Supporting Students' Probabilistic Reasoning Through the Use of Technology and Dialogic Talk

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Research has shown that pupils and many adults have intuitions about probability that are often at odds with accepted probability theory. Drawing on the literature on probabilistic reasoning, effective pedagogical approaches and the use of technology tools, our aim is to examine the relationship between students' talk together, their use of *TinkerPlots* software and the development of their reasoning about uncertain outcomes. In this paper we report on findings from the first iteration of a design study conducted in an afterschool club for Year 7 students in Exeter. More specifically we describe the trajectory of two students making conjectures about the fairness of some games involving combined events, testing and revising their initial theories based on simulation data. Our analysis shows that these students' use of dialogic talk in combination with the technology leads to a shift from intuitive reasoning to probabilistic reasoning.

Keywords: secondary, probability, technology, dialogic talk.

Introduction

The teaching and learning of probability is a difficult task since formal probability theory can be counterintuitive for students. Earlier studies conducted by psychologists and mathematics educators have identified and documented difficulties students encounter when making judgment about an uncertain event.

The research of Kahneman and Tversky (1972) suggests that students often use intuitive strategies when judging the likelihood of uncertain events. One such intuitive strategy is called the representativeness heuristic and refers to evaluating the probability of an uncertain event based on the degree to which it represents some essential features of its parent population. For example, when flipping a coin six times, students often consider the sequence TTTHHH more likely to happen than either HHHHHH or TTTTTT, because it has an equal number of heads and tails and, thus, is more representative of the expected 50-50 distribution. Another study by Tversky and Kahneman (1982) also reveals a misapplication of the law of large numbers to small samples, that is the tendency to regard small samples as highly representative of the population from which they are sampled and to use them as a basis for inference and generalizations.

Equiprobability bias is another intuitive approach to solving probability tasks. Students with this bias tend to view all possible outcomes of random events as equally likely (Lecoutre, 1992). In the example of rolling a five and a six and rolling two sixes from a pair of dice, Lecoutre describes how students come to the equiprobable responses. For instance, a majority of students believe that likelihood of getting a six and a five is equal to that of getting two sixes because "random events should be equiprobable by nature" or "it is a matter of chance" (p. 561).

Students come to classrooms with these intuitive ideas and strategies and we need to challenge them to facilitate the development of more formal ideas. In this paper we examine what mechanisms of dialogic talk combined with the use of technology tools support students' probabilistic reasoning in games of chance. In particular, we are interested in the following research questions:

1. How do *TinkerPlots* tools and peer-to-peer dialogic interactions foster students' reasoning about probability events?
2. What are the mechanisms that lead to a shift in students' probabilistic reasoning?

Dialogic talk

Dialogic talk is a way of talking together that helps groups solve problems creatively. It is very close to what Mercer and others call 'Exploratory Talk', talk in which partners engage critically but constructively with each other's ideas (Mercer, 2013). Calling this productive way of talking 'dialogic' puts the focus more on the quality of the relationships than on the explicit verbal reasoning. In dialogic talk there is openness to the other and to otherness in general such that individuals are able to listen and to change their minds. The general dialogic mechanisms that have been observed in groups leading to success in creative problem solving are: opening a space of dialogue (e.g. asking open questions); deepening the space of dialogue (e.g. questioning assumptions); expanding the space of dialogue (e.g. introducing new voices and perspectives); seeing from the perspective of a specific other (e.g. listening to and taking on board the comments of a colleague, this includes the mechanisms of 'appropriation' and 'co-construction' noted by Mercer (2013) for Exploratory Talk); seeing from the perspective of a generalized cultural other (e.g. invoking the perspective of the absent addressee or audience for the product or referring to the generalized other of the community) and finally evidence of reflection through taking an outside perspective (e.g. genuinely asking why something is happening without any presuppositions, looking at it in a new and unexpected way) (Wegerif, 2013).

Technology use in teaching probability

A traditional approach to teaching probability focusing solely on mathematical computations and procedures is not sufficient to help students develop probabilistic thinking and reasoning needed in making judgments in real life. Learners need additional resources to develop probabilistic ideas in a more conceptual way (see Watson, Jones and Pratt, 2013). The computer technologies with the speed, dynamic visualisations, and capability to carry out laborious manipulations give students opportunity to focus more on their thinking and making generalisations and abstractions about data and chance.

With the advances in digital technologies and emphasis on the use of ICT in school mathematics, software tools with features for exploratory data analysis and probability simulations have become increasingly valuable in supporting children's learning. One such software is *TinkerPlots 2.0* (Konold and Miller, 2011), a data analysis tool with simulation capabilities (see Figure 1). It is a distinct computer program compared to other graphing or spreadsheet programs as it builds on the intuitive knowledge learners have about data representations and analysis. Students actually construct their own graphs when progressively organising their data by ordering, stacking, and separating. It also includes a variety of tools, such as dividers and reference lines, to intuitively analyse data in making inferences. One of the new features in *version 2* is the probability simulation tool that expands its focus from data to incorporate probability. With the Sampler tool, students can build their own chance models using a variety of devices (i.e., mixer, spinner, bars, stacks, curve, counter) that can be filled with different elements to sample from. They can connect series of devices to create a sequence of independent or dependent events. This tool then allows students to collect outcomes and carry out a large number of trials quite quickly.

During the field testing of the development version of *TinkerPlots 2.0* Konold and Kazak (2008) engaged students in developing integrated set of statistical and probabilistic ideas. The findings indicate that *TinkerPlots* environment facilitates students' visual reasoning via dynamic graphs where the results accumulate as they are generated by the Sampler. Through observing the simulation data from multiple trials coupled with sketching graphs (drawing only the overall shape and relative heights of stacks), students can explore the fit between the expected distribution based on the sample space and the empirical data. As a result of these observations they begin to perceive data as a combination of signal and noise (by signal we mean a stable shape in the distribution whereas noise is the variability around it due to chance).

Description of the study

The study is part of a larger design experiment (StatsTalk Project) investigating the relationship between students' talk together, their use of ICT tools and their development of conceptual understanding of key concepts in data handling and probability. The data reported in this paper come from the first iteration of this design experiment conducted with five 11-12-year-old students voluntarily participating in an afterschool club at a private school in Exeter.

Procedure and tasks

Each class in the afterschool program, co-taught by the authors, met for an hour, once a week over the period of eleven weeks. Students worked in groups of two or three and each group's work around a computer was videotaped.

Through the sequence of tasks designed for the purpose of the project, students were engaged in analysing data sets, such as reaction times and backpack weights, data modelling tasks, and chance activities using *TinkerPlots 2.0* software. In the eighth session of the afterschool program students began to investigate chance events. The focus of our paper will be on this task, called The Chips Game.

In addition to the use of *TinkerPlots* as an ICT tool to explore data and chance in the study, we introduced certain ground rules to promote a dialogic way of talking in joint activity (Dawes, Mercer and Wegerif, 2000). To facilitate effective talk students were expected to (1) make sure that each person has an opportunity to contribute ideas, (2) ask each other why, listen to the explanation, and try to understand, (3) ask others what they think, (4) consider alternative ideas or methods, and (5) try to reach an agreement before they do anything on the computer.

The chips game

Following the discussion of a fair method or a fair game, we asked students whether they thought the following games were fair or not:

> There are two bags containing game chips of two colours-red and blue. To play the game, you will randomly select a chip from each bag. If the two chips are the same colour, you will win. If they are different colour, teacher will win.
>
> Game 1 - Bag one: 3 red chips, 1 blue chip; Bag two: 2 red chips, 2 blue chips
>
> Game 2 - Bag one: 3 red chips, 1 blue chip; Bag two: 1 red chip, 3 blue chips
>
> Game 3 - Bag one: 2 red chips, 2 blue chips; Bag two: 2 red chips, 2 blue chips
>
> Game 4 - Bag one: 4 red chips; Bag two: 2 red chips, 2 blue chips

The task involved iterations of making predictions, testing initial theories by collecting simulated data from models built in *TinkerPlots*, and making conclusions.

Students' reasoning about probability in the context of fair game

We now focus on an analysis of the episode where two 11-year-old boys, Chris and Jacob (pseudonyms), investigated their initial predictions about the fairness of the games described in the previous section, by building a model of each game in *TinkerPlots* and collecting data.

Making Initial Predictions

We presented the Chips Game to the students and asked them to make a guess about the fairness of each of the four games listed on the board. They made a prediction for Game 2 first and worked on it before they moved to the next game listed in Table 1.

Table 1. Students' joint initial predictions and their reasoning about the fairness of the games.

Games	Predictions	Explanations
Game 2 Bag one: 3 red chips, 1 blue chip Bag two: 1 red chip, 3 blue chips	"fair"	Jacob: …both bags equals to four chips a piece. Teacher: Right Jacob: So half of them are eq.(.) are more (.) fifty fifty chance of winning (.) And now go to the same as for game 3 because both bags have two same colours of chips which will also equal up to just similar results.
Game 3 Bag one: 2 red chips, 2 blue chips Bag two: 2 red chips, 2 blue chips	"fair"	
Game 4 Bag one: 4 red chips Bag two: 2 red chips, 2 blue chips	"not fair"	Chris: Bag two is fair. Bag one is all red chips. It is impossible to pick out a blue chip from there. It is all made up of red chips.
Game 1 Bag one: 3 red chips, 1 blue chip Bag two: 2 red chips, 2 blue chips	"definitely not fair"	Jacob: I agree. It is totally unfair because in the first bag you get ¾ chance of getting red and ¼ chance of getting blue. And the second bag you get a 50-50 chance.

As can be seen in students' responses in Table 1, they failed in their first predictions about the fairness of all games, but Game 3. Their initial reasoning about the outcomes of the games relied on their attention to the contents of each bag. They expected that Game 2 would be fair because there were four red chips and four blue chips in total when the two bags were combined. They also inferred that Game 3 was fair too for the same reason. Even though their prediction about Game 3 was correct, the reasoning behind it was problematic. They focused on the total number of red or blue chips rather than the combined outcomes, mixed and same colour. They expected that the symmetry in the combined bags would generalize to the combined outcomes.

In Game 4, both students were initially quite certain that the game was unfair because of the first bag with 100% red chips. Focusing on the single events in each bag, i.e., bag two is being fair because of equal number of red and blue chips while bag one contains only red chips and no blue ones, Chris stated that the game was certainly not fair. In the same line of reasoning, they expected Game 1, which is slightly different than Game 4, to be unfair too even after they explored the previous situation and saw that they were wrong about it.

Testing Conjectures in TinkerPlots

To further test their initial predictions, students used *TinkerPlots 2.0* to build a model of the game situation. Even though they had bags and chips available to play the game, they chose to skip this part after the first exploration (Game 2). After the predictions for Game 3, they moved to the computer and used *TinkerPlots* to quickly generate and analyse large samples together.

Figure 1 shows a model for Game 2 that students constructed in *TinkerPlots*. When they built the Sampler (upper left), they used two spinners, one with 75% blue and 25% red and the other with 25% blue and 75% red sections, representing the proportion of red and blue chips in each bag. Each spinner spins once to execute a trial of randomly drawing a chip from each bag. Repeat number is set to 1000 (students' choice). The results table to the right of the spinners displays the repetitions as they occur in the Sampler. The plot at the upper right shows the percentage of four possible outcomes in the sample of 1000. In the graph at the bottom, the individual outcomes, 'blue,blue' and 'red,red' then 'blue,red' and 'red,blue', are combined into a bin by dragging one into the other to display the percentage of "the same colour" and "the mixed colour" outcomes respectively.

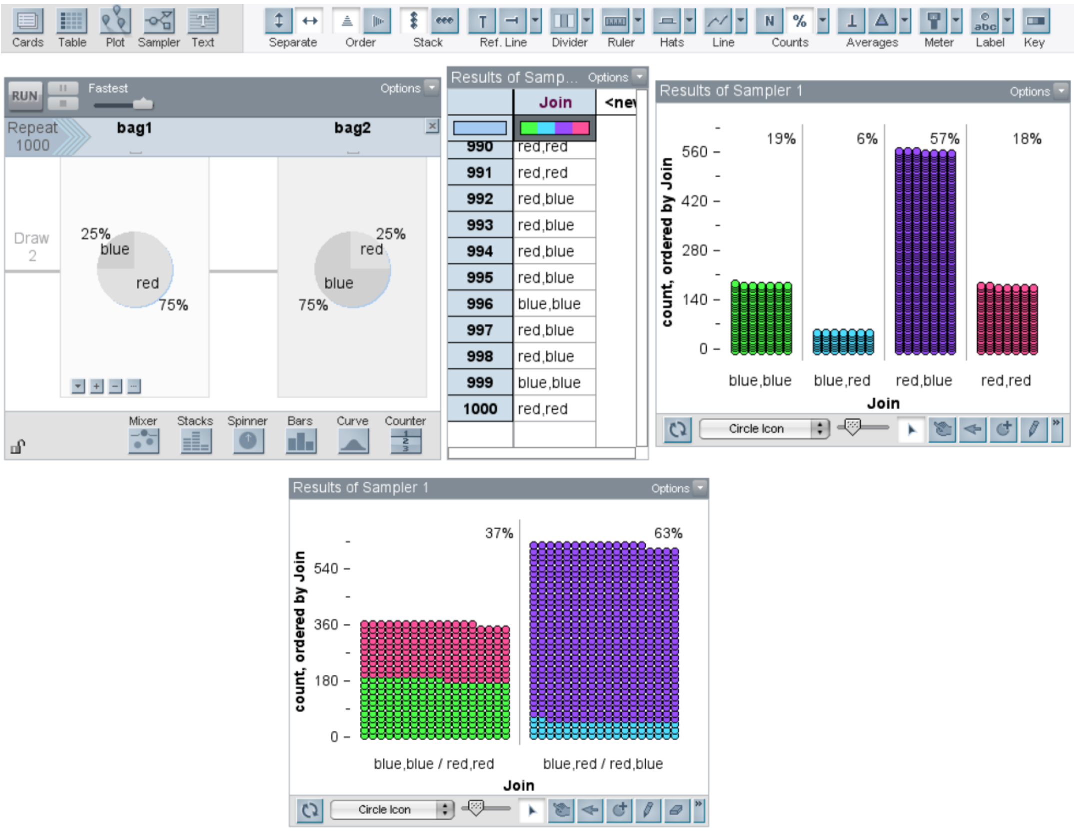

Figure 1. Computer model of Game 2 built by Chris and Jacob.

Seeing the results from a sample of 1000, Chris and Jacob realised that their initial prediction was obviously incorrect. Drawing large samples gave students more confidence in their conclusions based on the simulated data and helped them gain a new insight about the likelihood of outcomes in the game through the spinner model they built in the Sampler. For instance, to explain the data they observed in the computer simulation Chris further said, "I see what this is. There is so much for 'red,blue' [*pointing to the highest stack in the graph at the top*] it is because there is [*pointing to the first spinner on the screen*] that huge amount of red to choose from here, and there is loads of blue to choose from [*pointing to the second spinner*]."

Dialogic Talk around the Computer

After modeling game 4 in *TinkerPlots* and running 1000 repetitions of the game similarly to the example in Figure 1, the students completely changed their view of the situation again. In Kazak, Wegerif and Fujita (2013) we described the following exchange between the teacher [Sibel] and the pupils which happened right before they ran their model. When Jacob said "I am actually debating now" this implied that he was now aware of new voices and perspectives and beginning to *see the situation from more than one point of view*.

Sibel: Okay and you think that you guys will win most of the time, huh?

Chris: I think we will actually win most of the time.

Jacob: Actually, I am actually debating now [*while he presses the run button to collect 1000 data*]

The contradicting results from the simulated data led them to re-examine the chances in the game looking at their model.

Jacob: Oh yes, it is 50-50 because oh yeah!

Chris: Jeez, we got an entire army on our side!

Jacob: No, no, Chris you don't get it. The first one [*pointing to the first spinner on the screen*] you always get 100% red

Chris: Exactly

Jacob: Then the next

Chris: Then the next one you could get [*pointing to the second spinner on the screen*]

Jacob: It's a 50-50 chance of getting the same [*he is laughing and almost speechless*]

Chris: I don't get it!

Jacob: So basically the first time you will get a red, next time you got a 50-50 chance of getting the same or something different [*he is covering his face with his hands and laughing*]

Chris: I don't get this at all. Why are you laughing? Jacob, why are you laughing? Just calm down.

In Kazak, Wegerif and Fujita (2013), we argued that Jacob seemed to have a dialogue with an absent witness or addressee (Wegerif, 2013) when thinking aloud about why the outcomes turned out to be even. This relationship with an outside point of view enabled him to change his understanding. Chris's 'I don't get this at all' remark, suggested that he could not make sense of Jacob's utterances because he did not share in Jacob's new insight. Later, the exchange between the two pupils continued with Jacob's explanation of his insight addressed to Chris in a way that proved persuasive:

Jacob: [*now talking to Chris*] First one you will definitely get a red, so the next one you would get either a red or a blue. So basically you can either get 50% you will get red

Chris: Red, yeah. So it is

Jacob: 50% you will get blue

Chris: So it is 50-50.

Concluding remarks

In our study as well as others (i.e., Vahey, Enyedy and Gifford, 2000) the notion of fairness provided a motivating and productive area of inquiry for students investigating probability in computer-based activities. The Chips Game task involved students' active engagement in predicting, generating a large number of data through computer simulations, and interpreting the resulting outcomes in comparison to their initial predictions about the fairness of various games.

In chance situations, like the games involving compound events described above, students' probabilistic intuitions are not always reliable. In all but one game, their intuitive ideas led them to an incorrect judgment about the fairness of the game. When asked to give an explanation for their predictions, they seemed to focus either on the symmetry in the total number of red and blue chips in the bags (e.g., games 2 and 3) or comparing the likelihood of simple events (e.g., the chance of getting a red chip) in each bag (e.g., games 1 and 4). These findings are consistent with previous research in that the shift from simple events to compound events is difficult for students (Watson, 2005). One reason for this is that dealing with compound events entails considering all possible outcomes of an experiment and counting the number of ways that a combined event can occur. It is challenging for students to learn the need to consider the sample space in such situations; instead their thinking is often led by intuitive judgments (Konold and Kazak, 2008).

TinkerPlots played a significant role in fostering students' reasoning about chance in these games. Its capacity for simulations allowed students to construct models of the probability activities that they were given. These models meant that they could run many simulated iterations of each game thereby gathering far more data than they could have gained manually. The flexibility and speed of *TinkerPlots* enabled them to test and revise their initial predictions. The findings from this activity prompted them to rethink their early assumptions. Especially important to this process of prompting a re-think was the way in which the model they had to build in *TinkerPlots* showed that the process had two stages. Seeing the two stages of the game offered the new way of framing the problem that they needed to understand it.

The role of dialogic talk that emerged through the testing of initial predictions with *TinkerPlots* simulations was also noticeable in students' articulation of their thinking including their half-baked ideas (Kazak, Wegerif and Fujita, 2013). The exclamation "I don't get it!" by Chris was indicative of a certain humility and trust. This then led Jacob to explain his insight into the two-stage structure of the problem with reference to the model on the computer screen. The dialogic education approach that we had used here had as a key social norm or 'ground rule' that making mistakes and showing that you do not understand is OK. An atmosphere of trust was promoted in the class and this facilitated the admission of a lack of understanding and the asking of open clarifying questions. Without trust this important step might not occur.

In addition to the role of dialogic talk in helping each other understand and think, Jacob's initial switch in perspective can also be understood as dialogic. This switch in perspective is the outcome of an invisible dialogue with a projected external addressee or 'witness'. He is able to change to his mind because he looks at the problem again from the perspective of the projected external addressee. This dialogic mechanism at work is taking an outside perspective (Wegerif, 2013).

Overall, a combination of dialogic talk and the use of *TinkerPlots* helped students revise their initial conjectures and explanations as they reasoned about the chances in each game. However, it is worth noting that their understanding in a particular game is likely to be fragile. After the investigation of Game 4, they seemed to develop an insight about the likelihood of combined outcomes in the game but they were unable to use it in the next game, which required a very similar reasoning. This suggests that development of underlying probabilistic understandings is rather slow and we should begin teaching these ideas as early as possible and revisit them regularly in the secondary school years (Konold and Kazak, 2008).

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