Networking theories of society and cognitive science: An analytical approach to the social in school mathematics

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Debate about the interplay between social and individual aspects of mathematics teaching and learning remains at the cutting edge of theoretical understanding of mathematics education research. In trying to make sense of the insights of these divergent perspectives I ask: How is it that social reality exists? What are the merits and limitations of considering the students in our classrooms as only collections of individual minds, in contrast with perspectives that posit the primacy of the social in determining the identity of mathematics learners? Can each be accorded its relative legitimacy in a rigorous and rational manner? Recent developments in analytical social theory may have the potential to address this issue productively. This paper covers the conflict between social-constructivist and socio-cultural perspectives in the literature and the critical role of inter-subjectivity in communicating mathematics through interaction. The paper concludes by drawing on Searle's notion of collective intentionality to address the networking and complementary use of theories based in cognitive science and critical theory and the interplay of the individual and social in school mathematics.

Keywords: collaborative learning; critical theory; cognitive science; social reality; intersubjectivity; collective intentionality

Introduction

This article is a theoretical discussion about elements of analytical frameworks, situated in the context of research on small group interactions in mixed ability year seven mathematics classes in England adopting elements of 'complex instruction'. The analysis was focused on what Sfard (1998) might call the interplay between acquisitionist and participationist metaphors for learning. In the course of this research, I developed an analytical framework based in Habermas' (1987) Theory of Communicative Action (TCA). Consideration of sociological perspectives and cognitive aspects of the research on small group interaction led me to question how social reality exists from an analytical perspective using Searle's recent works on social theory (Searle, 2010; Searle, 1997).

The importance of 'intersubjectivity' as a concept in mathematics education is a sign of a significant shift in the historical conceptualisation of the knowing subject and thus also in the nature of knowledge. Signs of the change can be seen in the shift of the mathematics education literature from conceptualisations initially based more or less solely in psychological theories towards theories that attempt to re-conceptualise the field using social and socio-cultural ideas (Lerman, 2006). This shift has been driven by awareness that there are complex issues at play in mathematics education that cannot be accounted for using solely psychological perspectives. Intersubjectivity becomes an important concept in socially oriented theories that seek to understand the nature of learning and teaching mathematics. In seeking a more technical, empirically based, understanding of the role intersubjectivity plays in the

interactive constitution of knowledge in school mathematics teaching and learning, Habermas's theory of communicative action was found to be analytically productive (Kent, 2012).

I will describe the research context and aspects of analysis; discuss multiple theoretical perspectives on interaction, communication and mathematics education; discuss relationships between Habermas' TCA and Searle's Speech Act Theory; and make an argument for the critical potential and productivity of coordinating critical theories and cognitive theories using Searle's Theory of Social Reality.

The context: analysis of small group interactions in year 7 classes adopting equitable approaches to mathematics teaching in England

The case study focused on student interactions in the context of particular non-traditional mathematics pedagogy, complex instruction, in mixed-ability year seven classes. Three mixed ability year seven mathematics classes at three different school sites took part in the case study research. Participants included three teachers, three teaching assistants and four classes of students. Data included: audio recordings of preliminary interviews with participating teachers, data from a summer professional development workshop on complex instruction in mathematics education, video data from classroom observations and data from professional development workshops held at the schools prior to the case study. Teachers and researchers worked collaboratively to design tasks. Qualitative data was collected on student interactions around tasks using participant observation, and video recordings of whole class and small group interactions. Researcher memos supplemented video observations of the lessons. During the course of the case study, regular reflective interviews with teachers took place after every lesson observation focusing on the teaching of the planned tasks.

Analytical approach, codes and model of intersubjective small group interactions

The plan for data analysis was to adapt the constant comparative method as a rigorous way to address the iterative dialectical approach in Bassey's (1999) model for case study research. The integrated approach adopted combined the microanalysis techniques of open coding and constant comparison with the iterative analysis of Bassey's case study method. Codes and categories were developed out of the constant comparative method and open coding. These were brought into relation to one another theoretically through the development of a model of intersubjective small group interactions. The codes are heavily influenced by Habermas' TCA and the model is an attempt to articulate the dynamic relationships between different utterances and speech acts within the episodes of interaction (Kent, 2012). These tools were then used to develop analytical themes including coordination and communication; power in small group interactions; communicative breakdown; and the preconditions for communicative action in collaborative mathematics learning. The models and theories developed through the iterations of analysis as ideas generated were taken back to the data to be tested and refined. The analytical challenge of this endeavour is part of the motivation for this paper, which seeks to explore how social, communicative, and individual cognition may be related to one another.

Intersubjectivity and mathematics education: socio-cultural, situated and interactionist views

Intersubjectivity is raised by Lerman (1996) explicitly in an article that challenged the theorising of social constructivists (as based too much in radical constructivism and therefore incoherent with the primacy of the social that Lerman related to concepts of intersubjectivity) and in the same paper outlined several features of intersubjectivity in mathematics education. In particular, three features regarding intersubjectivity in mathematics education: the role of intersubjectivity in the consititution of subjectivity; cognition as situated in practices; and the nature of mathematics as cultural knowledge having a real but non-deterministic force.

The first is the idea that subjectivity is constituted through social practices and thus can be said to come into existence through intersubjective processes. This idea resonates with ideas from situated theory, which see identity as a process of coming to belong to a community of practice. Lerman uses an interpretation of Vygotskyian theories of enculturation, internalisation, and the zone of proximal development to address these questions. The second feature that Lerman identifies is the notion of cognition as situated in practices. Sfard and others (Sfard, 2008; Sfard and Kieran, 2001) have argued that cognition ought to be thought of as a process of communication[3].

The final feature that Lerman highlights is the idea of mathematics as cultural knowledge. In this situation Lerman again uses ideas founded in Vygotsky of the pre-existing social structures that are the force behind the development in participants of knowledge that is part of the cultural tradition of mathematics[4][5]. Lerman(1996) argues for the separation of cognitive traditions and sociocultural traditions, stating that it is incoherent to assert that a radical constructivism can be a primary foundation from which to understand the functioning of social processes as having force in the development of knowledge and subjectivity.

However, not all authors take such a strong position. Bauersfeld (1994) makes a case for interactionist perspectives as a middle way. The interactionist perspective as articulated by Bauersfeld seems at first to be quite close to the theories of communicative action as articulated by Habermas[6]. Bauersfeld, (1994), outlines the core convictions of the interactionists highlighting a series of interrelated conceptualisations of the nature and roles of: learning; meaning making; 'languaging'; knowing or remembering; 'mathematising'; internal representations; visualisations and embodiments; and teaching. This taxonomy represents a compromise between cognitive and social approaches emphasising which parts are about internal subjectivity and which are about social conditions and interaction.

In addition to these perspectives there is a literature of discourse in mathematics education that has developed over the last 10 to 20 years. Ryve (2011) undertakes a fairly comprehensive analysis of this emerging body of research. Ryve

[3] This claim is pointing to the strong relationship between the conditions of satisfaction of intentional mental states and the conditions of satisfaction of communicative utterances (Searle 2010).

[4] It is interesting to note that he sees this function as non-deterministic yet having a real force that has a more or less direct impact on the development of knowledge in participants.

[5] It should be noted that this is not necessarily a transmission concept but is related to the former idea of cognition as situated in practices.

[6] However it should be noted that the literature on interactionist perspectives does not make reference to Habermas' theory of communicative action (TCA). Rather there is articulation and reference to speech act theory and pragmatic semantics and other (primarily sociological) traditions that overlap with the interests of elements of Habermas' reconstructive approach to theorising social science.

addresses a number of questions in relation to this literature including: how and to what extent are the articles theoretically conceptualised; what data are used and how are the data analysed; and to what extent do the articles relate to or build upon one another? In his analysis Ryve notes that conceptual clarity of many of the studies is weak and the cumulative development of theoretical approaches is uncommon.

Ryve asserts that the priority should be on developing the sophistication of the theoretical perspectives that have already been developed rather than introducing further approaches from other fields. Ryve's analysis indicates that general features of theoretical development such as defining keywords, building on the work of others, and clearly positioning the article in epistemological perspectives are of great importance for future studies in mathematics education.

Lave and Wenger (1991) developed the ideas of 'situated learning' in the early nineties and they have gained widespread influence as a productive theoretical perspective for the analysis of learning and teaching in mathematics education (Greeno et al., 1997; Boaler, 1999; Boaler, 2000; Boaler and Greeno, 2000; Kumpulainen, 1999). The theoretical positions they articulated conceptualised communities of practice as the location of identity formation and learning; the concept 'legitimate peripheral participation' as a process of coming to belong to such communities; and learning as the process of forming an identity in the context of a community of practice (Lave and Wenger, 1991). Further, Cobb et al. (2000) also raise the importance of situated theory to the ideas of the interactionists, arguing for a pragmatic use of situated theory that rejects a purely psychological point of view in the attempt to formulate theory that can have practical application in the improvement of mathematics education practices by taking into account theories that address the social aspects of learning.

The issue of intersubjectivity in mathematics education and its relation to communication and learning has been well recognised in the literature. Interactionist, situated, and socio-cultural perspectives of mathematics education provide a useful framework for the analysis of the complex dynamics of classroom practice. These ideas seek to go beyond the limitations of psychological interpretations of learning, teaching, and understanding as an experience of the individual by considering the linguistic and social contexts in which meaning is constituted. Habermas' TCA (1987) and Searle's philosophy of social reality (1997, 2010) can inform theoretical frameworks focused on intersubjectivity and the associated constellations of cognitive, linguistic and social factors in mathematics education.

Habermas and Searle: Frankfurt School Critical Theory and a Theory of Social Reality based in Speech Act Theory

In Habermas' TCA, 'communicative action' is the coordination of action by multiple goal-oriented actors through a process of cooperative interpretation or 'intersubjective understanding' (Habermas, 1987). For Habermas meaning and understanding are inextricably linked such that understanding the meaning of an utterance implies: "1) The recognition of its literal meaning; 2) The assessment of the speaker's intentions; 3) Knowledge of the reasons which could be adduced to justify the utterance and its content and; 4) Acceptance of those reasons and hence the utterance" (Finlayson, 2005). Communicative action, like other models of action, has as its purpose the achievement of the goals of the actors involved. However, in communicative action this is achieved through understanding (as defined by Habermas' theory of pragmatic

meaning). This is a key insight for realising the usefulness of communicative action as a theoretical lens for evaluating interactions in the context of complex instruction.

Theories about the give and take of validity claims in pursuit of shared understanding and common goals can be related to ideas about cognition, language and society. Searle (1997, 2010) develops an analytical philosophy of social reality based in philosophies of mind and language, seeking to establish a perspective of the constitution of social reality. Searle argues that in order to understand social reality concepts such as 'intentionality' (the property of minds to be about things) and the ideas of speech act theory must be built upon with conceptions of collective intentionality, assignment of function, status functions, social institutions, and institutional facts. This is all done in the context of a weak realist position that seeks to avoid positivist reductions while maintaining a materialist position with regards to both social and physical reality.

Some main features of this theory include: a novel conception of the 'collective intentionality' and the role it plays in the establishment of language and institutional facts; the 'assignment of function' which is developed with recourse to the idea of collective intentionality in the context of analytical philosophies of mind and speech act theory and establishes the anatomy of social reality in an iterative and self-reinforcing manner. Collective intentionality is the idea that we have an innate capacity to have 'we-mental states' not just 'I-mental states'. Intentional states in this sense refer to the philosophical concept of intentionality as opposed to the common usage. The common usage of intentionality is a subset of intentional states. Intentional states are mental states that are about things. Essentially it is the property of thinking to be 'about' something. Thus desires, beliefs, knowledge, and even potentially perception or aspects thereof are all intentional states. Imagining a triangle on the surface of a sphere could be considered an intentional state. Thus collective intentionality involves mental states such that two people are more or less seamlessly coordinated on and through the same mental object. An example might be, "We are going for a walk."

This serves as the basis for Searle's description of the origin and anatomy of language. Searle asserts that the capacity for collective intentionality serves as the basis for the 'assignment of function' that is characterised generally as 'X counts as Y in context C'. This is the basic anatomy of language that builds the foundation up from non-linguistic coordination. Searle asserts that the rest of social reality is built up iteratively using the assignment of function. Searle asserts that this theory of social reality addresses a pressing need in philosophy about how to reconcile knowledge from the physical sciences with knowledge in the social sciences[7].

Networking Theories of Cognition and Critical Theory in order to deepen analysis of small group interaction and generate new avenues for investigation

Concepts of intersubjectivity based in Habermas' TCA can be coherently used alongside insights from other theoretical approaches, and in particular cognitive approaches. Similar attempts have been made by the interactionist researchers, and the profusion of theoretical points of view in the analysis of mathematics education has led to a situation wherein the knowledge being brought to bear in research often reflects insights from multiple perspectives. However, as is made clear by the issues raised by Lerman, Bussi, and Ryve, this can be a problematic undertaking and it is

[7] This explains Searle's articulation and emphasis of a weak realist position.

therefore essential to be clear theoretically and methodologically as to how this attempt will be made.

Bussi (1994) states that there is a need to 'look for' conceptual tools to deal with complementarity and the use of multiple theories, while Cobb (2006) raises two important points in relation to the use of multiple theories in mathematics education. First, in consideration of how various theoretical perspectives "…orient the types of questions asked and knowledge produced…" he suggests that the dichotomy between activity being viewed as primarily individual or primarily social in character fails to recognise the problematic nature of what is meant by the individual. Cobb suggests that, instead of positioning perspectives into these dichotomous categories, it makes more sense to compare and contrast the different characterisations and theoretical treatments of individuals[8].

Cobb's second point is that the use of multiple theories, and the relative validity of each, should be dealt with in a pragmatic fashion. This is not to say one should use whatever works in a non-reflective manner; rather it is based in Dewey's account of pragmatic justification such that a theory's ability to provide insight into empirical situations is a key factor in determining their truth. Using these ideas of complementarity and pragmatic justification one can see the development of models based in TCA as productive while Searle's ideas can be seen as having potential to contribute to networking of sociological, critical and cognitive theories of mathematics education[9].

Conclusions

By beginning to consider how the social realities of school mathematics might exist from the perspective of Searle's theory, researchers may be able to recognise and attend to technical features of the institutions and institutional facts of mathematics education and their relation to the cognitive development of mathematics learners in a coordinated fashion. Cognitive perspectives may be complemented by and deepen sociological perspectives (and in particular critical theory) in a reciprocal fashion.

Consideration of how sociological and cognitive aspects of mathematics education can be coordinated productively using Searle's theory of social reality that can strengthen the impact of research insights on mathematics learning and teaching. The actuality addressed by such by a critical approach to cognition and social reality in mathematics education research is the inequitable distribution of power, opportunity, and knowledge and the existence of conflict in mathematics classrooms. The critical context is the continuing reproduction of inequitable outcomes in England, especially in mathematics and sciences (Boaler, Altendorf and Kent, 2011) and the normative imperative of educating students of mathematics in a manner conducive to participation in a democracy (Boaler, 2008). The possible can be addressed using critical theories through the identification of principles conducive to more equitable communicative interactions. The use of Searle's theory can help explain how concepts of intersubjectivity and communicative action can be brought

[8] An important point as a large part of Habermas's project is a shift in focus away from philosophies of consciousness that consider rationality to be located primarily within the structures of the conscious subject, towards a treatment of rationality as inherent in the intersubjective features of communication.

[9] The validity of such conceptual tools would be primarily in their pragmatic application of deepening insight and understanding towards the development of new courses of action that may be useful in achieving the common goals of the mathematics education community, such as they are. This position is in line with the methodological arguments based in Habermas' (1987) philosophy, and focused on the rigorous use of multiple theoretical sources in the development of knowledge in the social sciences.

into relation with cognitivist and psychological theories without reducing one to the other. Addressing the question of how social reality exists using Searle's ideas may allow mathematics education research to address links between cognitive science, critical social theory and mathematics education to achieve deeper understanding about the relationship between institutional reality of school mathematics and the development of mathematical cognition.

References

Bassey, M. (1999) *Case Study Research in Educational Settings*, Open University Press.

Bauersfeld, H. (1994) Theoretical perspectives on interaction in the mathematics classroom. In Biehler, R., Scholz, R.W., Strasser, R. & Winkelman, B. (Eds.) *Didactics of Mathematics as a Scientific Discipline* (pp. 133–146) Dordrecht: Springer.

Biehler, R., Scholz, R.W., Strasser, R. & Winkelman, B. (1994) *Didactics of Mathematics As a Scientific Discipline*, Dordrecht: Springer.

Boaler, J. (2000) Exploring Situated Insights into Research and Learning. *Journal for Research in Mathematics Education*, 31(1), 113–119.

Boaler, J. (2008) Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal*, 34(2), pp.167–194.

Boaler, J. (1999) Participation, Knowledge and Beliefs: A Community Perspective on Mathematics Learning. *Educational Studies in Mathematics*, 40(3), 259–281.

Boaler, J. & Greeno, J.G. (2000) Identity, agency, and knowing in mathematics worlds. In Boaler, J. (Ed.) *Multiple perspectives on mathematics teaching and learning*, (pp.171–200) Westport CT: Greenwood.

Boaler, J. & Staples, M. (2008) Creating mathematical futures through an equitable teaching approach: The case of Railside School. *The Teachers College Record*, 110(3), 608–645.

Boaler, J., Altendorff, L. & Kent, G. (2011) Mathematics and science inequalities in the United Kingdom: when elitism, sexism and culture collide. *Oxford Review of Education*, 37(4), 457–484.

Bussi, M.G. (1994) Theoretical and empirical approaches to classroom interaction. n Biehler, R., Scholz, R.W., Strasser, R. & Winkelman, B. (Eds.) *Didactics of Mathematics as a Scientific Discipline* (p.121) Dordrecht: Springer.

Cobb, P. (2006) Supporting a Discourse About Incommensurable Theoretical Perspectives in Mathematics Education. *Philosophy of Mathematics Education Journal*, 19.

Cobb, P., Yackel, E. & McClain, K. (2000) *Symbolizing and Communicating in Mathematics Classrooms: Perspectives on*, Lawrence Erlbaum Associates.

Cobb, P. & Bauersfeld, H. (1995) *The emergence of mathematical meaning: interaction in classroom cultures*, Routledge.

Delanty, G. & Strydom, P. (2003) *Philosophies of Social Science: The Classic and Contemporary Readings*, Open University.

Finlayson, J.G. (2005) *Habermas: a very short introduction*, Oxford University Press.

Greeno, J.G. & Group, T.M.-S.M. through A.P. (1997) Theories and Practices of Thinking and Learning to Think. *American Journal of Education*, 106(1), 85–126.

Habermas, J. (1987) *The theory of communicative action*, Harper Collins Publishers.

Kent, G. (2012) Intersubjectivity and Groupwork: Examining Student Interactions in Year 7 Mixed Ability Mathematics Classes using Complex Instruction in England. *Unpublished doctoral thesis* University of Sussex. Available at: http://sro.sussex.ac.uk/45256/1/Kent,_Geoffrey.pdf.

Kumpulainen, K. & Mutanen, M. (1999) The situated dynamics of peer group interaction: an introduction to an analytic framework. *Learning and Instruction*, 9(5), 449–473.

Lave, J. & Wenger, E. (1991) *Situated learning*, Cambridge University Press.
Lerman, S. (2006) Theories of mathematics education: Is plurality a problem? *ZDM*, 38(1), 8–13.
Lerman, S. (1996) Intersubjectivity in Mathematics Learning: A Challenge to the Radical Constructivist Paradigm? *Journal for Research in Mathematics Education*, 27(2), 133–150.
Ryve, A. (2011) Discourse Research in Mathematics Education: A Critical Evaluation of 108 Journal Articles. *Journal for Research in Mathematics Educati*on, 42(2), 167–199.
Searle, J.R. (2010) *Making the social world: the structure of human civilization*, Oxford University Press.
Searle, J.R. (1997) *The construction of social reality*, Simon and Schuster.
Skovsmose, O. (1994) *Towards a Philosophy of Critical Mathematics Education* (Mathematics Education Library), Springer.
Sfard, A. & Kieran, C. (2001) Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture and Activity*, 8(1), 42–76.
Sfard, A. (2008) *Thinking as communicating: human development, the growth of discourses, and mathematizing*, Cambridge University Press.
Sfard, A. (1998) On two metaphors for learning and the dangers of choosing just one. *Educational researcher*, 27(2), 4–13.
Wenger, E. (1990) Toward a theory of cultural transparency. *Unpublished doctoral dissertation*, University of California, Irvine.