BSRLM Abstracts, Sheffield Hallam University, 8 June 2013

Janet Duffin Lecture

Aron **Samkoff**
On the different ways that mathematicians use diagrams in proof construction
based on the Research in Mathematics Education (RME) paper (with Y Lai and K Weber) which won the 2012 Janet Duffin award (judged by the Editors, Editorial Board and International Advisory Board, to be the most outstanding contribution to RME in 2012)

Research presentations

Alshwaikh, Jehad & **Morgan,** Candia
Institute of Education, University of London
c.morgan@ioe.ac.uk
Analysing Palestinian school mathematics textbooks: A multimodal perspective
In this project, funded by the British Academy, we aim to produce an analysis of Palestinian school mathematics textbooks, focusing on the nature of mathematics portrayed in the texts and the nature of mathematical activity expected of students, and to compare these with English textbooks. We will share some results of an early analysis of two extracts of textbooks (Palestinian and English). We have developed a framework of analysis in which we focus on (a) the nature of mathematics and mathematical activity, (b) the image of learners and their relationships to mathematics and (c) the nature of the mathematical text. In this presentation, we will focus on the extent to which a specialised mathematical discourse is used in the texts and the presence of human agents in mathematical processes. We will discuss how differences in these aspects shape the nature of the mathematics that students are expected to experience.

Ayalon, Michal; **Lerman**, Steve &**Watson**, Anne
Weizmann Institute of Science; South Bank University; Oxford University
anne.watson@education.ox.ac.,uk
Development of students' understanding of functions throughout school years
Functions is a central topic in school mathematics around the world, including Israel and the UK. Several decades of intensive research on functions have yielded much information about functions learning and teaching, including accumulating research that reveals substantial difficulties, and ways of addressing them. However, little is known about the overall development of students' understanding of functions throughout school years, and there is no agreed 'best' curriculum order for teaching the subject. Our project addresses these issues. It aims at examining and mapping the development of students' understanding of functions throughout school years in two different curriculum systems: in the UK and in Israel. The research uses a survey including various tasks related to functions that was developed in collaboration with a group of teachers, both from Israel and the UK. In this meeting, the process of the research as well as initial findings will be presented and discussed.

Cable, John
King's College. London
johnliz.cable@talk21.com
Rank order and how-many-ness - a first draft for discussion
The attributes of how-many-ness and, to a lesser extent, rank order underlie the whole of elementary arithmetic, but their separate identities and the precise relation between them are obscured under the language of "number". Sometimes they are called "cardinal number" and "ordinal number", but the degrees of rank order lack any operation of addition, as expected of

number. Sometimes they are referred to as the ordinal and cardinal "aspects of number", but it is hard to find any third thing of which they might be considered aspects. Sometimes they are wrapped up together and labeled "natural number", but this term has proved elusive under close investigation. Like other fundamental issues, this is of special professional interest to philosophers of mathematics and teachers of young children.. Mathematics education literature provides a penetrating analysis of the counting process and the seeds of a new conceptual analysis based on usage. Matheamtics philosophy literature is largely obsessed with defining "natural number" and requires critical re-evaluation. The structure expected of "natural number" is displayed by how-many-ness, but the name to be given to this concept depends on one's view of attributes generally. I propose a classification into qualities, quantities and quotity. (How-many-ness is quotity.)

Clark-Wilson, Alison; **Hoyles**, Celia & **Noss**, Richard
LKL, Institute of Education
alison.clark-wilson@ioe.ac.uk
Cornerstone Maths – Designing for scale
This session will build on the outcomes of the Cornerstone Maths pilot study, which was reported at the BSRLM Day Conference in June 2012. The project, a USA/UK collaboration, is now in a phase of scaling to over 100 schools. We will describe the vision for the project vision in addition to its iterative design, which have both been informed by a twenty-year history of research on dynamic digital technologies by Jim Kaput and ourselves. The resulting intervention builds on our understanding of some of the constraints to the widespread use of dynamic digital technologies by pupils in mathematics, which relate to accessibility, teacher development, curriculum fit and the need to overcome the instrumentation phase. The accompanying research agenda is concerned with evaluating models for scaling that are mindful of the 'grain size' of analysis and the need to maintain the 'implementation fidelity' of the overall intervention.

Cockerham, Fiona**; Dickinson**, Paul; **Pawlik**, Pete & **Timlin**, Rob
Manchester Metropolitan University
F.Cockerham@mmu.ac.uk
University Schools: A Collaborative Approach to ITT
In response to the changes proposed for Initial Teacher Training by the government, MMU has piloted a collaborative approach to teacher training, similar to those used in European countries such as Finland. The schools involved in the pilot are referred to as 'University Schools.' We have placed nine mathematics trainees into each of the six University schools, and a University tutor works in the school for 1½days a week. At the end of the first placement, the schools exchange trainees and the two schools work closely together to deliver a cohesive teacher training programme. The main focus of this pilot was to strengthen our relationships with the schools we use for ITT and to bridge the gap between the theory delivered at University and the application to the classroom. We also aimed to support schools with their delivery of ITT by working closely with the subject mentors and professional mentors with issues such as observing lessons, planning of lessons, professional issues and developing pedagogy. The focus of this session is to discuss how we have run the model, our findings after two years of this pilot, and to discuss the impact this model has on teacher training.

Dalby, Diane
University of Nottingham
ttxdd4@nottingham.ac.uk
The connections and contradictions for students of contextualised tasks
Classroom mathematics and the mathematics of real life or work often appear to be unrelated from a student viewpoint and tasks that are intended to be realistic rarely resemble the real tasks that might actually occur in life. Research highlights the difficulties of crossing the boundaries between the classroom and the world outside, referring to the problems of

transferability and the situated nature of learning. In this session qualitative data from discussions with post-16 students about a range of contextualised functional mathematics tasks will provide some insight into their perceptions of relevance and relationship to life. The results indicate ways in which they make personal connections to the context, the activity or the mathematical content at different levels or reject tasks because of the contradictions they present. Their judgments lead to interesting distinctions between tasks that they believe remain firmly situated in a mathematics classroom and ones that may belong in a real life situation. The research involved vocational students in colleges but has implications for much wider consideration across mathematics teaching.

Demosthenous, Eleni
University of Cambridge
ed342@cam.ac.uk
Engaging students with generalisation tasks in elementary school classrooms
There is growing interest in preparing elementary school students for algebra and developing students' algebraic habits of mind. This study investigates the teaching of algebra-related topics and in particular, teachers' practices when engaging elementary school students with generalisation tasks. Generalisation tasks intend to engage students in "passing from consideration of a given set of objects to that of a larger set, containing the given one" (Polya, 1954, p.12). Data was collected from five elementary school classrooms (fifth and sixth grades) in the Cypriot educational context. Classroom lessons were observed and post-lesson interviews were conducted with the teachers. The classroom data was analysed using discourse analysis and coding the classroom interactions while the post-lesson interviews provided teachers' rationale for their practices during the lessons observed. This session introduces the study and presents preliminary findings.

Didis, M. Gozde**; Erbas**, A. Kursat; **Cetinkaya,** Bulent & **Cakiroglu,** Erdinc
Middle East Technical University
mgozde@metu.edu.tr
Prospective Secondary Mathematics Teachers' Interpretations of Students' Thinking
Teachers' understanding and interpretations of students' mathematical thinking are among the important components of knowledge for teaching as they are often stressed by the mathematics education community. Thus, teachers should acquire and enhance their knowledge and skills for understanding and interpreting students' thinking even before they start in the profession. In teacher preparation programmes, using documentation of instructional practices such as students' written works and video records of classroom lessons provide prospective teachers with opportunities for in-depth exploration of students' thinking. Thus, the purpose of this study was to investigate to what extent prospective secondary mathematics teachers enhance their interpretation of students' thinking when they first work on non-routine tasks themselves as students, and then examine solutions produced by high school students. Twenty-five prospective teachers were the participants of the study. The data sources consisted of individual reflection papers, focus group interviews and notes from the prospective teachers while working on students' work and field notes. The data analysis suggests that as a result of investigating student thinking manifested in the students' written works and video cases, prospective teachers started to question and tried to examine the details of students' thinking and to understand students' ways of thinking in depth.

Fielding, Helen
Nottingham Trent University
helen.fielding@ntu.ac.uk
Fractions: A Piece of Cake? An Exploration of Student Teachers' Understanding, Attitudes and Beliefs in Relation to Fractions
This session will discuss the findings of my doctoral study which focused not only on the aspects of fractions which student primary teachers found difficult but also explored their perceived areas of strength. This study adopted a phenomenographical approach in order to

provide further insight into each student teacher's subject knowledge. The purpose of this study was to discover the individual and distinct ways in which each student teacher understood fractions and the strategies they adopted to address any problems they encountered.
This study involved detailed study of six self-selected small groups, which enabled a range of rich and honestly reflective data to be collected. Groups were filmed working on two collaborative tasks involving the sequencing of fractions by magnitude, this was followed in each case by reflective discussions/group interviews. Each group also undertook a diagnostic interview. The session will present some of the data gathered and discuss these findings, especially the individual and collaborative strategies employed by the participants.

Forsythe, Sue
University of Leicester
skf6@le.ac.uk
Linking dragging strategies to levels of geometrical reasoning in a Dynamic Geometry Environment
Students working in Dynamic Geometry Environments interact with geometric figures by dragging constituent objects on the computer screen. A number of researchers have described different dragging modalities and linked them to cognitive activity. This paper draws on data from recordings of students working with a dynamic figure based on fixed length perpendicular diagonals. The diagonals can be dragged in the figure thus generating a number of quadrilaterals and triangles. Studetns have been observed to use two new dragging strategies. Refinement dragging is when students check and review side and angle properties of shapes they have generated. Dragging maintaining symmetry is when students drag so that one diagonal bisects the other, generating what could be termed a 'dragging family' of shapes. This paper describes these new dragging strategies and relates them to the Van Hiele levels of reasoning. The students' innate sense of symmetry also emerged as an important aspect of how they conceptualise 2D shapes.

Hough, Sue & **Dickinson**, Paul
Manchester Metropolitan University
S.Hough@mmu.ac.uk
Exploring the challenges for trainee teachers in using a Realistic Mathematics Education (RME) approach to the teaching of fractions
We report on the second part of a study into the subject knowledge of Secondary Mathematics trainee teachers enrolled on a Subject Knowledge Enhancement course prior to their PGCE. In the first part of the study, trainees revealed a predominantly procedural knowledge of fractions. Most used the procedure as the authority over their answers, and few were able to make sense of the fractions as numbers or represent the fractions pictorially. The trainees then studied the teaching of fractions, examining alternative learning trajectories based on RME, after which they taught the topic in schools.
We focus here on the challenges faced by trainees attempting to adopt a classroom approach that did not concur with the nature of their knowledge of fractions or with their own experience of learning fractions. Many trainees were able to adopt some of the surface features of RME, and they began to appreciate the value of being able to visualise fractions and the important role of discussion. However, the need for the trainee to have knowledge of a learning trajectory through fractions that is not dominated by procedures, and a belief in this trajectory, emerged as critical features.

Inglis, Matthew & **Aberdein**, Andrew
Loughborough University; Florida Institute of Technology
m.j.inglis@lboro.ac.uk
The Personality of Mathematical Proofs
What do mathematicians mean when they use terms such as "deep", "elegant", "explanatory", etc? Traditionally this question has been approached philosophically, by proposing and

critiquing accounts of what counts as "an explanatory proof", "a deep piece of mathematics", and so on. Here we adopt a different approach, using methods derived from those used by social psychologists to study human personalities. We report a study where 255 mathematicians were asked to characterise a proof of their choice using 80 different adjectives that have often been used to describe mathematical proofs. We found that there are only four broad dimensions on which the 'personality' of a mathematical proof varies. We suggest that these findings may offer an explanation of why it has been so hard to pin down what mathematicians mean by an "explanatory" proof, and argue that bottom-up data-driven analyses of mathematical practice may offer a valuable counterpoint to traditional top-down approaches.

Jones, Keith
University of Southampton
dkj@soton.ac.uk
Diagrams in the teaching and learning of geometry
Diagrams are generally taken to be an integral component of doing and understanding mathematics. In the teaching and learning of geometry, the use of diagrams is not only because of the nature of geometrical objects, but also because a diagram is often a particularly effective problem representation that enables
diagrams. This session provides an overview of research on the affordances and limitations of diagrams in the teaching and learning of geometry.

Joubert, Marie
University of Nottingham
marievjoubert@gmail.com
Mathematics education: messages from "official" reports
This presentation draws together the threads of a six-month project which involves collecting, analysing and synthesising the content of "official" reports related to mathematics education in the UK, published since the beginning of 2011. Whereas at a previous BSRLM day conference, I discussed the methodological approaches I adopted in the project, and invited feedback and comment from the audience, this present talk focuses more on the reports themselves, looking at what their focus is, who has written them, why they were written, what they say (explicitly and implicitly) and what they recommend.
For example, the majority of reports suggest that change is necessary in mathematics education for reasons which range from a population which is not sufficiently numerate to mathematics education generally not taking advantage of the full potential of digital technologies. A number of reports explore reasons for the 'problems' such as a shortage of qualified teachers and negative attitudes towards mathematics.

NicMhuirí, Siún
St. Patrick's College
siun.nicmhuiri@spd.dcu.ie
Responsibility for Learning in a Mathematical Discourse Community
A mathematical discourse community is one which engages in discourse of a mathematical nature. In a classroom mathematical discourse community, students engage in developing, explaining and evaluating mathematical ideas (Hufferd-Ackles, Fuson & Sherin, 2004). In effect, students share mathematical authority with the teacher. This stands in contrast with traditional teaching approaches where students are often positioned as 'received knowers' (Boaler, 2003). It also had implications for the ways in which students may demonstrate responsibility for learning. I will present details of a teaching experiment which aimed to create a discourse community in an Irish primary mathematics classroom. In particular, details and analysis of one lesson will presented and the ways in which students demonstrated responsibility for learning will be examined. Finally the teachers' role and responsibilities within the discourse community will be discussed.

Parish, Alison
University of Warwick
alison4maths@gmail.com
Mathematics Teachers and use of ICT
In this presentation, I report on some of the findings from a small scale narrative research project that looks at the use of ICT by English secondary mathematics teachers. In early days of computers in schools, there was a vision of mathematics being a key user of IT, and many mathematics teachers became responsible for IT. However, reports into mathematics teaching by Ofsted and Becta for many years have reported that use of ICT is limited. Why is this? Many studies have looked at the constraints facing teachers in the classroom, but less have focussed on the teachers themselves. My study includes aspects such as what software is available, the software the participating teachers use and their beliefs about benefits for teaching and learning in their classroom. The obstacles that face even keen teachers are discussed, with a few suggestions as to how they might be overcome. Teachers are recognised as learners when finding out how to use available digital resources, so there is consideration of implications for professional development, and whether it is possible to up-skill all teachers in the use of digital technologies.

Rempe-Gillen, Emma
Liverpool Hope University
gillene@hope.ac.uk
'It's not my place': lesson observation in the professional development of mathematics teachers
Teacher collaboration, and teacher professional development within this context, has become an area of interest in recent years. In particular, mathematics teacher education has seen the rise of collaboration as an effective school-based professional development activity, where in-service teachers plan, observe and reflect on lessons together. This presentation will focus on some of the findings of my PhD research - a year-long case study of two cross-phase and cross-school teacher pairs, where the principal aim was to investigate professional development in a collaborative setting and the role that technology played in this collaboration. Although the teachers were encouraged to jointly plan lessons and peer observe, they were reluctant to do this. In this session I will present their reasons for choosing to work alone and the implications this has for collaborative development.

Trubridge, Nicola
Plymouth University
nicola.trubridge@plymouth.ac.uk
Exploring the features of a collaborative connected classroom
There are a range of theories and opinions from the mathematics education community extending back to the 1970s and up to the present day as to what is meant by mathematical knowledge and understanding. In this session I will present the findings from my literature review. I will consider the various dichotomies between types of student understanding for example relational versus instrumental and procedural versus conceptual. My study concludes that whilst the different categorisation is useful, it is the interplay and connections between these types of understanding that is more beneficial. I then consider the features that might be apparent in the secondary mathematics classroom and propose the Collaborative Connected Classroom Model. Finally I will explain the next steps of my research study.

Warburton, Rebecca
University of Leeds
mm06rkp@leeds.ac.uk
'Mathematical Knowledge for Teaching': do you need a mathematics degree?
Two concerns for UK government are: the shortage of mathematics teachers and the poor mathematics results of school-leavers. In order to increase the supply of teachers, the government have sponsored 'subject knowledge enhancement' (SKE) courses to graduates

from numerate disciplines to enable those without mathematics degrees to train as mathematics teachers. Additionally, policy makers and researchers have seen defining and codifying the subject knowledge required by teachers as central to improving student learning. The University of Michigan have developed measures to test teachers' "mathematical knowledge for teaching". These measures have primarily been used within the US but have been adapted for use in a small number of other countries (Ireland, Norway, Ghana, Indonesia and Korea). For this study the measures were utilised with a sample of trainee secondary mathematics teachers in England. In contrast to existing studies, an alternative approach to selecting measures was employed. The selection process and preliminary results from administering the measures in a pre- (teacher-training course) questionnaire will be discussed. In particular, the responses of mathematics graduates and SKE students will be compared.

Working groups

Alcock, Lara; **Gilmore**, Camilla; **Inglis**, Matt; **Evans**, Jeff; **Monaghan**, John; **Noyes**, Andy & **Pope**, Sue
Loughborough University, Middlesex University, University of Leeds, University of Nottingham, University of Manchester
J.Evans@mdx.ac.uk
Using statistics in mathematics education research
BSRLM's journal 'Research in Mathematics Education' recently published a special issue (SI) on experimental methods in mathematics education (vol. 15, issue 2). The bulk of this session will consist of a discussion of this SI.
Experimental methods are substantially under-represented in mathematics education research: in 2012 only 3% of studies published in mathematics education journals used experimental methods. But a well-designed experiment offers a powerful method with which to approach causal research questions.
The editors of the SI will present a short overview of the rationale of experimental methods, present a brief example of a basic experimental study, and outline the various methods used by authors of SI papers. It would be an advantage if delegates coming to the session were able to have a look at the SI, in particular the Editorial, in advance. This is available at the RME website (http://www.tandfonline.com/loi/rrme20) (in the week before the meeting) or by contacting Matthew Inglis (m.j.inglis@lboro.ac.uk).
The session will conclude with a discussion of possible topics for the next meeting.

Archer, Rosa; **Wake**, Geoff & **Ryan,** Julie
University of Manchester, University of Nottingham, Manchester Metropolitan University
rosa.archer@manchester.ac.uk
Lesson study in research and CPD in mathematics education
Lesson study was established in Japan as an important and effective means of professional development. The systematic collaborative approach to lesson design, planning, implementation and review has found to promote student engagement and learning and cultures of professional development. Many countries are adopting and adapting lesson study as a means of professional development and enhancement in mathematics, and this has become a subfield for research (Hart et al. 2011).
Our aim in this workshop is to share experiences of using lesson study in various ways in various contexts in schools, in HE, and in ITE/CPD in mathematics education. The plan is to ask lead participants to present brief reports of their practices of lesson study, which will include primary school, secondary school, and HE/ITE contexts. But we will reserve most of the session for discussion of issues arising such as:
How and why does the Japanese model evolve in different cultural contexts, and phases?
Is there an 'essential, sine qua non' to lesson study?
How can lesson study adapt to performative institutional cultures (such as in the UK)?

How is lesson study constituted as research, or synergised with research in mathematics education?

Clarke, Nichola
University of Nottingham
nichola.clarke@nottingham.ac.uk
Sustainability and Mathematics Education
In the first part of the session, we will continue to work on a literature review. We will then work together to develop our current work on sustainability in mathematics education, including work on tasks (proportional reasoning, measures, problem solving) and the theory of task design (situativity, relevance, and realism), with reference to the new mathematics National Curriculum.

Mendick, Heather; **Morgan**, Candia & **Smith**, Cathy *
Brunel Univeristy; Institute of Education, London
c.smith@ioe.ac.uk
Curriculum, Assessment and Society
Assessment is important in positioning mathematics as a discipline and students in relation to mathematics, schools, universities and other educational institutions. Research shows that it is also a source of considerable inequalities. We hope that this session will be the first of a new BSRLM working group focused on curriculum, assessment and society, and we start by exemplifying our own research and the theoretical approaches we have taken to understand the social functions of assessment.
Research into assessment has looked at the relationship between examination questions and mathematics and at the impact of assessment systems on student engagement. However, wider research in the sociology of education shows the impact on student identities. In this session we look at the identities available to students within examination questions and within their classroom use. The session will be split into two sections. The first will explore how examination questions position students and the second how students position themselves in relation to examination questions. Each section will begin with a short presentation of research data, followed by guided discussion.
We welcome ideas on how the working group could develop.