## BSRLM – MEI Abstracts, St Patrick's College, Dublin, 27 & 28 February 2015

### 27 February Plenary presentations

## Shiel, Gerry & Dooley, Therese

Educational Research Centre, Dublin; St. Patrick's College, Dublin

Developing Mathematics Learning and Teaching in the Context of Curriculum Renewal Recently, the Irish National Council for Curriculum and Assessment (NCCA) announced its intention to revise the current mathematics curriculum for primary schools and commissioned two research reports on teaching and learning mathematics in early childhood and primary education (3-8 years). One of the reports focused on theoretical aspects of mathematics education, and the other concerned teaching, learning and assessment. In this plenary, two of the reports' authors draw on key findings in three areas: aims, foci and goals; learning paths; and language and problem solving. First, we will provide a brief overview of the current mathematics education context in Ireland across early childhood, primary and post-primary levels. Then we will address each of the three areas with reference to existing research evidence. Implications for practice and future research possibilities will also be discussed.

### Gifford, Sue

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A good foundation for early years mathematics education in England?

What is the likelihood that the current English mathematics curriculum policy for the Early Years will increase children's achievement in mathematics? Recent expectations are not supported by research, which suggests focusing on number sense and developing this in playful ways. What might such a curriculum look like in the English context? Alternatives and implications will be reviewed.

## 28 February Research presentations

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An analysis of pre-service teachers' reflections of 'good practice' teaching videos

Video clips of mathematics lessons are used extensively in initial teacher education and continuing professional learning activities. Given course time constraints, an opportunity to critique these videos is not always possible. Because of this, and because pre-service teachers make extensive use of material found during internet searches, much of it purporting to exemplify 'good' practice, we were interested to know what sense they make of such material. By encouraging pre-service teachers to reflect and comment on the practices being promoted in this way, we wanted to hear what they focused on, their initial views of the teaching and learning shown in the video, and how their views were formed and affected by engaging in discussion.

#### Barclay, Nancy

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Encouraging productive mathematical noticing

Mason's (2002) statement 'what you do not notice, you cannot act upon' provides a stimulus for this doctoral research. This classroom based study explores the role of dialogue in encouraging and influencing primary children's 'mathematical noticing'. It further considers how what is noticed is built on in the dynamics of pair interaction.

This small scale study is currently ongoing; data comprises video recordings of mathematical activity in three primary school classrooms over the course of the current academic year. In this session I will present a small sample of data drawn from one research lesson. I invite comment, ideas and advice on the analytic tools and framework currently being developed and used.

## Bisson, Marie-Josée; Gilmore, Camilla; Inglis, Matthew & Jones, Ian

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Measuring Conceptual Understanding Using Comparative Judgement

The importance of improving students' understanding of core concepts in mathematics is well established. However, assessing the impact of different teaching interventions designed to improve

students' conceptual understanding requires the validation of adequate measures. We propose a novel method of measuring conceptual understanding based on comparative judgement (CJ). Contrary to traditional instruments, the CJ approach allows test questions for any topic to be developed rapidly. In addition, CJ does not require a detailed rubric to represent conceptual understanding of a topic, as it is instead based on the collective knowledge of experts. In the current studies, we compared CJ to already established instruments to measure three topics in mathematics: understanding the use of p-values in statistics, understanding derivatives in calculus, and understanding the use of letters in algebra. The results showed that CJ was valid when compared to established instruments, and achieved high reliability. We conclude that CJ is a quick and efficient alternative method of measuring conceptual understanding in mathematics and could therefore be particularly useful in intervention studies.

#### Breen, Sinead & O'Shea, Ann

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Transition through Mathematical Tasks

The transition to university level mathematics is often problematic for students. Clark and Lovric (2008 and 2009) have written about some of the differences between mathematics at school and at university. These differences include the type of mathematics taught and the way mathematics is taught. Students at this stage also have to contend with social and cultural changes. As part of a project on task design, 10 first year students at two different universities in Ireland were interviewed. We discuss the students' experience of mathematics at school and at university. In particular, we will consider the differences in the types of mathematical tasks encountered at both levels and the students' views of the influence of such tasks.

#### Brown, Tony

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Rationality and belief in learning mathematics

The pursuit of economic ambitions sometimes seems to result in school mathematics favouring the performance of skills and procedures rather than nurturing student's intuitive powers of mathematical rationality. I argue that this apparent dichotomy conceals a common belief that mathematics exists independently of its past or present applications. I propose a perspective where the material points of reference that characterise school mathematics are seen as supporting a speculative belief in ideal mathematical entities. These privileged entities (e.g. geometric objects, iterations, counts, equations) are seen as deriving from human experience of physical or social worlds and the rationalities that have been created to connect them. As a consequence I contribute to recent debates on the politics of mathematics education by arguing that rational mathematical thought necessarily rests on beliefs set within a play of ideological framings. School mathematics then presents not so much a distortion of "genuine" mathematical thought as a particular mode of thinking that enables the inclusion then selection of learners according to arbitrary curriculum or assessment criteria. An interface of material and ideal is illustrated by some student teachers reporting on their shared experience in a spatial awareness exercise concerned with the speculative apprehension of geometric objects.

#### Burroughs, Elizabeth

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Empirical Evidence of Knowledge for Coaching Mathematics Teachers

I present results from a five-year longitudinal experimental study in the field of mathematics classroom coaching. The study examined a sample of 56 coaches who work with mathematics teachers in classrooms with children aged 5 to 13. I will describe the construct of coaching knowledge and present our findings about relationships between coaches' skills, practices and beliefs about coaching and their coaching effectiveness. Coaching effectiveness is defined by positive gains in measures of teachers' mathematics knowledge, classroom practice, or attitudes about teaching mathematics. Participants will be invited to discuss those elements of the practice of coaching that can be empirically studied and those questions about coaching that this study has so far been unable to answer..

Cronin, Anthony

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Lecturing a large university mathematics class – can mathematics support centre feedback help? The Mathematics Support Centre (MSC) in University College Dublin (UCD) is a busy centre with almost 6000 student visits recorded in 2013-2014. Over the last 18 months we have been developing a system that captures the visiting student's mathematical difficulty. One aim is to enable the MSC to offer evidence-based support to students. Another aim is to provide feedback to lecturers. The (anonymised) feedback is uploaded to a dedicated page of the School of Mathematical Sciences website where it is organised by module code, and can be accessed by the lecturer(s) of the module. We were keen to find out how useful lecturers of large first and second year groups found this MSC feedback, and in particular, if this feedback had, or had the potential to have, an impact on their practice. Thirteen academics agreed to participate in the research. They accessed the feedback regularly and were interviewed three times during the first semester. We present some preliminary findings. Almost all lecturers in the study found the MSC feedback useful and for some it impacted their practice.

### Curley, Nuala & Meehan, Maria

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The Challenge of Collecting and Analysing Useful Qualitative Data on Students' visits to a Mathematics Support Centre at a University in Ireland

Since September 2008, the Maths Support Centre (MSC) in University College Dublin (UCD) has kept an electronic record of each student visit to the centre. By September 2013 there had been 21 200 visits and an analysis was planned to identify the mathematical topics and concepts that cause persistent difficulties for students inorder to better target the provision. However we found nature of the data (tutors' comments) lacked specific detail to enable the proposed anlaysis to take place.

We realised that in order to identify the mathematical topics students experienced difficulty with, we needed to identify the nature of the data we required and then work with the tutors to find ways that this could be done efficiently. We describe our efforts, and those of the tutors, over the last 18 months to collect this data as efficiently as possible.

In September 2014, we commenced our data collection proper. This involved eight weeks of intensive collaborative work with the tutors to ensure the quality and authenticity of the data collection. During this period there were 2012 visits to the MSC. We present a preliminary analysis of the most prevalent mathematical topics that are emerging from this eight-week data collection.

### Dalby, Diane & Wake, Geoff

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Structuring cycles of teacher classroom inquiry

There is evidence that effective teacher professional development is contingent upon sustained activity over substantial periods of time and collaboration within mathematics teams. However it also benefits from being informed by outside expertise, being evidence-based and attentive to the development of the mathematics itself (Joubert & Sutherland, 2009). Working with these guidelines, as part of the professional development strand of the EU-funded project MaSciL (Mathematics and Science for Life: http://www.mascil-project.eu), we have designed a toolkit for professionals who are leading teachers in developing inquiry approaches to learning mathematics and science in ways that connect to "the world of work". To ensure that participant engagement is sustained over substantial periods of time, the professional learning is designed to connect closely with teachers' day-to-day classroom activity. Consequently our model involves establishing professional learning communities that take part in repeated cycles of inquiry. We provide insight into our design of two online toolkits, for in-service and pre-service teachers, by describing the theoretical underpinning of the design and examining a sample of the tools.

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Developing Mathematical Knowledge for Teachers

The concept of Mathematical Knowledge for Teaching (MKT) was introduced by Ball et al. (2008) building on Shulman's (1986) notion of pedagogical content knowledge. MKT is 'the mathematical knowledge needed to carry out the work of teaching mathematics'. In this project, a team of researchers

at DCU and Maynooth University studied the development of MKT in two groups of pre-service teachers

The project aimed to help students develop their own MKT thorugh a series of workshops designed and delivered by the authors. The workshops involved a broad discussion of MKT using the Ball et al. (2008) paper, video analysis of teaching situations and discussion, mathematical task analysis and design, sessions on content knowledge relating to fractions and algebra and analysis of pupils' work related to these concepts, and readings from the Mathematics Education literature. The students' awareness of MKT was investigated using pre- and post-intervention questionnaires and focus group interviews. We describe the intervention and present the findings from the analysis of the data collected. In particular, we describe how the group's view of the mathematical work of a teacher changed over the course of the project.

### Deshler, Jessica

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Using Modified Lesson Study with Mathematics Graduate Teaching Assistants
Researchers estimate that 37% of undergraduates will have a graduate student as a mathematics instructor sometime during their programme, probably early on (Speer, Murphy, and Gutmann, 2009) and that novice instructors play a critical role in undergraduate mathematics education: graduate students are instructor of record for as many as a half-million undergraduates each year in the US (Lutzer, Rodi, Kirkman, & Maxwell, 2005). Many of these inexperienced instructors interact with many students early in their college experiences and they influence undergraduates' feelings and decisions regarding mathematics.

In a recent study, seven graduate teaching assistants (GTAs) worked together to develop, implement and critique two lessons for a Calculus I course using modified lesson study. A GTA was chosen randomly to teach the lessons during a calculus class while the other GTAs observed. A preliminary analysis of the GTAs' critiques reveals the reflections of the GTAs to be focused superficially, supporting previous work on K12 pre-service teacher reflective abilities. The specific topics of the reflections will be discussed as well as possible ways to improve the reflective abilities of GTAs and current efforts in research and support of mathematics graduate teaching instructors in the US.

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Using Narrative to Access Mathematical Identity: An overview informed by an empirical study in five third level institutions in Ireland

We present the findings of research into the mathematical identity of students using narrative as a tool. It is the work of a team from five institutions on the island of Ireland and was part of a project entitled "Mathematical Identity using Narrative as a Tool" (MINT) funded by the Standing Conference on Teacher Education North and South (SCoTENS). The chief aim of MINT was to propose an efficient protocol for third level mathematics educators to explore the mathematical identities of their students. We set the scene with a discussion and overview of the central notions of mathematical identity and narrative, taking cognisance of different perspectives from the literature. We then present an overview of the methodology and the themes arising from the qualitative data collected from groups of students in the various institutions, and compare these with themes identified previously in similar studies. We offer evidence that narrative enquiry can produce meaningful insights into mathematical identity and that the themes, although requiring some refinement for different cohorts, provide a robust framework on which to predicate analysis. We conclude with proposals for future work in this area.

# Freemyer, James; Johnson, Patrick & Fitzmaurice, Olivia

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Motivating Young People to Seek Careers in Science, Technology, Engineering, and Mathematics: Research Conclusions from Interviews and Observation in Ireland and the U.K.

We need fresh ideas. James Stigler, in The Teaching Gap (1999), noted that "looking across cultures is one of the best ways to see beyond our blinkers and sharpen our view of ourselves" (page xviii). He provides an effective blueprint for international research to compare and contrast approaches in different countries, stimulating ideas for new and more effective teaching approaches. This research

sought to compare and contrast current teaching practices in Ireland and the United Kingdom by a neutral educator from the United States.

The research results include teacher observation and interviews from more than 14 schools and 50 teachers from Ireland and the United Kingdom during the fall semester of 2014. A research paper will be provided which includes summaries of the interviews, observations, and survey results from 144 teachers in Ireland. The survey results provide insights into what Irish teachers feel encapsulate effective teaching practices for mathematics education.

## Gurhy, Ann Marie

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Assessment and Mathematics Learning: The Learner's Perspective

Growing evidence suggests that Assessment for Learning (AfL), aka Formative Assessment (FA), is a warranted strategy that positively impacts student achievement (O'Leary, Lysaght and Ludlow, 2013). During the academic year 2012-2013, the impact of AfL practices on the teaching and learning of mathematics in fourth class in a vertical, all-girls primary school in Ireland, was investigated. Underpinning the potential of AfL to impact learning is the belief that learners know where they are in their learning, where they are going and how they can close the gap (Willis & Cowie, 2014; Chappuis, 2005), and so understanding participant perspectives in AfL interactions is important. Specifically, I focus on one aspect of my doctoral research, exploring the findings from the perspective of learners, the oft-neglected voice in education. Quantitative and qualitative findings reveal that by the end of the intervention, students had increased their use and understanding of AfL practices during mathematics lessons, and were well able to articulate their perspective, thus offering valuable insights about the project. Findings revealed a statistically significant increase in students' confidence, motivation, beliefs and attitudes towards mathematics, although results on standardised tests were similar to the control group.

## Harbison, Lorraine & Harbison, Joseph

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How good at mathematics do students need to be on entry to primary school initial teacher education? Student primary teachers might be expected to be competent in primary school mathematics (PSM). In Ireland, mathematics is graded at Leaving Certificate (LC) level with a point score allocated by the Central Application Office (CAO). There is controversy about setting a threshold in LC mathematics for initial teacher education (ITE). It is unclear how LC results correspond to competence in PSM. 95 student teachers completed a standardised attainment test (SAT, SIGMA-T) for 11-12 year olds in the first term of college. Results were compared with their CAO scores for LC mathematics. A 'pass' mark of mean-1 SD SAT score was derived as a potential threshold for LC attainment.

The mean SAT score was 84.6 (SD 13.9). The mean CAO score was 43.4 (SD 14.5) corresponding to B2 grade at Ordinary Level. The SAT score correlated moderately with CAO score (r=0.59, p<0.001). 16 out of 17 students unsuccessful in the SAT scored 40 or less in CAO. However, 40% of students successful on the SAT also scored 40 or less.

LC mathematics achievement correlates moderately with PSM competency but may an unsuitable measure for establishing a threshold entry requirement for ITE.

## Hough, Sue; Solomon, Yvette; Gough, Steve & Dickinson, Paul

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'Joel's getting left behind' - dilemmas of teaching and learning in post-16 GCSE resit
Students who have not achieved at least grade C in GCSE Mathematics by 16 are now required to work towards this during their 16-19 study. However, prior experience means that they frequently lack motivation, confidence and understanding, and resit success rates are often poor. Working within the constraints of a nine-month course, teachers feel a tension between covering content and taking time to develop understanding: consequently, teaching is largely transmission-based, and focused on examination practice and memorisation of rules and procedures. We report on an intervention based on Realistic Mathematics Education which aims to enhance achievement through support for deeper and longer-term understanding, and to change students' negative perceptions of mathematics and their own ability. Teaching employs sustained use of context and models to support the process of formalisation while retaining 'sense-making'. Interim performance, interview and 'smartpen' data indicate that while there is evidence for progress towards 'giving it a go', some students, especially those on the D/C

grade boundary, can be unwilling to engage with methods that they see as unnecessary and too slow, when faster learned alternatives are available. We discuss the dilemmas for both teachers and students, focusing on pace, coverage, and formal methods versus making meaning.

#### Howard, Emma

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Mathematical Identity and the Transition to University – quantifying the impact of the transition on the mathematical identity of university students

Society tends to use external indicators such as examination grades to judge mathematical prowess. However, mathematical identity is also a suitable indicator. Students choose third-level education paths based on their sense of self, future professional ambition and what gives them enjoyment. The transition from post-primary to third-level education has a significant impact on the mathematical identity, and sense of belonging to maths, of students. It can cause students to change their major, or even withdraw from their degrees.

I discuss quantifying the impact of the transition from post-primary to third-level education on mathematical identity using a questionnaire with a sample of first year students in an Irish university who have a strong mathematical background at post-primary level and are enrolled in a degree programme where they have the choice to pursue mathematics to degree level. Statistical analysis was used to check for statistically significant changes to aspects of mathematical identity. Clustering was applied to the survey results in order to 'group' students according to how their mathematical identity was affected in the transition. I hope to use statistical analysis, along with interviews with students, to gain further insight into how this transition impacts the mathematical identity of high-achieving students.

## Iannone, Paola & Simpson, Adrian

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*University students' perceptions of assessment: the role of context* 

We report results of a comparative study of students' perceptions of assessment. Participants were studying mathematics or education at one of two research-intensive UK universities and we employed a mixed method approach. A survey aimed at mapping students' preferences of assessment methods, employed the Assessment Preference Inventory used in our previous research. The survey was followed by semi-structured interviews with volunteers. Results of the survey show that students in both disciplines prefer to be assessed by methods they perceive as a good discriminator of academic ability. Mathematics students prefer closed book examination and education students prefer extended essay. Thematic analysis of the interviews reveal students' epistemological beliefs play an important part in shaping their perceptions of assessment. Education students perceive critical thinking, analytical reading and academic writing as important for knowledge acquisition in education which may account for their preference. Mathematics students perceive identify understanding and problem solving as important in knowledge acquisition which may explain their assessment preference. We conclude that more research is needed across Biglan's (1973) spectrum to discern the extent to which epistemological beliefs influence assessment preferences.

## Jones, Ian & Hunter, Jodie

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Teacher awareness of children's conceptions of the equals sign

Children's conceptions regarding the meaning of the equals sign in arithmetic and algebra have been widely studied. However, little is known about teachers' awareness of how children view the equals sign. We report results from a specially-designed instrument administered to practicing primary teachers (N = 197) in New Zealand and England. Confirmatory factor analysis revealed that the instrument performed as expected, with items loading cleanly on to "operational", "sameness" and "relational" factors. Overall, teachers seemed to be more aware of sameness conceptions than of operational and relational conceptions. Cluster analysis suggested teachers varied widely in their responses, and four clusters were apparent in the data. Cluster 1 prioritised sameness over other views, Cluster 2 prioritised relational views and to an extent sameness views, Cluster 3 seemed relatively unaware of students' views of the equals sign and Cluster 4 appeared to have a high all-round

awareness of students' views. We consider how teachers' backgrounds and experiences might impact on their awareness of children's understanding of equivalence, and identify future research avenues.

#### Kavanagh, Lauren; Millar, David & Shiel, Gerry

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Mathematics Performance Outcomes from the 2014 National Assessments of English Reading and Mathematics

National assessments of literacy and numeracy at primary level have been conducted periodically in Ireland since 1972. We outline mathematics performance outcomes from the 2014 National Assessments of English Reading and Mathematics. The 2014 assessments were administered to representative samples of over 8000 pupils in second and sixth classes in 150 Irish primary schools. Trends in performance from the last round of national assessments, conducted in 2009, are also outlined. Breakdowns of performance by content area and process skill are presented. Analyses of performance by gender and by school disadvantaged status are also summarised. Findings are considered in the light of performance targets for numeracy outlined in the National Strategy to Improve Literacy and Numeracy among Children and Young People, 2011-2020.

## Lowe, Edmund & Pope, Sue

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Investigating expected progress in mathematics in an English secondary school

England is notorious for its high stakes performativity culture and a school inspection regime that strikes fear and dread into the heart of many teachers and school leaders. A key driver in any school inspection is academic outcomes (exam results) and, more recently, progress. Children are tested at the end of primary schooling (age 11) and awarded a level, level 4 is the national expectation. Secondary schools are expected to secure three levels of progress for all learners by the end of the subsequent five years of schooling. Children who achieve level 4 at the end of primary are expected to progress to achieve GCSE grade C which has been matched to level 7. Drawing on data for two cohorts of students from a school in a relatively deprived area of the country, this research found that three levels of progress was unlikely to be achieved by students who failed to meet the national expectations at the end of primary, but was probably insufficiently challenging for students who had exceeded national expectations at the end of primary. Only prior attainment, eligibility for free school meals and being on the school's SEN register were found to produce statistically significant progress outcomes.

### Martin, Helen & MacDonald, Catriona

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Valuing Diversity, Developing Flexibility: teachers' understanding of children's mathematical thinking We have been developing a research design based on the work of the Maths Recovery programme (Wright et al, 2006, 2012) involving both their Schedule for Early Number Assessment (SENA) and small group teaching approaches. Student teachers on a teacher education programme at the University of Aberdeen have been collecting video recorded data of children engaging with mathematical tasks in a wide variety of primary school classrooms across the north east of Scotland. As a result, we have a growing collection of recorded video conversations that will be added to each year of the project creating an exciting and rich data set to reveal the wealth of mathematical diversity among children. This project investigates how previous research focused on intervention can be used to enhance mathematical understanding of student teachers, teachers and therefore children. Initial findings (MacDonald, 2012) suggest when analysing interviews to explore mathematical reasoning, teachers seemed to move beyond narrow 'school-based' representations, recognising and valuing a broader range of reasoning. Our overriding contention is that there are many more young mathematicians in our schools than are recognised.

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Troubling Mathematical Concepts

"A chair or a star is not in the least like what it seems to be; the more we think of it, the fuzzier its outlines become in the haze of sensation which surrounds it; but '2' or '317' has nothing to do with sensation, and its properties stand out the more clearly the more closely we scrutinize it". In this extract

from A Mathematician's Apology, G. H. Hardy (1940) articulates the dominant view of mathematical concepts – that they are clear and bounded, asocial and beyond feeling. We see this view enacted in everything from education policy to popular representations of mathematics. In this presentation I will begin to trouble this dominant conceptualisation of mathematical concepts. I will suggest that the issue is not how closely we scrutinise them but how we scrutinise them and I will use queer theory to do so. Queer theory's aim is to show that where what we thought were clarity and boundaries, are disruption and chaos; that the construction of things as asocial and beyond feeling, requires and impacts socially and emotionally.

## Moffett, Pamela

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Reforming the algebra classroom through research lesson study

Algebra is regarded as a 'gatekeeper' course in the mathematics curriculum (Kaput, 2002) and the 'new civil right' that all students must experience success with in order to compete economically in society (Moses, 2001). Traditionally, the teaching of algebra has focused on procedures for manipulating symbols at the expense of a more conceptual understanding (Stephens, 2008; Walkoe, 2010). One of the greatest challenges for mathematics teacher educators is to provide preservice teachers with experiences that will broaden their vision of algebra so that they can effectively promote algebraic reasoning in the primary classroom (Stump & Bishop, 2002). Furthermore, research shows that teachers who pay attention to pupils' mathematical thinking increase their own knowledge of mathematical content (Fennema et al. 1996) and are in a better position to support pupil learning and increase pupil achievement (Jacobs et al., 2007).

This study focuses on a 'lesson study' project involving twelve preservice teachers who collaboratively planned, implemented, observed, analysed and reflected on an actual algebra lesson. Findings suggest that participation in the project helped the preservice teachers to attend to pupil algebraic thinking in ways that will foster a more conceptual understanding of algebra.

### Monaghan, John

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Digital tools in academic stream 16-19 mathematics curricula

My presentation is in three parts. The first part sets the scene with some questions: What is academic stream 16-19 mathematics? What role could/should digital tools play in an academic stream 16-19 mathematics curriculum? How are national curricula responding to the previous question? The second part summarises work from a recent report which examined 16-19 curricula in England, France, the Netherlands, New Zealand, Singapore and Victoria (Australia). The third part looks to future research that can build (and improve) on the work described in the second part and also link curricula innovation with what is (and could be) happening in classrooms.

## Ní Ríordáin, Máire; Paolucci, Catherine & O'Dwyer, Laura

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Examining the Mathematical Knowledge of Irish Out-Of-Field Mathematics Teachers

Teacher quality is believed to be one of the most important factors affecting student learning, and research has demonstrated that students learn more from skilled and experienced teachers who know what and how to teach (Darling-Hammond & Youngs, 2002). However, out-of-field mathematics teaching is prevalent in the Irish context and accordingly, a two-year part-time Professional Diploma in Mathematics for Teaching (PDMT) has been established nationally to up skill these out-of-field teachers. This research project aims to examine the content and pedagogical knowledge of out-of-field teachers, on commencing the PDMT (September 2013). The sample comprises 202 out-of-field teachers from across Ireland who are enrolled in the PDMT and who completed an entrance exam on commencement of the programme. All test items are aligned to key strands and associated sub-topics of the Irish post-primary curriculum, which these teachers are required to teach and assess. Findings indicate wide variations and significant areas of weakness in out-of-field teachers' conceptual understanding of key topics relating to the Irish mathematics curriculum. We will discuss these findings and their implications for understanding areas in which mathematics teachers need support and for the design of effective continuing professional development programmes.

#### Ni Shúilleabháin, Aoibhinn

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Developing PCK through lesson study: building teachers' knowledge of content and students (KCS) and knowledge of content and teaching (KCT)

This research investigates the development of teacher pedagogical content knowledge (PCK) through iterative cycles of lesson study. The research was carried out over one academic year as a multiple case study in two post-primary schools with twelve mathematics teachers. Qualitative data was generated through audio recordings of all teacher meetings around lesson study and through multiple individual interviews with participating teachers. Data was thematically analysed utilising a framework of mathematical knowledge for teaching as suggested by Ball, Thames, and Phelps (2008). From the analysis of the data as a temporal development of lesson study features of knowledge of content and students (KCS) and knowledge of content and teaching (KCT), developed through teachers' participation in iterative cycles, were identified. Not all teachers were affected in uniform ways through participation in the model and newly qualified teachers began to include more of these features of PCK in their planning and reflection conversations than their more experienced counterparts. This paper should provide a useful discussion point around utilising a framework of pedagogical content knowledge for in-service teachers and in highlighting the relationship and links between KCS and KCT as distinct elements of mathematics teachers' knowledge.

#### O'Sullivan, Brendan

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Textbook analysis: using textbook tasks to examine curriculum implementation in Ireland
Textbooks are an important resource in Irish mathematics classrooms, which can have both a positive
and negative impact on teaching and learning. The mathematics curriculum at post-primary level in
Ireland was reviewed in 2005. The *Project Maths* initiative was introduced to reform the mathematics
curriculum, bringing about changes to what students learn, how they learn and how they are assessed.
Publishers have produced new texts in response to the expectations of the revised curriculum and the
changed needs of the classroom. I present a framework to consider how tasks found in mathematics
textbooks are meeting the objectives of this new curriculum. Sections of textbooks currently being used
in Irish classrooms at second level have been analysed using this framework and the results indicate
that, while all textbooks incorporate a significant number of these objectives to some extent, key
aspects are being neglected.

### Rodd, Melissa

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Teaching Post-graduate Research Students in Mathematics about Teaching Mathematics to Undergraduates

I describe a new project (running from October 2014 - March 2015) with colleagues from the University College London (UCL) mathematics department which aims to help post-graduate research students teach undergraduates mathematics. Most post-graduate research students in the School of Mathematical and Physical Sciences at UCL have some teaching responsibilities while they pursue their own doctoral research and most of them are prospective university mathematics lecturers. However, until now, opportunities for their 'teacher education' were either generic UCL-based courses that did not address issues in mathematics teaching/learning or they were mathematics teaching courses run outside UCL (which were optional and rarely taken). Hence the motivation to provide a context-specific mathematics teaching course so their undergraduate teaching would be enhanced. There will be two main parts to the presentation:

- 1. Challenges in course design and delivery: what should a curriculum look like for these post-graduate research student participants? Especially as the course is anticipated to take less than 10 hours of their time.
- 2. Details of post-graduates' teaching the topic area: first year Real Analysis will be presented together with a discussion of what are good principles for teaching these post-graduate students to teach undergraduates.

#### Sani, Naomi

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How can we get more (good) teachers of mathematics - in our primary schools, secondary schools and F.E. colleges?

One way to get more teachers of mathematics in schools and colleges is to 're-train' the teachers we already have. Re-training teachers, of other subjects and from other phases, to teach mathematics has been happening in the UK for some years. The current Post-ITT Subject Knowledge Enhancement (SKE) programme has recently been renamed Teacher Subject Specialism Training (TSST). The UK government has pledged £67 million for new programmes to train up to 17 500 teachers of mathematics and physics over the next Parliament. With the reformed GCSE and the expectation that most post-16 students will engage with some mathematics – retaking GCSE, studying for a Core Maths qualification as well as A and AS levels, many more mathematics teachers will certainly be needed. How viable is TSST?

My research centres on case studies from the 2014 cohort of students with a view to tracking them for four years. The TSST course combined 100 hours each of face-to-face tuition and e-learning provision with participants from primary, secondary and F.E. I focus on one case study: a science teacher who is now Head of Mathematics; she claims the course has changed her life.

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Raising girls' participation in A-level mathematics: how do 'good practice' case studies match the research?

Fewer girls than boys participate in post-compulsory mathematics and the recent increase in popularity of Mathematics and Further Mathematics A-levels has not changed the balance. Previous studies have shown the significance to girls of their mathematics lessons and teachers, of discursive co-constructions of masculinity and mathematics, of the range of careers associated with mathematics and science, and family 'science capital'. This study identified four case-study schools and one FE college that had unusually high participation by girls in mathematics A-level. Focus groups with staff and students, and lesson observations, explored factors relevant to girls' participation. Initial findings are that schools had not engaged in gender-based initiatives. Common factors were: preparation for demanding mathematics during key stage 4, a departmental ethos where girls could ask questions in and out of lessons, teachers who explicitly and repeatedly confirmed that they would succeed at mathematics A-level, appreciation of mathematics as opening doors to many careers. Messages about further mathematics were more restrictive but emphasised interest over unusual ability.

### Stafford, Patsy; Oldham, Elizabeth & O'Dowd, Valerie

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Ratio and Proportional Thinking: A Study in an Irish Context

The concepts of ratio and proportional thinking can be problematic, not only for students but also for some teachers. In Ireland, an analysis has shown that the focus on these concepts in the (intended) school curriculum is rather sparse compared with that in many other countries. Also, evidence from Irish participation in an international project on ratio in teacher education (the ATEE Ratio Project) has pointed to some deficiencies in the understanding of the concepts held by student teachers for second-level schools, suggesting that the next generation of teachers is not well placed to address these shortcomings.

We build on the earlier work using data collected from student teachers in primary teacher education courses participating in the ATEE Ratio Project and using the instrument developed for that project. The curriculum analysis is extended to include an examination of the treatment of ratio in primary textbooks (which have been shown in earlier research to have a strong effect on the implemented curriculum). Data collected from the student teachers were examined in the light of the textbook analysis. Findings indicate areas of the curriculum that should be emphasised more in teacher education courses and considerations for textbook presentation of the concepts.

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Analysing student feedback about assessment of their progress on a Mathematics Enhancement Course A Mathematics Subject Knowledge Enhancement Course (MEC) is a government funded course designed for non-mathematics graduates with at least an A level in mathematics who wish to teach mathematics in English secondary schools. The course aims to develop knowledge and understanding of mathematics to a standard which enables participants to hold their own with graduates who have at least half a degree in mathematics or a mathematics related subject when they embark on an Initial Teacher Education programme, typically a PGCE.

Using pre- and post-course questionnaires comprising open-questions, linked with semi-structured interviews I found that participants changed their views during the course about how they assess their progress from reliance on correct answers and marks to reliance on tutor feedback. Similar outcomes were found for a second cohort. I am unsure as to whether reliance on correct answers and marks and tutor feedback as a means of assessing progress on the course impact the extent to which students take responsibility for their own learning.

I will share my approach to analysing the responses and welcome feedback on what can be reasonably inferred from them, and how it might be possible to collect responses that give more depth.

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Investigating the Normative Role for Intellectual Virtue in Mathematics Education

In educational philosophy, much has been written on virtue ethics and its role in m

In educational philosophy, much has been written on virtue ethics and its role in moral education, with an emphasis on the moral virtues in the development of character. There is, moreover, a growing literature on the intellectual virtues in education, with emphasis placed predominantly on their role in critical thinking and the cultivation of dispositions essential to the education of critical thinkers. However, little has been written on how the intellectual virtues, as good habits of the mind, might apply to specific curriculum areas and the role they ought to play to foster intellectual engagement and, hence, excellent teaching and learning.

In this paper I start from an account of the intellectual virtues developed by Hugh Sockett (2012) in which he stresses the overall importance of truthfulness, accuracy, open-mindedness and impartiality. These virtues can be considered as the enabling traits that dispose one to think critically and to engage intellectually with one's learning. In investigating how these virtues might apply to mathematics education, I consider the normative implications that flow from a commitment to the premise that their cultivation is a key attribute of intellectual engagement in that field.

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English Pre-Service Primary School Teachers' Beliefs concerning the Intergration of Children's Literature in Mathematics Teaching and Learning

I present the findings of a study which explores English pre-service primary school teachers' beliefs concerning the integration of children's literature in mathematics learning and teaching. This research project is part of a larger comparative study that explores the beliefs of both pre-service and in-service primary school teachers in England, Finland, Hong Kong and Australia. Through story narratives of children's literature, children discover that mathematics is, in fact, part of their everyday life, and as such it provides a meaningful context for them to explore and investigate mathematical concepts and develop mathematical skills. While much has been written about how children's literature can be integrated into mathematics instructions, very little has been written about teachers' perceptions of such integration. Such research gap is worrying given how it has been argued that teachers' beliefs can influence their instructional practices, and that any attempt to improve the quality of their teaching must begin with an understanding of their beliefs. This paper will present the analysis of open-ended survey responses concerning the research topic of over 100 pre-service primary teachers in a university in the South East of England.

**Twohill**, Aisling St Patrick's College, Dublin City University aisling.twohill@spd.dcu.ie Constructing generalisations from visual spatial patterns Fundamental to the development of children's thinking in mathematics are opportunities to construct generalisations, and to work with generalisations (Kaput, 1998; Mason, 1996). Lannin (2005) suggests that generalising through patterning activities may create a bridge between students' knowledge of arithmetic and their understanding of symbolic representations. Equally, patterns may serve as "dynamic representation of variables" when children are being introduced to algebra and algebraic thinking (Lannin, 2005, p. 233). The Irish Primary School Mathematics Curriculum does not propose facilitating children in generalising, and visual spatial patterns are limited to repeating patterns in infant classes (Government of Ireland, 1999). Children may not be facilitated therefore in developing an 'explicit' approach to constructing a generalisation (Lannin, 2005). Also, their opportunity to consider a pattern from the figural aspect may be limited. In my research I am exploring the responses of children attending Irish primary schools when requested to generalise from visual spatial patterns. Of the children I interviewed some found the consideration of a general term very challenging, but others succeeded in utilising 'figural modes' and explicit methods to construct valid generalisations from a variety of patterns (Rivera and Becker, 2011; Lannin, 2004).

## 28 February Working Groups

### Harbison, Lorraine & Halpin, Audrey

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Enhancing and enriching the primary school child's experience of learning mathematics Recent curriculum developments in the UK and Ireland are underpinned by shared fundamental ideologies that inform and shape the range of good-quality mathematical experiences and outcomes that we seek to achieve for each child. Effective curriculum implementation guarantees that children have a rounded and fulfilling educational experience by:

- ensuring that the starting point for learning reflects the learner's needs and interests
- enabling children to develop a deeper conceptual understanding in mathematics
- applying mathematical concepts in real-life situations
- infusing language and thought into mathematics teaching
- using open-ended challenging tasks that motivate children to engage in a meaningful way
- inspiring positive attitudes, appreciation and enjoyment of mathematics.

In this workshop we will:

- 1. consider the use of the educational framework, Universal Design for Learning (UDL), to fully exploit opportunities for mathematical progression
- 2. examine how UDL guides the development of flexible, accommodating, mathematics-rich environments from the outset
- 3. study how UDL can complement the required expertise of teaching skills and systematic planning of mathematical experiences
- 4. reflect on how UDL provides multiple scenarios for advancement and excellence in mathematics
- 5. explore how we might develop video samples exemplifying the use of UDL to support quality mathematics teaching and learning in the classroom.

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Using Digital Tools within a Constructionist Learning Environment to Facilitate the Development of Geometric Reasoning in a Primary Classroom

The primary aim of my research was to develop a learning environment that facilitated the development of pupils' geometric reasoning abilities with regard to 2D shapes. Research has demonstrated that geometric reasoning is key in the development of an understanding of shape and space. It has also been established that there is a link between exploring shape and space in dynamic geometric contexts and the development of geometric reasoning.

As part of the research the pupils explored a carefully designed series of 2-D shape problem solving tasks using dynamic geometric software. The study involved 23 primary school children, aged between 11 and 13 years. The results from this study showed improvements in geometric reasoning skills of all the participants, with considerable improvements in a majority of cases.

During this working group, participants will use dynamic geometric software (DGS) to engage with a range of tasks to explore the potential advantages such environments can create for the development of geometric reasoning.