate an exploration of children's thinking as their skills develop. The clinical interview is an instrument which allows access to children's emergent thinking (Ginsburg, 1997; Piaget, 1929) and in this paper I discuss the design of a clinical interview with specific relevance to children's skills in algebraic reasoning.

Keywords: algebraic reasoning, clinical interview, assessment, primary school.

Introduction

Kaput, Blanton and Moreno (2008) suggest that algebraic reasoning comprises the skills of "generalising; expressing generalisations, and using specialised systems of symbols to reason with the generalisations" (p. 21). Algebraic reasoning relates to children's ability to think logically about quantities (known or unknown) and the relationships between them. Carpenter and Levi (2000) define algebraic reasoning by identifying two central themes, namely "making generalisations and […] using symbols to represent mathematical ideas and to represent and solve problems" (p. 2). Algebraic reasoning skills are a precursor to children succeeding in the study of formal algebra. For formal algebra to be accessible to all children, rather than to a high-attaining minority, it is necessary for instruction in algebraic reasoning to commence early in primary school (Kaput, 1998). Additionally, while in primary school, the engagement of children in algebraic reasoning supports their understanding of the structure of number and provides tools for successful engagement in problem solving (Kaput, 1998; Schifter et al., 2008; Carpenter and Levi, 2000).

In Ireland, the recommendation to commence study in algebra early is satisfied as the 1999 Primary School Curriculum Mathematics (PSMC) advocates that the study of algebra begin in the first year of primary school (Government of Ireland 1999). In the 2003 review of the implementation of the revised Primary School Curriculum the Inspectorate of the Department of Education and Skills (Government of Ireland, 2005) reported that "[i]n the majority of classrooms there was good

practice in relation to the implementation of the algebra strand" (p. 28). On the whole the inspectorate seemed satisfied with how the algebra component of the new curriculum was being taught. Considering that algebra commences early in the primary curriculum and that the inspectorate is satisfied with how it is taught, the findings of research into children's attainment in algebra are surprising. The National Assessments of Mathematics and English Reading 2009 ((NAMER 2009) (Eivers et al., 2010) assessed the mathematical attainment of children in 2nd class (aged 8 or 9 years) and 6th class (aged 12 or 13 years). The research found that 90% of pupils in the second class cohort did not "show understanding of the associative property of addition; the connection between two-step word problems and their corresponding numerical expressions; and the correct use of the symbols =, <, >" (p. 39). An examination of the sixth class cohort of NAMER 2009 shows that only 10% of pupils in this study achieved a mastery of algebra that enabled them to evaluate "linear expressions and one-step equations" (ibid. p. 42). NAMER 2009 utilised a set protocol of questions to assess children and is thus limited in identifying whether children used algebraic reasoning in arriving at solutions to test items. The findings are thus taken as an approximation of the level of skill development in algebraic reasoning of the population of sixth class pupils in Irish primary schools.

The purpose of my research is to examine to what extent children are developing skills of algebraic reasoning as they progress through primary school in Ireland. To this end, I am aiming to develop a clinical interview based upon a framework of growth points. The framework of growth points is structured around five broadly defined developmental steps, incorporating interim learning trajectories, to monitor children's skills as they develop proficiency (Twohill, 2013). This discussion paper outlines the theoretical perspective for the clinical interview design and also the relevance of a clinical interview as an assessment instrument in the area of algebraic reasoning.

Theoretical perspective

In order to assess and design assessments, it is imperative to have a clear idea of what constitutes mathematical proficiency. Milgram (2007) discusses the difficulty encountered in attempting to clinically define 'mathematics' and suggests instead a description of the most important characteristics of mathematics, namely "precision" and "stating well-posed problems and solving them" (p. 33). In the same volume, Schoenfeld (2007) suggests that the key element of proficiency in a subject is the ability "to use it in the appropriate circumstances" (p. 59). While Schoenfeld and Milgram adopt differing perspectives on defining mathematical proficiency, their assertions complement each other. Thus, underlying this paper is a theory of assessment as a measure of a participant's ability to apply mathematical knowledge and skills appropriately in solving new problems while always retaining a focus on precision.

Kaput (1998) discusses the development of algebraic reasoning and considers the need to nurture and encourage the 'roots of algebraic reasoning' over the primary school years. Mason (2008) identifies skills of algebraic reasoning which are evident in very young children as imagining and expressing, focusing and de-focusing, specialising and generalising, conjecturing and convincing, classifying and characterising. The immature skills of young children as highlighted by Mason (2008) develop over a length of time into strong broadly applicable skills of algebraic reasoning. To assess algebraic reasoning during this time requires an understanding

that the skills under assessment are in development. In discussing the tools and artefacts of assessment, Smagorinsky (1995) warns against presuming that evidence of children's thinking represents a "cystallized, fully formed state of development independent of the artefact's cultural significance and the means through which the learner has appropriated an understanding of how to produce it" (p. 199). In designing an assessment, it may be preferable to engage participants in non-routine tasks that are not derivatives of a rote-learned approach.

Children, whose skills are emergent, may present with a variety of skills while complete solution of a problem remains beyond the range of their ability (Radford, 2012). When operating within their Zone of Proximal Development (ZPD), children are capable of demonstrating skills which they are currently developing (Vygotsky, 1978). In assessing algebraic reasoning skills therefore, an assessment instrument is required which will facilitate children in engaging with tasks within the highest cognitive ranges of their ZPD and also facilitate the researcher in observing, as far as is possible, the mental processes underlying the approach each participant adopts. In this paper, I aim to discuss the characteristics of the clinical interview and how it may be suited to the assessment of young children's skills in algebraic reasoning.

Assessing algebraic reasoning skills

The development of algebraic reasoning skills involves more than the rote-learning of routine algorithms or processes. To reason algebraically, it is necessary for children to develop appropriate strategies and habits of mind (Cai and Knuth, 2011). Kieran (2007) discusses the shift in thinking which is required for students to progress from a purely arithmetical approach to developing algebraic thinking. Among other skills, she suggests that it is necessary to focus on relationships and not simply on the calculation of an answer. In considering methods for assessing children's algebraic reasoning, it is necessary to develop a strategy for assessing such constituent elements as foci, skills and habits of mind. The clinical interview creates an opportunity to assess the reasoning that underlies decisions a participant makes in the solution of tasks (Ginsburg, 1997). Van de Walle (2004) suggests that algebraic reasoning is more pervasive than the specific skills of pattern spotting, equation solving or variable use and as such requires assessment across many areas of mathematics. Students with strong skills in algebraic reasoning may seek to identify generalisations in many areas of mathematics while children with less robust skills might experience difficulty in identifying and expressing relationships (ibid.).

There is an inherent challenge in skills-based assessments of algebraic thinking, particularly among young children. It is not sufficient to score participants on the basis of correct or incorrect answers, but rather assessments are required to incorporate questions that afford insight into how the participant is thinking. Participants may arrive at a correct answer in an item that purports to assess algebra without applying algebraic reasoning. For example, Osta and Labban (2007) found that there was a need to alter the test question in their study of seventh graders' pre-algebraic problem solving strategies. In the solution of a numerical sentence with an unknown, children reverted, when possible, to a trial-and-error application of arithmetic, rather than adopting an algebraic approach. In interpreting the findings of research into Irish primary school teachers' mathematical knowledge for teaching Delaney (2010) expressed concern regarding findings in the area of algebra. There existed a possibility that in solving a question designed to assess algebraic skills,

some teachers may have adopted an arithmetical approach that was sufficient to solve the problem correctly.

Equally, as suggested by Van de Walle (2004), in problem-solving items that are not designed to assess algebraic reasoning skills specifically, some children may utilise an algebraic approach but their doing so may not be identified by the assessment. An alternative to a standardised assessment is that of the clinical interview. Ginsburg et al. (1983) assert that "to establish different aspects of competence, it is useful to use flexible, nonstandardised procedures" (p. 14). The clinical interview method affords a researcher insight into the level of cognitive competence at which the child is operating (ibid.). In the following section I will discuss the clinical interview and why I consider it to be a more appropriate form of assessment of algebraic reasoning skills.

The clinical interview

Piaget (1929) presents a view that assessments with a set protocol of questions are destined to skew the possible information about the subject. Piaget suggests that within assessment there are occasions when a fixed questionnaire is not an appropriate instrument as it may yield insufficient information regarding the internal reasoning of the participant. In solving a task that is purporting to assess algebraic reasoning, a student may use trial and error or purely computational strategies (Osta and Labban, 2007; Delaney, 2010). Piaget's clinical method "claims to unite what is most expedient in the methods of test and of direct observation, while avoiding their respective disadvantages" (p. 19). Utilising a clinical method of assessment, the researcher allows the child to guide the direction of the assessment, while constantly maintaining the focus on the area of research. Rather than a single linear sequence of tasks, the clinical interview consists of a number of pathways, along which the researcher will guide the participant depending on his/her reaction to each item. In designing a clinical interview to assess algebraic reasoning skills, I intend to include, among others, patterns presented in Radford (2011) and Cooper and Warren (2011). Tasks will include extending the pattern to the following term, describing the pattern and also explaining how the pattern will continue to a far term. If the participant fails to complete a task, the researcher will choose a subsequent task based upon the nature of the participant's error, creating a scenario similar to a "choose your own adventure" story (Clarke, 2013). The methods involved in the administration of a clinical interview are sensitive to the child's interests and concerns, are interpretive in nature and involve an ethic of caring (Ginsburg, 1997).

Zone of proximal development

Vygotsky (1978) discusses the Zone of Proximal Development (ZPD) within the context of assessment and advises examiners to remain cognisant of the exact focus of assessment and of what assessments inform us about children. Without support from a teacher or more able peer children may only complete tasks for which they have already acquired the requisite knowledge and skills. The ZPD deals with the knowledge and skills that the children are in the process of acquiring, which may be as yet not fully formed but are in gestation (ibid.). It is very probable among children in primary school that algebraic reasoning skills will be partially developed and children's failure to complete a task independently may not offer a true reflection of their algebraic reasoning ability. In solving a task therefore each participant will be supported in progressing to the most challenging task that he/she is capable of solving

with mediation. Whilst it is necessary to maintain a differentiation between the tasks that a child completed independently and those completed with assistance, the supported successes should also be recorded. Vygotsky (1978) asserts that there is a distinction between tasks of which the child is capable with support and those that are beyond his/her ZPD. It may be necessary for task design, researcher-subject interaction and analysis to reflect the continuum of independent success to supported success to unfulfilled task. In analysing the performance of children on the clinical interview, there is a necessity to examine the level of mediation required in order to complete a task.

If the aim of an adult-child interaction is to work within the child's ZPD it is necessary to pay attention to the balance of autonomy within the researcher-child dynamic (Jordan, 2004). Jordan contends that in order for teachers to gain insight into how young children are thinking, it is necessary that there is a co-construction of understanding between the two parties. Co-construction occurs when adult and child are "interpreting and understanding activities and observations as they interact with each other" (p. 33). It is necessary therefore that the activity be meaningful and that the child and adult are engaged in interpreting information in the process of acquiring information. Not only must the adult remain vigilant to the pre-existing understanding of the child but also he/she must maintain a position of shared autonomy in the activity at hand. Jordan explains that co-construction depends on the extent to which a shared understanding is developed, and that this in turn depends upon the metaphorical distance between researcher and participant and on how power is shared between them. A questioning paradigm which best supports co-construction involves questions to which the adult does not have ready access to the answer, where silence is allowed and the child's lead is followed. In designing a clinical interview where the aim is to uncover a child's thinking at the highest cognitive level within his/her ZPD, questioning and progress of the interview will be mandated by the necessity of co-constructing understanding.

Mediation

Smagorinsky (1995) discusses Vygotsky's assertion that assessment when positioned after learning produces very limited and often misleading information regarding a participant's ability. In order to ascertain current skill-level within a field, it is preferable to engage the participant in a learning activity positioned within the upper cognitive range of his ZPD. While the clinical interview underpinning my research will be an assessment rather than a teaching activity, the tasks presented to the students should be sufficiently challenging so as to facilitate the child's learning. Indeed, Schoenfeld (2007) proposes that participation in an assessment should always involve learning. Smagorinsky (1995) concurs and adds that rather than viewing a researcher as potentially contaminating an assessment by his/her presence, it is more beneficial to view a researcher as a mediator in the participant's operating at the upper end of his/her ZPD.

Replicability of findings from a clinical interview

In discussing the use of the clinical interview in assessing children, it is pertinent to mention concerns regarding the level of finesse required in administering this assessment instrument. Piaget (1929) suggests that only after a year of daily use of the method may practitioners move beyond "the inevitable fumbling stage of the beginner" (p. 20), which for teachers may involve excessive talk and suggesting

answers. Piaget cautions that there is a challenge involved in striking the balance between preconceived ideas that may skew the direction of the interview and approaching the interview with too little knowledge about the subject matter to form a reasonable hypothesis for research. Equally, it is imperative to adopt a balanced approach to the utterances of the child, which may express factors extraneous to algebraic reasoning, such as a willingness to impress, an inclusion of spontaneous imagination, or fatigue. Piaget (1929) cautions that the greatest risk to the clinical interview is the researcher who considers every utterance of the child as either 'gold' or 'dross' and thereby the findings are rendered relatively meaningless. There is sophistication required in the approach taken to consider and analyse the findings, incorporating information regarding the child's temperament and mind-set at the time of interview.

In order to assess fully children's proficiency in their emergent skills of algebraic reasoning, it is preferable that children be facilitated in engaging with tasks within their ZPD and to that end, mediation is required. There are implications however for the reliability and validity of research when the role of the researcher is so embedded in the research design, in design of tasks, mediation during assessment and analysis of results. The social background of the researcher and his/her understanding of how children think will play a powerful role in the results obtained. Smagorinsky discusses the implications of mediation and cautions that "higher mental processes are culturally shaped rather than universal in nature" (p. 203). He suggests therefore that assessments of children's ability will in effect measure the extent to which the participants' high mental processes mirror that of the researcher. Some highly developed cognitive strengths may be undetected such as those identified by Moll and Greenburg (1990) in their study of Latino students who displayed great prowess in many skills but were deemed to be of low cognitive ability in school settings.

In order to maximise the replicability of results, Goldin (2000) suggests ten methodological principles for designing quality interviews. Among his suggestions are the development of explicitly described interviews and established criteria for major contingencies; deciding in advance what will be recorded and recording as much of it as possible; a robust pilot-test of the interview; and leaving a margin for compromise when appropriate. There is a need for researcher mediation to be scripted precisely preparing for all predictable contingencies and for the mediation to be recorded as much as possible in order to identify when mediation may have been inappropriate or excessive. There may be a need for the researcher to compromise and she/he should therefore be prepared to record such interactions and discuss their implications in the analysis (ibid.).

Conclusion

Among primary school children skills of algebraic reasoning are emergent and not fully developed. Skills-based assessments are optimised when the researcher has some access to the thought processes underlying the participant's solutions to tasks. Administering a clinical interview requires the researcher to apply her/his personal judgement and interpretation during the interview and rigour is required in both design and administration to maximise validity. Analysis of the data gathered can be both difficult and time-consuming. The clinical interview, however, affords participants an opportunity to engage in tasks at the highest cognitive range of their ZPD and also allows the researcher insight into the thinking underlying a participant's

solutions to a task. As such, the clinical interview offers a model of assessment that will allow me to investigate to what extent children in Irish primary schools are developing algebraic reasoning skills.

Adopting the theoretical perspective outlined in this paper, I intend to pilot a clinical interview designed to assess children's algebraic reasoning skills. The sample of participants for the pilot study will be taken from a single school situated in a small rural town. The socioeconomic background of the children attending the school is mixed and sixteen participants will be selected at random from four different class levels.

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