

Flemish mathematics teaching: Bourbaki meets RME?

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The Programme of International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) create much international interest in those countries perceived as high achieving. One such system, rarely acknowledged, is Flanders, the Dutch-speaking region of Belgium. In this paper I present the results of focused analyses of four sequences of video-taped mathematics lessons taught to students aged 10 to 14 years. These confirmed a mathematics education tradition drawing on two well-known curricular movements. The first presents mathematics as a Bourbakian set of interconnected concepts. The second exploits realistic problems in its presentation of mathematics.

Keywords: PISA, TIMSS, Flemish mathematics education, Bourbaki, RME

Introducing TIMSS, PISA and their impact

For nearly two decades educational policy makers, teacher educators, and researchers have been scrutinising the results of international assessments of students' mathematics achievements. Alongside the Trends in International Mathematics and Science Study (TIMSS), repeated every four years since 1995, has been the Programme of International Student Assessment (PISA), repeated every three years since 2000. Each report provokes a flurry of activity, frequently misplaced, focused on understanding how other systems, not always culturally and demographically similar to one's own, have appeared to achieve better results.

One European system whose students have shown higher than typical European achievement on both forms of test has been Flanders, the autonomous Dutch-speaking region of Belgium. It has participated in three of the five reported TIMSS and all reported PISAs. Its rankings on mathematics, not always well-known because PISA reports Belgium as a whole¹, are unparalleled in Europe, as shown in table 1.

TIMSS (Grade 8) Mathematics			PISA (Age 15) Mathematics				
1995	1999	2003	2000	2003	2006	2009	2012
1	1	1	2	2	1	1	2

Table 1: Flemish Europe-related rankings on TIMSS and PISA

On Flanders and Flemish education

Despite its unique successes Flanders remains largely unexplored as a research site, prompting the question, what characteristics of Flemish education in general and

¹ Until the reporting of PISA 2012, the Flemish authorities have published their own PISA summaries. These have been produced at the University of Gent by Inge de Meyer and her colleagues (De Meyer, 2008; De Meyer et al., 2002, 2005; De Meyer and Warlop, 2010).

Flemish mathematics teaching in particular have precipitated these successes? In this paper I examine the second half of the question and, as a way of framing my analysis, draw on anecdotal accounts describing Flemish mathematics teaching as a juxtaposition of the formalism of Bourbaki and the informalism of Realistic Mathematics Education (RME). However, before summarising these seemingly incompatible conceptions of mathematics education, I present a case as to why English policy-makers - and here I focus deliberately on England - should be interested in Flemish mathematics education.

Firstly, not only is Flanders economically comparable to England, but it has experienced similar socio-economic and ethnic segregation, due mostly to post-war immigration from Southern Europe, Turkey and North Africa (Agirdag et al., 2012a). Such communities typically congregate in working class districts, leading not only to higher proportions of immigrant children in particular schools but lower achievement in comparison with that of native Europeans (Dronkers and van der Velden, 2013) and increasing feelings of isolation and helplessness (Agirdag et al., 2012b).

Secondly, like England, Flanders operates a complex system of school types, although, unlike England, these are typically fully funded by the state (Cherchye et al., 2010). Significantly, the core secondary curriculum is the same for all. However, with guidance students choose either a vocationally oriented, a humanities oriented or a classically oriented track (Op 't Eynde et al., 2006). This right to choose initially puts more students in higher tracks than in selective systems, leading to high expectations and achievement (Prokic-Breuer and Dronkers, 2012). Interestingly, school type and track influence significantly the mathematical learning of Flemish students (Pugh and Telhaj, 2007), with Catholic schools outperforming municipal schools, which outperform national schools (Cherchye et al., 2010).

Thus, in the light of Flemish mathematical success and cultural resonance with an English context, I argue that an analysis of Flemish mathematics classrooms may yield insights relevant to curricular and pedagogical developments in England. Also, as indicated above, there is anecdotal evidence that Flemish mathematics teaching is an unlikely juxtaposition of Bourbakian mathematics and realistic mathematics education (RME) (Dyckstra, 2006), traditions summarised briefly in the following, making it a particularly interesting research site.

Bourbaki and Realistic Mathematics Education

The realistic mathematics education (RME) movement was an alternative to the deductive mathematics experienced by Dutch students, whereby learning entailed the acquisition of isolated and decontextualised knowledge and skills (Wubbels et al., 1997). Such knowledge, divorced from children's experiences, is rapidly forgotten and "children will not be able to apply it" (Van den Heuvel-Panhuizen, 2005a: 2). Based on a belief that "what humans have to learn is not mathematics as a closed system, but rather as an activity" (Freudenthal, 1968: 7), RME comprised three key elements; learning mathematics necessitates doing mathematics; the subject matter of mathematics should draw on solutions to problems derived from reality (Van den Heuvel-Panhuizen, 2005b); and through the processes of mathematising, students come to reinvent mathematics (Gravemeijer, 2004).

Nicolas Bourbaki, a pseudonym adopted by group of French mathematicians, published material intended to remedy a lack of structural integrity and intellectual rigour in university mathematics (Munson, 2010; Weintraub and Moroski, 1994). The group promoted axiomatic mathematics, based on set theory, from which all topics are

derived (Guedj 1985; Landry 2007). Such perspectives, on mathematics as the study of “objects as sets-with-structure” (Awodey, 1996: 211), not only influenced how Piaget came to view mathematics and mathematical learning (Munson, 2010) but provided the intellectual underpinning of the *New Math* movement, which, in an attempt to bridge the gap between school and university mathematics, privileged principles over procedures, highlighted structures, sets and patterns, and emphasised experiential over rote learning (Klein, 2003).

Method

This paper draws on data derived from a video study of mathematics teaching in five European countries. Focusing on how four Flemish teachers, each considered locally as effective, present mathematics to their students, data are sequences of five lessons taught on standard and previously agreed topics. Filming sequences reduced the possibility of showpiece lessons and agreed topics facilitated comparative analyses. All four teachers were involved in initial teacher education, while one had been filmed as part of a national project aimed at improving mathematics teaching quality.

Tripod-mounted cameras were placed discretely at the side or rear of project classrooms and videographers instructed to capture all teacher utterances and as much board-work as possible. Teachers wore wireless microphones, while static microphones captured most public student talk. The first two videos in each sequence were transcribed and translated into English by English-speaking colleagues. This enabled the production of subtitled videos that colleagues from all countries could view and analyse. The accuracy of the transcripts was verified by a Dutch-speaking graduate student at my former university.

Analysis was undertaken in three phases. Each subtitled lesson was viewed several times to obtain a *feel* for how it played out; each of these subtitled lessons was viewed again, usually several times, to identify episodes characteristic of either Bourbakian mathematics or RME; the remaining lessons - three in each sequence - were viewed several times for additional illustrative evidence.

Results

The analyses yielded several themes, two of which are reported here. The first concerns teachers’ exploitation of realistic problems, while the second teachers’ attention to the rigorous development of those conceptual structures that underpin mathematics typically associated with Bourbakian mathematics.

Realistic mathematics education

While not a common occurrence, the analyses indicated that realistic problems were not only conceived very broadly but were used strategically by all four teachers to provide pedagogical and affective motivation for their teaching. For example, as they arrived for their first lesson on linear equations, Pauline informed her students that the lesson would be based around a problem involving the cartoon family, the *Simpsons*. This brought forth discernible gasps of excitement and many smiles. Pauline then questioned the class to establish the names and ages of both the children and their mother before posing, very slowly and animatedly, the following question.

Bart is 7 years old and Lisa is 5 years old when their little sister Maggie is born. Their mother, Marge, 34 years old, wonders if there will be ever a year in which she will have exactly the same age as her three children together.

Having posed the problem it then provided the impetus for three different episodes of the lesson. Firstly, Pauline asked her students to copy a table she had prepared on the board (see figure 1), which comprised three rows - Marge's age, the sum of the children's ages and the difference between the two. The first three columns were completed publicly before she invited the class to complete the rest. She allowed several minutes before initiating a public discussion of the results. However, before attending to the answer she focused on the ways in which the figures in each row changed, highlighting the growth in the two rows relating to age and a decline in the difference between them, which she annotated in red. Next, after eliciting the answer of 11 years, she asked what figures would be in the final column and received the answers 45, 45 and 0, which she added to the table. Further questioning found that zero was a consequence of subtracting 2 eleven times from 22. Finally, she amended the table, as shown below, to highlight what the public conversation had yielded.

Marge's age	34	35	36	45
Children's total age	12	15	18	45
Difference	22	20	18	0

Figure 1: The table of values publicly constructed for the solution of *the Simpsons* problem

Secondly, although students were not asked to do anything similar for themselves, Pauline switched on the overhead projector and, by means of three prepared transparencies - axes and two straight lines - demonstrated how the straight lines representing the two growth patterns intersected after 11 years. This episode of the lesson also involved much questioning.

Thirdly, Pauline initiated a public discussion on an unknown as a means of representing the number of years that would pass before the two sums would be equal. This extensive bout of questioning led to her writing on the board that if x represented the number of years necessary for the two sums to be equal then Marge would reach $34 + x$ years, while the children would reach $7 + x$, $5 + x$ and $0 + x$ respectively, which was written very slowly and neatly. Finally, $7 + x + 5 + x + x = 34 + x$ was written on the board, at which point Pauline announced that the topic under scrutiny was equations, which was then written in red and underlined. However, she did not simplify the equation to the $12 + 3x = 34 + x$ or attempt, at this point, to solve it.

A second example occurred during Emke's introduction to her sequence of lessons on percentages. Percentages is one of a small number of topics to have little meaning when taught independently of its role as a mathematical tool for understanding and managing many aspects of the real world. In this respect, her introduction demonstrated well this understanding. She had, prior to the lesson, asked students to bring in artefacts relating to percentages before spending the first ten minutes of the lesson discussing them. The first part of this public discussion went as follows:

Emke Who can give me an example? A large one, so that everybody is able to see it. Yes, Katrijn?

Katrijn A yoghurt pot.

Emke A yoghurt pot, yes. Please show it. ... And?

Katrijn With nine percent fruit.

Emke Nine percent fruit. What would that mean? Katrijn has a yoghurt pot containing nine percent fruit. Sofie?
Sofie There are nine percent of one hundred fruit in it.
Emke Nine percent of one hundred fruit in it. I do not understand this very well. Who can explain it? Nine percent fruit are there are nine pieces of fruit in it?
in it. Does that mean
Several No.
Emke Maybe nine grams?
Several No.
Emke Would it make a difference if it is a large or a small pot? Or would it remain nine percent?
Joost It depends. Yes, I think it would stay the same.
Emke That those nine percents will always remain, yes. The amount will change. Afterwards, we will learn what those nine percent actually mean.
Emke Does anyone have still a similar example concerning a part in respect of a whole? So the whole is the pot of yoghurt and there is a part in that pot of yoghurt. Is there anyone with a similar example?

Following this, Emke spent much of the next two lessons posing a series of problems that exposed the multiplicative structure of percentage calculations. However, at no point did she undertake any calculations, but, through the use of number base blocks, focused attention solely on the conceptual underpinning.

Towards the end of a sequence of lessons on plane shapes, in which tangrams had been used to highlight the fact that polygons are typically irregular and frequently concave, Peter wheeled his bicycle into the classroom and asked how students might predict how far he would travel if his wheels underwent one full turn. This particular problem then motivated an investigation into the constancy of the relationship between diameter and circumference, which Peter managed as a collective activity.

Bourbaki

Throughout their lessons project teachers were observed not only to focus extensively, and frequently to the exclusion of procedural skills, on the conceptual underpinnings of mathematics but also to make explicit the links both within and between topics. Moreover, a key element of this tradition seemed to lie in the encouragement of students to acquire and use the formal vocabulary of mathematics. For example, as indicated above, following her introduction, Emke invited students to use base blocks to model the conceptual underpinning of percentages. The first task, publicly posed as were all such tasks, went as follows:

Emke Everybody should place four hundred squares in front of him. Not stacked, but next to one another. There should be some distance between them, so that you can see very well that you have four times one hundred, right?... Now put five in front of each hundred (she waits while students follow her instructions)
Emke So, what did you do? What have you done? Yes, Elke?
Elke I've put five unit cubes in front of each hundred square.
Emke And which value has a hundred cube?
Elke One hundred.
Emke One hundred. So what have you done?
Joost I've put five unit cubes in front of each hundred.
Emke You can also say, I've put five per 100, or five at 100. Or you can also say, I've put five on 100. These are all different ways of saying the same thing.
Emke How should we write it down? You have put five units for every hundred. Of how many? How many do I have in total?

Frederick Of four hundred.

Following this she wrote on the board, including brackets; (five for every hundred) of four hundred. In so doing, she exposed, something she later came to exploit, the multiplicative basis of percentages. Moreover, she ensured that students were able to follow the procedure correctly, wrote what she later came to call a formulation and, finally, highlighted the linguistic arbitrariness of positional prepositions.

A more prosaic example was seen in Heleen's teaching of grade seven polygons. During her second lesson she invited three girls and a boy to come to the board and, respectively, draw a right-angled trapezium, a rectangle, a parallelogram and a rhombus. As they worked, using set squares and lengths of string as compasses, she drew two intersecting but arbitrary line segments and began questioning the remainder of the class about the ways in which various configurations of diagonals define different quadrilaterals. Her questioning yielded a set of relationships pertaining to parallelograms, rectangles, kites, rhombi and squares. Finally, turning her attention to the now completed diagrams, Heleen initiated a discussion of the properties of each of the four quadrilaterals, asking about their angles, sides and diagonals. In so doing she focused attention on how different ways of analysing quadrilaterals' properties created different hierarchies, each of which started with the most general form of quadrilateral but, through ever more tightly defined structures, led to different conclusions. In so doing, she made her students aware not only of the role of classification in mathematical structures but also how they may or may not be equivalent according to, essentially, arbitrary decisions as to the focus of attention.

Shortly before she began her exposition on the solution of linear equations, Pauline initiated a discussion on some of the fundamental relationships of arithmetic. She wrote the statement, $a = b$, on the board and questioned her students as to what could be inferred if, for example, c was added to both sides. Her students responded appropriately, telling her that the equality would remain. After two or three minutes, the following was completed on the board.

$$\begin{aligned} a = b &\Rightarrow a + c = b + c \\ &a - c = b - c \\ &a : c = b : c \\ &a \cdot c = b \cdot c \end{aligned}$$

Such insights, written very formally but always discussed publicly, permeated Pauline's work. Later the same lesson, after she had begun to discuss the solution of $6(x - 5) - 8 = x - 3$, which was the focus of her exposition, students introduced words like associativity, commutativity and distributivity to justify the various actions they proposed. In such vocabulary lies a deep-seated and conceptually sound, awareness of mathematical structures and their interconnectedness.

Discussion

Acknowledging that the data are limited to analyses of just twenty lessons, the juxtaposition of two substantially different traditions is interesting. Many of the teachers' approaches were consistent with the traditions of Bourbaki. For example, they encouraged high levels of mathematical rigour consistent with Bourbakian expectations that "mathematics itself should rest on a bedrock of unshakable fundamental principles" (Munson, 2010: 18). Moreover, teachers' structural emphases accord with the Bourbakian belief that pedagogy should "elucidate the fundamental structures" to the extent that "underlying ideas must be elevated above the examples

that illustrate them” (ibid: 19). Also, their ambitions were clearly commensurate with Bourbakian perspectives on school mathematics, in that what they did could be construed as attempting to bridge the gap between the mathematics of school and university, privileging principles above calculations, emphasising structures and autonomous experimentation (Corry, 2007).

Secondly, if realistic mathematics entails “taking students’ initial understanding as a starting point, providing them with problem situations which they can imagine, scaffolding the learning process via models, and evoking reflection by offering the students opportunities to share their experiences” (Van den Heuvel-Panhuizen, 2005b:36), then Flemish project teachers clearly appeared to work within such frameworks. However, typical RME problems encourage multiple solution strategies allowing students to compare approaches and gain new insights (Treffers, 1987). Moreover, typically in the early stages of a topic’s development, RME exploits students’ informal solution processes as an important first step in the process of mathematisation (Van den Heuvel-Panhuizen, 2005b). Indeed, project teachers, despite extended periods of public discourse, encouraged neither the sharing of multiple solutions nor informal methods; the tasks they exploited were always directed towards well defined goals closer to the aims of Bourbaki than RME.

In conclusion, it is probably more accurate to describe Flemish mathematics as Bourbaki moderated by RME than an equal partnership of the two, although there is clearly more to Flemish teaching than such descriptions suggest. For example, the analyses highlighted two other characteristics in need of further research. Firstly, all lessons were largely teacher-centred with relatively little individual work. Secondly, the pace of lessons was slow, in that teachers seemed content to take several lessons to develop the conceptual basis for any procedures they introduced later. However, this is only an initial analysis and much further research will be needed before we understand the construction of Flemish students’ mathematical achievements.

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