

Problem solving tasks in mathematics classrooms: An investigation into teachers' use of guidance materials

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This paper reports on a design research study undertaken as part of the Mathematics Assessment Project (MAP). The project aims to support teachers in implementing a new curriculum in US schools, through the use of formative assessment lessons (FALs) designed by the MAP team. Here we report results of our research into teachers' use of the accompanying guidance materials as they implement problem-solving FALs, drawing on detailed case studies of lessons from a sample of UK teachers. Although we observe much variation in the ways in which teachers use the guidance, both when in and out of the classroom, we identify the provision of a 'Common issues' table, outlining likely responses from students together with advice of potential ways to respond, as one of the most valued and used aspects of the guidance materials provided.

Keywords: Problem-solving; formative assessment; guidance materials; teacher guidance; curriculum.

Introduction

Problem-solving lessons can often be challenging for teachers (Ball, 2001; Chazen and Ball, 2001; Evans and Swan, 2013; Lampert 2001; Leinhardt and Steele, 2005; Sherin, 2002). In solving a non-routine, unstructured problem, students may select one of a range of mathematical methods to apply and teachers cannot always be sure which method they will choose. Not only do teachers have to try to understand how students are making sense of the problem, they also have to try and begin to align students' disparate ideas and approaches with canonical understandings about the nature of the mathematics (Stein and Kaufman, 2010). Teachers frequently need to adjust the way they typically teach and well-designed curricular guidance has been shown to be effective in supporting teachers to make a pedagogical change (Remillard and Bryans, 2004).

As Remillard (2005) pointed out, the factors influencing how teachers use materials is critical to their development and iterative designs of curriculum materials take into account the ways in which teachers respond to and use the resource. It is known, for example, that when guidance materials have been provided, teachers often read the guidance, without necessarily engaging fully with the content. However, while some teachers do "little more than check for the materials that they need to carry out the lesson ... and the activities that students are supposed to do" (Stein & Kaufman, 2010: 671), others "actively look for the mathematical point of the lesson and for information regarding how students might respond to the various tasks" (Stein and Kaufman, 2010: 671). Unsurprisingly perhaps, the latter group of teachers tend to enact the lesson more closely to the designers' intentions. In terms of enacting the lesson, Remillard's studies (1996; 2000) showed that minimal teacher learning results from merely reading the guide and that the most significant professional learning occurs when the teacher enacts the lesson in the classroom.

The challenge for designers of curriculum guidance is to promote teacher engagement with the resource, whilst resolving some of the tensions described by Davis and Krajcik (2005), such as designing for different types of teachers and determining an appropriate amount of guidance. This study addresses this issue by investigating teacher use of curriculum guidance materials when enacting problem-solving lessons in the classroom. In particular we explore which features of the guide teachers use and ignore, in an endeavour to understand more fully what teachers value most in guidance materials.

Mathematics Assessment Project

During a four-year design research project beginning in January 2010, a team of mathematics educators at the Centre for Research in Mathematics Education at the University of Nottingham, designed mathematics Formative Assessment Lessons (FALs) to support US teachers in implementing the Common Core State Standards in Mathematics (CCSSM). The lessons were developed, from research-informed draft designs to 'final' products, through an iterative process of piloting and refinement. A third of the lessons designed were 'problem-solving' lessons, aiming to engage students in the solution of non-routine tasks.

Each problem-solving FAL follows a similar structure including the following phases:

- Prior to the FAL, students attempt the problem individually. Rather than grading these scripts, which tends to promote competition between students (Black and Wiliam, 1998) and distracts attention away from the mathematics, the teacher formulates questions for students to consider, in order to improve their work on their next attempt.
- Students begin the FAL by individually reviewing their initial solutions to the problem, in light of the questions raised by the teacher.
- In small groups, students share and evaluate their initial attempts, with the aim of producing a joint solution that is better than their individual efforts, usually in the form of a poster.
- After discussing as a class the range of strategies used by different groups, students are given some sample responses to analyse. These are based on student work, but carefully written by a designer to introduce a range of problem-solving strategies. Students evaluate the sample responses, commenting on their strengths and weaknesses.
- The lesson ends with a class discussion of the various strategies seen and used and students are encouraged to compare different approaches.
- In a follow-up lesson, students engage in a final reflection on what they have learned.

Guidance materials

As expected in a design research project, the content of the guidance materials included in the FAL units, alongside the task, evolved during the course of the project, based on feedback from teachers' enactment of the lessons in US trials. In the first drafts, the suggested lesson outline section of the teacher guide was about three to four pages of text. As the project developed, the number of guidance pages increased significantly to between 10 and 12 pages, as the extent of support for the shift required in many US teachers' pedagogic practices through the introduction of

the CCSSM became more apparent. For example, in February 2012 a US observer wrote:

In working with high school teachers it seems clear to me that their vision and beliefs about teaching and learning in math classes is far from the Standards, which is why I support giving more detail and more script in the summary, as it serves as a means of presenting a different vision and challenging their beliefs.

Such requests asked for more examples of possible questions to use within the lesson, as well as recognition of the need for additional explanations for US teachers to aid their understanding of the intended use of the resources.

At the time of the research reported here each FAL teacher guide comprised:

- an overview page (outlining mathematical goals of the lesson, CCSSM addressed, a summary of the lesson structure with proposed timings and materials needed);
- advice on introducing the task *before* the lesson and assessing students' initial responses, including a 'Common issues' table (see below);
- a lesson outline detailing specific suggestions for what the teacher might do at each phase of the lesson, including questions they could ask and
- a summary solution and outline of possible approaches to the problem.

In addition to the teacher guide, PowerPoint slides containing an outline of the problem, directions for students as they work in the different phases of the lesson and the sample responses to support class discussions are provided for use during the lesson enactment.

The design team were conscious that producing the much lengthier guidance document that was emerging could result in teachers choosing not to read it all. Feedback from the US trials highlighted this: for example, a member of the US observation team reported, "the teacher stated that the lesson materials packet was 'long' and he didn't read it all the way through" (Observation Report, Dec 2012).

The 'Common issues' table

When the problem-solving FALs were first designed, the students were given the problem at the start of the lesson. However, US trials showed that it was difficult for teachers to understand and follow students' reasoning in the moment of the lesson and to be able to follow it up immediately. As a result, the teacher guide was revised with the direction to give the problem to students to tackle on their own, in class or for homework, *before* the lesson. Feedback from the teachers and observers involved in the US trials suggested that teachers would welcome more guidance with the formative assessment of students' initial attempts and to assist teachers in doing this a 'Common issues' table evolved.

At first draft stage, the contents of the table were informed by research findings of common misconceptions and the designers' own classroom experience. The table was revised and refined during the design process, based on feedback and analysis of students' attempts at the task and common difficulties students had with the task, during the trials.

The aim of the table is to help teachers identify common problem solving strategies that students might be expected to employ and the difficulties that may arise. The table exemplifies how these might be evidenced in students' work with each issue linked to suggestions of follow-up questions and prompts that teachers may use as feedback to their students. The aim is for the teacher to select questions from those provided, or devise their own, to direct students' attention to the strategies for problem-solving and help them to make further progress on the task. Teachers can use

these questions and prompts to provide feedback on students' work, both at the start of the FAL and as the lesson progresses.

Central to our research study was the opportunity to examine in what ways the 'Common issues' table could be a useful mediating device, providing teachers with insight into the big ideas of the problem-solving task and practical advice on how to proceed in the classroom.

UK Research Study

In the 2012 - 2013 academic year, eight teachers with a range of teaching experience, from three different UK secondary schools, used some of the problem-solving FALs regularly with a class of their choice. During the course of the year, a team of researchers (who had been involved in the design of the FALs) followed the teachers' journeys as they enacted the lessons in their classrooms. Each of the 66 FALs taught during the course of the academic year was videoed and teachers were interviewed prior to and after each lesson.

Whilst the FALs had been used in a trial phase in the US to provide iterative feedback for the design of materials, including teacher guidance, their use in British classrooms was in a research phase to better understand their use and in particular, teachers' use of the guidance materials. We expected that the UK teachers would view the teacher guide, not as a prescriptive set of instructions to be followed closely, but as suggestions for classroom enactment. This study provided an insight into their engagement with the different features of the guide and an opportunity to observe the extent to which the pedagogical principles suggested within the guidance were adopted.

The analysis in this study of the UK teachers' use of the guidance materials accompanying the problem-solving FALs draws on Remillard's framework for examining teachers' curriculum development (1999) where she models teachers' construction of mathematics curriculum in the classroom. The model includes three arenas in which teachers engage in curriculum development: design, construction and curriculum mapping. The design arena focuses on the decisions teachers make before the lesson, whilst the construction arena is concerned with the actual enactment of the lesson in the classroom. In the context of our study it is these first two arenas that are of relevance, as we explore teachers' use of the guidance materials both prior to and during the lesson, using them to frame our analysis. Evidence is taken from pre- and post-lesson interviews, an interview conducted near the end of the study and comments made during a final meeting, which involved teachers coming together to reflect on their experiences. The lesson videos allowed for triangulation of data.

Findings

The ways in which the UK teachers chose to engage with the teacher guidance provided with the problem-solving FALs varied from one-off decisions to patterns of behaviour that became a regular feature of the teachers' lessons.

The pre-lesson interviews provided an insight into the teachers' preparation for the lesson (the design arena) including how much time they had spent and what their focus had been. The post-lesson questions offered a way of following up from the observed FAL and an opportunity to explore the decisions the teachers had made when enacting the lesson in the classroom (the construction arena) and how these had been informed by the teacher guidance.

Use of the guide in the design arena

We found that the level of engagement with guidance materials in preparing for the lesson varied considerably from teacher to teacher. At one extreme, one of the teachers, although reporting he “found all parts of the teacher guide useful” (final interview, 1/7/13) and read the details of the ‘Materials required’ and ‘Time needed’ sections, only briefly scanned the lesson outline. On the other hand, at the other extreme, another teacher reported on one occasion, spending half an hour reading the suggested lesson outline and described how the guide “provided [him] with a starting point, enabling [him] to understand where to begin with the lesson” (pre-lesson interview, 19/6/13). He later described how he “used [the teacher guides] a lot in [his] planning ... [and] always used them to give [him] an idea of what’s expected” (final interview, 1/7/13).

We also found that use of the materials varied over time for individual teachers. For example, another of the teachers reported that over the course of the project, she began to understand the reasons behind the structure of the lesson and pedagogical principles expressed in the guide and as a result endeavoured to follow the guidance more closely in her lessons (final interview, 11/7/13). Other teachers also reported an increasing interest in the use of guidance materials, as their suggestions in dealing with issues likely to arise in the lessons proved to be helpful in preparing for the lesson.

The ‘Common issues’ table appears to have provided an important factor in engaging teachers with the guidance materials and signalling key intentions of the lesson in relation to problem solving. As one teacher commented, he found it to be “the most helpful part of the guide” (final interview, 1/7/13). Another described how he would “choose some of the questions when marking students’ initial attempts at the task” (final interview, 1/7/13) and another described how he would “print off [the ‘Common issues’ table] to use when assessing the pre-lesson work” (pre-lesson interview, 25/6/2013). All eight teachers reported making attempts to annotate students’ initial work on the task prior to the observed lesson, making reference to the ‘Common issues’ table.

Use of the guide in the construction arena

In the UK research study teachers’ use of the guidance materials during the enactment of the lesson varied significantly. For example, one teacher always had a printed copy of the guide with him in the classroom and commented that he would use it to “[think] about the timings of the lessons and just [remind] myself [of] what’s coming up next ... it’s nice to just have those prompts for when your mind goes blank” (final interview, 1/7/13). Another teacher would print out the teacher guide but scale it down to four pages per printed page, (likely to make it difficult to engage with, at a glance during a lesson) and another teacher would have the teacher guide accessible on his iPad “so that [he] could refer back to it” (final interview, 1/7/13). Others would print certain sections of the guide for classroom use or not print it out at all. One of the teachers regularly printed out the ‘Common issues’ table and had this with her in the lesson. Whilst another teacher did not print out the guide, he would often access the guide on his computer in order to be able to display the ‘Solutions’ section to students at the end of the lesson. Two other teachers also displayed the ‘Solutions’ section of the teacher guide to students during the lesson, a surprising use of the summary solution, which had been designed as guidance for the teacher, rather than a resource for students.

Some further issues also arose, that demonstrate how the teachers' enactment of the lesson in general deviated from intentions signalled in the guidance materials:

1. The guidance suggests that teachers make sure students understand the context of the task, prior to attempting the problem and gives suggestions of how to do this. In the research study, the teachers would often distribute the pre-assessment quickly at the end of an earlier lesson, refraining from giving *any* information, in an attempt to allow students to work things out on their own.
2. The guidance asks teachers to 'note different student approaches to the task'. Teachers are advised to listen and watch students carefully, to note different approaches to the task and the assumptions students make and to note any common mistakes. This information can then be used as a focus for the whole-class discussion towards the end of the lesson.

In relation to this second point, when questioned about the use of the teacher guide, one of the teachers commented:

One thing I haven't really done, because the guide talks about things like making notes about what the pupils are saying to each other, to then sort of come back and influence the whole class discussion. I've kind of really just been listening to what they've been saying, and giving them prompts at the time. And then when it's come to sort of a class discussion, I've let them dictate ... There are a couple of times when somebody's said something that's vastly different to the rest of the class and I've wanted to make sure that they've shared that, but I haven't really used the things that they were saying to each other to form a basis of the class discussion as was suggested.

When questioned as to the reason for his deviation from the suggested guidance he explained:

I think maybe if I'd thought about having something physical where I could jot these things down. Part of it is remembering it, especially if it's like the first group you see, and you know you're going to see another five or six groups. Remembering who's said what, so it probably would have helped to have something to write those down on ... Also, I wanted the discussions to come from them, not led by me. (final interview, 1/7/13)

Whilst the teacher made this deviation from the guide explicit, some teachers, on occasion, when questioned about decisions they had made during the enactment of the lesson, would assume that they had been acting upon the suggested guidance, when at times this had not been the case. An example of this was seen when one of the teachers decided to ask the students to work in groups of four during one of the lessons. To facilitate this, the teacher spent considerable time rearranging the tables in his classroom and when questioned as to his motivation for doing this described how he wanted all groups to present their work to the rest of the class, commenting that he had "followed the suggestion from the teaching notes regarding this lesson, to allow pupils to discuss in pairs then come together in a four" (post-lesson questions, 13/12/13). On inspection of the teacher guide for this particular lesson, it was recognised that this was not actually the case and yet the teacher was confident that he had been following the lesson guidance.

Conclusions and discussion

It was evident during the UK study that teachers valued the guidance for the insight it provided into the lesson as a whole, but did not tend to use it to guide their minute-by-minute interactions in the classroom. Whilst all teachers referred to the teacher guide both prior to and during the lesson, their engagement with it varied.

It would seem that the ‘Common issues’ table proved to be the most useful aspect of the guidance, providing a valuable resource to the teachers within both the design and construction arenas. The inclusion of concrete examples of anticipated responses, with underlying misconceptions identified and suggestions on how to respond to the range of possible student approaches to the task, encouraged teachers to think about their practice *prior* to the lesson as well as informing teacher actions *during* the lesson. The inclusion of material on students’ thinking in teacher guidance requires substantial enquiry by designers and then by teachers, into student responses to particular topics and tasks. As Ball and Cohen (1996) suggest, by designing guidance materials in this way, with the construction arena in view, it is possible to see opportunities to use teacher guidance to assist teachers’ learning and practice, offering them more opportunities to learn in and from their work.

We note that the enquiry to inform the development of the ‘Common issues’ table in the guidance materials reflects the important ‘kyozai kenkyu’ phase of the Japanese lesson study cycle, in which teachers engage in classroom research into the critical role of stimulus material in generating student mathematical activity. Doig and Groves (2012) point to how this is often overlooked outside of Japan and perhaps the ‘Common issues’ table provides teachers with an alternative means of gaining the insight that their own research, through the use of a lesson study approach to examining their practice, might provide. It appears that the role of the ‘Common issues’ table in anticipating likely student responses provides an effective means of mediating important decisions within the design arena, in a practical way, that has utility and purpose to classroom teachers, both as they prepare for the lesson and in their moment-to-moment decision making during its enactment in the classroom. As remarked earlier, we found repeated use of the guidance materials and enactment in the classroom led to deeper engagement with the guide, with teachers then reporting a better understanding of the design principles for lessons.

The finding that teachers were prepared for students to engage in a problem-solving task without support in understanding the context in which the problem arises, has some significance. Although the ‘Common issues’ table does give advice about how to proceed in response to students’ initial and subsequent attempts at problem solving, it appears in this initial phase of the FAL, teachers provide minimal support. Research has shown that lack of specific guidance about what teachers could do to support students has led some teachers to conclude that they should never directly tell pupils anything (Leinhardt, 2001; Smith, 1996) and it may be that the teachers involved in the study were so focused on not telling students *how to do the task* that they omitted to check that students *understood the context of the task*.

Producing guidance materials that encourage teacher engagement is challenging and as has been outlined in this paper, often requires a lengthy process of revision and development over time. As the materials are developed, the question as to how such teacher guidance can be made accessible to teachers in a way that is useful, without being lengthy and onerous still remains. The role of online versions, that enable teachers to be selective about what they read, may provide a solution. Whilst the use of technology in presenting guidance materials was not a feature of the design of the FAL teacher guides, its potential in this area is recognised. The content of the materials is still of paramount importance, regardless of the presentation format. Whilst it is the enacted lesson that counts, the design of guidance that prompts teachers to think about the classroom enactment and possible responses from students *in advance* of the lesson, has an important role to play in encouraging teachers to

present, rich, multi-approach problems for students to solve in their mathematics classrooms.

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