

How working on mathematics impacts primary teaching: Mathematics Specialist Teachers make the connections

Jenny Houssart and Caroline Hilton

Institute of Education, London

We draw on analysis of assignments by primary teachers as part of the assessment for the Mathematics Specialist Teachers programme (MaST). In the assignment teachers are asked to work on some mathematics themselves, write up the mathematical part of their work then write about how this experience has impacted on their practice as a primary teacher. We focus first on case studies of teachers who included algebraic work in the first part of their assignments and look at what they say about the connections between this and their practice as primary teachers. Connections are made in a range of ways, but an overall finding is that teachers tended to focus more on the process of *doing* mathematics and the consequences this had for their practice rather than *knowing* mathematics. A further theme was *feelings* about mathematics, entailing positive consequences for practice, even where the initial feelings included negative dimensions. Examination of assignments on other aspects of mathematics confirms the presence of these three themes. Across all the assignments there was strong evidence that this experience of doing mathematics impacted positively on how teachers worked mathematically with their primary classes.

Keywords: Mathematics Specialist Teachers Programme (MaST), primary mathematics, subject knowledge

Introduction

The purpose of this paper is to explore the connections that teachers make between their experience of doing mathematics themselves and their practice as primary teachers. Our interest in this is linked to the idea of the importance of ‘profound understanding of fundamental mathematics’ (Ma, 1999) in relation to the teaching of mathematics in primary schools. We hoped that analysis of the connections teachers made themselves, as a result of engaging with mathematical activities, would shed some light on how this profound understanding might impact classroom teaching. This paper is based on analysis of written accounts by primary teachers, drawn from assignments they submitted as part of the assessment on the Mathematics Specialist Teachers Programme. We therefore start by looking briefly at the thinking behind this programme and the related issue of teachers’ subject knowledge.

Background

The Mathematics Specialist Teachers Programme (MaST)

The MaST programme is a national initiative in England conceived following recommendations by the Williams review of primary mathematics (Williams, 2008). The review envisaged a Masters level programme which would enhance both the subject knowledge and pedagogical knowledge of participants. In particular it is

specified that teachers on MaST programmes should ‘... explore some mathematics in its own right’ (Williams, 2008: 25). Following the review, MaST programmes were developed by eight consortia across the country. A key requirement was that each programme should contain three strands, matching the three key objectives of the programme to develop subject knowledge, pedagogy and skills in working with colleagues (Walker et al., 2013). For the purposes of this paper, we focus mainly on two of these three strands, subject knowledge and pedagogy. The development of teachers’ subject knowledge in mathematics and the relationship between this and their pedagogy has been the subject of extensive research and debate, which we consider briefly below.

Mathematics subject knowledge

The connection between teachers’ mathematical knowledge and their teaching has been the subject of a considerable body of research across the years, much of it drawing on the work of Shulman (1986, 1987). He established seven categories of teacher knowledge, four of which are general and three that are subject-specific. The latter consist of subject matter knowledge, pedagogical content knowledge and curriculum knowledge. These categories have been adapted and developed over the years, for example by Ball and her colleagues in the USA (Ball and Bass, 2000; Ball, Hill and Bass, 2005) who distinguish between common content knowledge and specialised content knowledge, with the latter being required for teaching mathematics. Rowland and colleagues in the UK (Rowland, Turner, Thwaites and Huckstep, 2009) also develop categories of knowledge and put forward the Knowledge Quartet consisting of foundation, transformation, connection and contingency. Ways of conceptualising teacher knowledge are numerous as considered by Petrou and Goulding (2011) and by Ruthven (2011). Despite the volume of work in this area, Watson and Barton (2011: 67) point out that “The question of connection between knowledge and teaching is still open.” Our intention is not to attempt a direct answer to this question, but rather to examine in detail the connections that a group of teachers make.

Teachers doing mathematics

In contrast to the extensive literature on teacher knowledge, there has been less investigation of teachers engaging in mathematical activity themselves and how this might impact their practice. One example of such work is a professional development project with secondary teachers in New Zealand carried out by Barton and Patterson (2009). The teachers engaged in this project gave a range of examples of how learning mathematics themselves directly affected their teaching practice. The authors suggest that one impact is that the teachers increase the variety of learning opportunities offered to their students in mathematics lessons. They also point to the effect of students seeing their teachers as learners and the importance of teachers thinking deeply about their classroom practice. Another key piece of work in this area is by Davis and Renert (2014) who looked at ways teachers worked together to develop their mathematical understanding through sustained collaborative investigation which they call ‘concept study’. The authors argue that this impacts what they call the ‘pedagogical problem solving’ of the

participants. They conclude by developing the idea of 'profound understanding of emergent mathematics'.

Methodology

We started with an initial sample of 57 assignments written by teachers following their experience of working on some mathematics themselves. In the first part of the assignment they wrote up the mathematics, accompanied by diagrams, calculations, tables and formulae as appropriate; in the second part they wrote about the connection between working on mathematics themselves and their practice as primary teachers. Our first step was to categorise essays according to the aspects of mathematics carried out, based on the first part of each assignment. A second step was to look more closely at the second part of the assignments. As a starting point we did this with those assignments identified in our first sorting as containing algebraic work. This was because our informal reading of the assignments suggested many teachers had made use of algebraic notation and formulae and hence we expected some interesting examples in this category. The original intention was to look at aspects of algebraic subject knowledge, for example finding unknowns or looking for relationships and see how teachers made use of their increased knowledge of these aspects in their practice.

The sub group identified for initial, more detailed, analysis consisted of assignments which either included the word algebra in the title, or announced a clear focus on algebra in the first part of the work, for example in stating the search for a formula. Looking at the second part of each assignment and focusing specifically on connections made by teachers, we were able to identify the category knowledge and understanding of mathematics that the literature led us to expect. However, this accounted for only a small part of the teachers' discussion of connections and hence further broad themes were identified. The second of these concerned ways teachers changed their practice in response to their preferred approach to doing mathematics themselves. This category contained approaches such as the need for time, the place of talk and approaches to getting stuck. The third broad category concerned connections made based on how the teachers felt about doing mathematics themselves and their increased awareness of affective factors in the classroom.

Findings

Summary of Findings

The three categories are described in detail below, drawing on extracts from the work of four teachers who explored algebraic ideas. Pseudonyms are used throughout.

Knowledge and understanding

Trevor is chosen as an example here because the connections in his essay included explicit mentions of knowledge and understanding of algebraic ideas. Trevor had elected to work on sections of a book on algebra aimed at teachers that offered readers tasks to try (Mason, Graham and Johnston-Wilder, 2005). In the first part of his assignment, headed 'Improving my own algebra' he presents his work on these tasks, using algebraic notation and identifying themes in his own approach such as specialising and generalising, reflecting the approach to algebra implicit in the book

he was using. When Trevor moves on to write about connections between his experience of doing this and his practice as a primary teacher, he initially focuses on algebraic themes identified in part one of his assignment.

I am trying to introduce generalisations explicitly into my lessons ... During feedback from a recent observation, my head expressed surprise that I had asked pupils for generalisations relating to multiplication facts, but delighted that they knew exactly what I meant and were frequently able to respond appropriately. So far, I have only tentatively introduced the idea of symbols to represent values. One area in which pupils have indicated that they understand the concepts is in two-part problem solving. The pupils are encouraged to use a cloud symbol ... to represent the link between the parts of the problem, the answer to the first number sentence becoming part of the second part. One advantage, in this situation, of the cloud, is that the pupils are able to fill the cloud with the appropriate number when they have found it. This may not strictly be algebra, in that answers are not general, but it is an introduction to the idea that the unknown can be worked on.

In the extract above, Trevor shows a deep understanding of key algebraic ideas, such as generalisation and the ability to transfer this to his practice at a level appropriate for the pupils he teaches. He also shows awareness of the fact that symbols can be used for both unknowns and for variables. However, having acknowledged these aspects, Trevor moves on to suggest that changes in his approaches to teaching are much broader than the incorporation of basic algebraic ideas.

I feel that the algebra I have done so far has only slightly improved the level of algebra at which I am working. However, it has made a significant difference to the way that I look at and teach mathematics, I now expect the pupils to look at maths in a much more open way, to focus on why particular problems produce the answers that they do and to be moving towards making far more connections between different aspects of mathematics. ... The pupils appear to find the maths that they are doing to be more challenging, but also more fun and interesting. There is frequently a cheer when I introduce a numeracy lesson!

This second extract from Trevor's writing, includes examples of the two themes present in many of the essays, namely changes to practice based on approaches to doing mathematics and affective factors. These are a small part of Trevor's writing, but other teachers, such as Elaine (below) considered them in more detail.

Changes to practice

Elaine started her assignment by outlining her approach to the problem 'Counting triangles' which started by asking the total number of triangles in an equilateral triangle with edge length four drawn on an isometric grid. A key algebraic aspect of Elaine's work was her search for number patterns and ultimately for a formula. There was a particular focus on square numbers and when Elaine started writing about connections to her practice she suggested that this new depth of understanding would impact positively on her teaching.

These ideas made me reconsider the thinking process I went through as I was thinking about square numbers during my study of the number of triangles. I absolutely had to go back to the root of what a square number is. The impact of this has been to force me to challenge my own assumptions and this has given me a fuller, deeper, more meaningful understanding of what a square number is. I had to *deconstruct* to go to the *physical* root of square numbers. I have no doubt that this new depth of understanding will serve me well in my teaching and will have a positive impact children's learning.

Although Elaine acknowledges her new depth of knowledge, she writes in much more detail about other connections between her own investigation and her practice. Her account of her investigation made several references to the approaches she had taken, including those that suited her and those that did not. For example, in the early stages of working on the problem she showed enthusiasm for a ‘lift the flap’ approach using coloured paper, scissors and sticky tape, contrasting this to an approach based on looking at diagrams which others had used but which she described as ‘not enough’ for her. In later stages of the problem-solving process, she discovered with some amusement that “... there are actually ‘how many triangles’ facebook pages with hundreds of fans!” However she rejected this possible source of advice, in favour of working in person with an individual. When it came to making connections in the second part of her assignment, the focus for Elaine was on different approaches to working on mathematical problems and how the pupils she taught should be given the opportunity to approach mathematical problems in ways that suit them. She considers this below:

In reflecting on my experience of trying to solve the triangle problem at the local meeting (in which I didn’t have the time, equipment or freedom to explore the problem in the way I needed to) ... By exploring the triangle problem in a way which suited me, it supported my development, I learnt and it was challenging but thrilling! This has implications for the classroom. Children need to be given time to explore a problem in their own way and there needs to be an atmosphere in which children’s thinking and ways of working are encouraged and valued ... this draws me to conclude that we absolutely must allow our children the time to explore problems in a way that suits them in order that they will **fully** understand.

Elaine’s consideration of her own deeper understanding is reminiscent of the profound understanding of fundamental mathematics identified by Ma (1999) but she goes beyond this to draw clear messages about the practices she will employ in her classroom which relate directly to her own experience of working on the triangles investigation.

Enjoyment and motivation

Both Trevor and Elaine make some reference to enjoyment of mathematics. Trevor does this when he talks about his class, Elaine refers to her own pleasure when working in a way that suited her, although she also acknowledges the frustrations of working in a way that did not suit her. For some teachers, affective factors were a key part of their discussion about connections. One teacher, Phillip, talked explicitly about how he tried to make links to his practice and in doing so moved from connections about approaches to doing mathematics towards those related to motivation. Phillip had been looking at magic squares in his investigation, with a particular focus on identifying general formulae. Phillip is a confident mathematician and drew extensively on algebraic symbols and terminology in the first part of his work. In the second part of his assignment, he started to consider the connections between this experience and his role as a primary teacher.

So what can I gain from this investigation that can positively influence my teaching? Initially, I thought that I could use this to help understand the process of problem solving in order to teach it more effectively. I often hear from colleagues that the using and applying aspect of the maths curriculum causes problems, as the children find it difficult to apply what they have learned. Typically this related to word problems...

Phillip moves on from this to acknowledge a mismatch between his experience of working on the magic squares problem and the way children are

sometimes encouraged to tackle word problems. He briefly talks about what he calls ‘a more generalised understanding of problem-solving within mathematics’, but then moves on to discuss affective factors.

This started taking me into a different direction. During the investigation I was highly self-motivated, I reviewed my work and thinking, I engaged in a high level of meta-cognition, I used and applied a range of knowledge and strategies and developed a problem solving approach subconsciously. So perhaps I need to focus on the use of problem solving as a valuable and enriching way of teaching mathematics and move away from the idea of teaching problem solving explicitly.

Although Phillip explicitly mentions motivation and engagement, he also considers knowledge and approaches to doing mathematics; hence the three strands we identified are closely interwoven in his account. Phillip’s initial feelings about doing mathematics were positive, but some teachers were more apprehensive, as illustrated by our final example, Karen.

Karen starts her account by acknowledging negative feelings about algebra and this is part of her rationale for embarking on an investigation that draws on algebraic notation. Karen elected to work on ‘Tring squares’ an investigation that explores the total of sequences of digits in growing squares.

I have found that returning to undertake mathematics at a level that was going to be a personal challenge has recreated feelings towards mathematics that I used to experience at school. Whist at school, if a piece of mathematics was linked to algebra, I would develop this whole body experience of sweats, rising panic, surges of adrenaline, to the point that I would frequently drop my pencil and find any avoidance strategy to avoid completing the mathematics.

Karen provides more detail on these memories and feelings, but then indicates her determination to tackle this issue.

It wasn’t until I sat down in our first local meeting to share the task of Tring Squares that I had ever admitted this mathematics anxiety to myself or anybody else. Therefore I decided that it was time to challenge this anxiety by unpicking what algebra I could do, to learn new methods to solve problems and to improve my teaching as a result of my new subject matter knowledge.

In the sections following, Karen tackles the Tring squares investigation and also works explicitly on her own understanding of aspects of algebra such as use of symbols. Later she reflects on the usefulness of her experience in her teaching. As part of this discussion, she picks up again on the idea of maths anxiety.

I now have an understanding of what maths anxiety is which has resulted in me challenging myself to ensure that I don’t pass this onto the future children that I teach ... but provide them with strategies to cope whenever they become stuck or they reach the panic stage when faced with something new or challenging.

Karen was not the only teacher who acknowledged some negative emotions as part of her description of how she went about the mathematical task. The encouraging thing was that teachers doing this, like Karen, were able to draw positive messages for their teaching from their own negative emotions, often focussing on what they wanted to avoid as far as their pupils’ experiences were concerned. Our findings about the importance of emotions in relation to doing mathematics, are in line with the work of Evans (2000) based on his study of adults. He points to the importance of the roles of affect and emotion in learning and suggests that they cannot be separated from cognitive engagement during learning activities.

Other aspects of mathematics

Having started to categorise the types of connections teachers made, our next step was to consider whether this categorisation still appears to be valid for those writing about aspects of mathematics beyond algebra. Our findings suggest a similar pattern in essays dealing with other aspects of mathematics. For example, a teacher exploring divisibility rules wrote about depth of understanding and about engagement with the investigation and approaches to getting stuck. As well as acknowledging knowing and feeling, this teacher drew on experience of doing mathematics to draw a range of messages for his own practice which included focussing on the mathematics as well as the patterns, focussing more on the journey and less on the answer and allowing children the space to make mistakes.

Another teacher acknowledged how she had deepened her understanding of arrays and multiplicative reasoning as a result of working on this aspect and at the same time experienced frustrations and developed mathematical resilience, linked to the 'willingness to struggle' identified by Johnston-Wilder and Lee (2008). The latter was done partly by the way she worked with others and this led her to consider the place of talk in her mathematics classroom and a consideration of what is needed for meaningful talk to take place.

A teacher who explored operations with negative numbers was led to considering the difference between the operation of subtraction and the meaning of a negative number, as well as developing her use of conjectures. She also identified perseverance as important to her in this work. Her connections to her practice included reviewing the meaning and usefulness of informal jottings and supporting children to use mathematical language as well as the importance of building positive attitudes to mathematics and resilience.

Summary and discussion

In making connections between their experience of doing mathematics and their practice in primary schools, the teachers did show evidence of developing and drawing on their understanding of mathematics. Sometimes this was about meeting new ideas, but often it was about developing a deeper understanding of aspects of mathematics they were already aware of. However, they also made other connections concerning approaches to doing mathematics and feelings about mathematics. All three are put forward by teachers as impacting on their practice and in many cases the three are written about in a closely linked way, giving further evidence of connections. These findings are in line with those of Barton and Paterson (2009), who noted that the teachers in their project drew parallels with the way they engaged in mathematical activities as part of their own learning and the ways they encouraged their classes to work subsequently. Our findings are consistent with the ideas advanced by Davis and Renert (2014) in their consideration of ways teachers have worked together to develop their mathematical understanding and how this has impacted on their practice.

Our findings suggest that if teachers are given the opportunity to engage in mathematical activities themselves, they are likely to make connections between this and their practice that includes a consideration of how the children they work with are enabled to do mathematics. With continuation of the MaST programme currently uncertain (Walker et al., 2013) and with the mathematics curriculum in England

undergoing changes, it is to be hoped that opportunities for doing mathematics will still form part of professional development for primary teachers and that they will have enough freedom in implementing the new curriculum to make connections which encourage the children they teach to work mathematically.

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