

‘To chunk or not to chunk’: learning division, the *why* before the *how* or vice versa

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In this small-scale study, I focus on the mathematical area of division (particularly the chunking and standard algorithms). The study takes place in a larger than average-sized, state-funded primary school in the south-west of England where the percentage of free school meals is lower than the national average. For one group of 17 low achieving students, having been taught chunking and getting confused, the standard method for short division was taught successfully. Six months later, when given free choice every child chose the standard method and they got the questions correct. 9 higher achieving students were taught chunking successfully but not taught the standard method. Six months later, given a free choice, they were still using the chunking method successfully. With the current focus on fluency and mastery, I am interested in whether there is a need for pupils to learn ‘why’ before ‘how’ (conceptual or procedural) or vice versa.

Keywords: division, procedural, conceptual, mastery, fluency

Introduction

Since I began teaching in the late 1990s, there have been significant changes in the way mathematics is specified to be taught. In 1999, the National Curriculum included, for the first time, a statement on aims, values and purposes. In the September of that year, the government also launched the National Numeracy Strategy (NNS) - introduced as part of their commitment to raising standards. According to Ofsted (2011) the aim of the NNS calculation element was to develop a series of strategies, both mental and informal, to help a child’s achievement in number. This would enable children to call upon a range of methods and approaches before they moved on to the more standard and traditional methods. In 2003, the government also introduced the idea that ‘Every Child Matters’ to emphasise that education at a primary level was not only to do with standards but also enjoyment and the individual needs of a child. Most recently, in 2014, a new National Curriculum (2013) has been introduced with a stronger emphasis on modelling in mathematics and problem solving.

With the introduction of this new National Curriculum also came the concern that it was tough and certain elements were being taught at an earlier age than before. The National Curriculum (2013) states that mathematics is a subject with interconnections and children will need to be able to move fluently between the mathematical representations and ideas within it. The curriculum continues by noting that children will need to be fluent in the fundamentals of mathematics, meaning developing an efficiency, accuracy and flexibility within the subject area. The curriculum states that children will need to develop their skills to make rich connections and they will need to develop fluency, mathematical reasoning and competence. McClure (2014) says the key to fluency is 1) ensuring children are able to make the connections and 2) doing so at the right time in their learning. The

National Centre for Excellence in the Teaching of Mathematics (2014) refer to mastery as having a competence and confidence within a subject area and they also note that the aim of the 2014 curriculum is that a large majority of children will achieve mastery - acquire a deeper knowledge and a capacity to use effective strategies.

With all these ideas in mind, and also coupled with the experiences I have had when teaching mathematics, I began to read around the area of division. Richards (2014) actually begins his work stating that he believes that if you talk to a primary aged child about aspects they find difficult in mathematics then they will, more often than not, say division. In my opinion and experience, it is not just the children that say this but also the parents and sometimes also the teachers (especially when the long division algorithm is mentioned). In agreement with this, Ofsted (2011) also comment that children can find efficient chunking difficult if they have a poor knowledge of times tables. When I read this, I wanted to look deeper into the area to find out how it affects the children in my classes and more widely in the school in which I teach.

My initial interest in the area was developed last year when I taught mathematics to a group of lower achieving children. When it came to the time to cover division I felt my lessons never went to plan and I found myself making split-second decisions in order to support those in the class. The plans, following the guidelines set out in the school's progression chart, stated that the class needed to divide using the chunking method. We looked carefully at the steps we had to take and the understanding we needed. After a while, I was faced with a class where 5% of the children were happy that they thought they knew what they were doing and the rest were feeling depressed and unsuccessful. They believed the chunking algorithm to be impossible. Comments made were that it was too long winded and the children said that they found that they made numerous errors on the way to completion of the problem. What was really frustrating was that they lacked confidence even when they were getting it right! This led to questions around mastery, fluency and understanding. Was there another way that division could be taught to help learners feel successful? It also made me wonder about how teachers can expect the children to learn if they just do not get it in the first place?

In response to their anxiety, I changed approach and showed them the standard short method. I told them the method was 'illegal' as we should not yet be using it – this appealed to the boys in the class. After only a few minutes the children were engaged and feeling successful. I remember one child even asking 'Why can't we use this one all the time, it is so much easier!' It was this statement that got me thinking and wondering about division and the progression of it. It made me ask questions such as: Why do we have to learn the *why* before the *how*? Should we not encourage success first and then develop a deeper understanding? Surely mastery and fluency are not dependent on following a predetermined order of progression and should work more around the individual?

The *why* and the *how*

When reading around the topic, I stumbled across an article by Ian Thompson (2012) titled 'To chunk or not to chunk'. This work, obviously influencing the title of mine, reports on another document called 'Good practice in primary mathematics: evidence from 20 successful schools'. Thompson (2012) states that his article particularly focuses on the parts of the document relating to the algorithm 'chunking' as it is this method that receives the worst press. Thompson notes that he sees the document as

one that has a bias. He comments upon the work of other researchers and states that all evidence should be referred to before a judgement is made, implying that not all areas have been covered in the document. Again, I am left asking questions such as: why does chunking receive such a bad press? What is its value and why is it used at all?

When I teach chunking, I see it as a method that teaches the children to understand the strategy of division rather than being just a procedure. However, it must be presented and used in a way that enhances a pupil's understanding rather than being just a procedure to follow. I therefore label chunking taught for understanding as a 'conceptual approach' – the *why*. To me this is in line with what Hiebert and Lefevre (1986) distinguish conceptual knowledge as being. They say that a conceptual approach is rich in relationships. Researchers Eisenhart et al.(1993) state that a conceptual approach is where there are relationships and interconnections of ideas that explain and give meaning to mathematical procedures.

To make my point clearer it is worth stating that I see the standard (short) method as being a procedure – this is *how* you do it! Hiebert and Lefevre (1986) describe procedural knowledge as having an emphasis on symbolic representations and algorithms. Eisenhart et al. (1993), however, say the procedural approach refers to mastery of computational skills.

Investigation and summary of findings

A small scale investigation was set up in my current school and through it 26 year six children were presented with just one division problem $72 \div 3$. They were asked to solve the problem using two different written methods: 1) the standard method and 2) the chunking method. Apart from teaching input earlier in the year, there was no specific teaching input before giving the children the actual problem as I wanted to ascertain whether the children could: 1) remember how to solve a division algorithm and 2) use both methods. Borthwick and Harcourt-Heath (2007) wrote that children often find it difficult to choose the most efficient and effective method in order to answer a question and I wanted to find out more about this. After solving the problem, the children also had to comment upon the method they preferred and why.

The lower achieving children (17) all chose to use the standard method as they saw it as less confusing, quicker and easier – all but one used it successfully. Only one child in this group actually attempted both methods – they were successful with the standard method but made errors in the chunking method.

Figure 1: Examples of the 'standard method' algorithm used by the children.

Figure 1 shows some examples of the children's work. In the first example, it is evident that the child has become confused – they seem to know that there are two 3s in seven but instead of putting two, they have put a six (2×3). However, they continue by carrying over the left over 'ten' and realise that there are four threes in twelve – certainly they seem to be working toward an understanding. The second example shows a correctly completed algorithm.

Interestingly the higher achieving children all chose the ‘chunking’ method. Within this method there was evidence of different levels of sophistication in that the chunks being removed were of different sizes (efficiency) and hence the number of steps was also different. With this method all but one were successful; Figure 2 shows several different possibilities for confusion and error. The last example, although the answer is incorrect, the working is correct – the child has just muddled their presentation of the chunks and hence arrived at an incorrect answer.

Figure 2 shows four examples of the 'chunking' algorithm for dividing 72 by 3. Each example starts with $3 \overline{) 72}$.

- Example 1:** Shows three steps: subtracting 30 (labeled 3×10), then 42 , then another 30 (labeled 3×10), and finally 12 (labeled 4×3).
- Example 2:** Shows two steps: subtracting 60 (labeled $\times 20$) and then 12 (labeled $\times 6$).
- Example 3:** Shows three steps: subtracting 30 (labeled $10 \times$), then 42 (labeled $10 \times$), and then 12 (labeled $4 \times$), resulting in a remainder of 00 (labeled 24).
- Example 4:** Shows four steps: subtracting 33 (labeled 3×11), then 39 (labeled 10×3), then 30 (labeled 10×3), and finally 09 (labeled 3×3), resulting in a remainder of 00 (labeled 3×3).

Figure 2: Examples of the ‘chunking’ algorithm used by the children.

When questioned about their approaches and why they had chosen the chunking method in preference to the standard short method it was interesting to note that the children commented that they had never been taught the standard method. Within the higher achieving group only one child had ever come across the standard method and they commented that although they thought it was a more compact and easy method they preferred chunking as it helped the layout of their calculation.

Interested in their responses and after introducing them to the standard short method, I then posed a problem to the higher achieving children with regards to whether they thought chunking should be taught first or not – the *why* before the *how*! Their responses were an eye opener. Thinking they would just like to get the problem solved, I was surprised to see that they actually preferred the chunking method. They commented that the chunking method supports the understanding of place value, it is easier to keep track of where you are and also allows the teacher to locate problems more quickly. They followed on by suggesting that, although the short method saved space, was quicker and there was less opportunity to go wrong, it was probably better used when a child has developed a better recall of their times tables.

Discussion and further questions

My intention, throughout this research, (although initially questioning the need for chunking – I never did it when I was at school and I consider myself to have a good understanding), was not to make a decision as to whether we should chunk or not but focus on the order things should or could be taught in for a better understanding and mastery of the area.

This study indicates that there are many things to consider when developing suggested lines of progression in mathematics with regards to the division algorithms. It is necessary to remember that all learners are individuals and what suits one might not suit another. Teachers perhaps need to ascertain a clearer picture of how the individuals have developed their learning before they arrived in their present class and adapt their teaching style accordingly.

Graebar (1999) and McClure (2014) both believe that children need both procedural and conceptual fluency / multiple approaches – from this I take that children need to be able to explore a topic in a plethora of ways to develop a concrete understanding but at the same time they need to feel successful. If this means they need to do the *how* before the *why* or vice versa then teachers should take action accordingly. If a child feels successful, in my opinion, they are more open to changing their methods and learning more about the same topic.

Chick (2003) indicates (also see Ofsted, 2009), that a key challenge is for teachers to address both procedural and conceptual practices. Ofsted (2008) also suggest that the fundamental issue for teachers is how better to develop a pupil's understanding. They write in their report that conceptual approaches and practical activities promote understanding as they allow misconceptions to surface and hence be tackled accordingly. Interestingly, they also note that by only using one method at a time and not linking them that the subject becomes fragmented, becoming a series of rules that can be confusing and incomplete and hence a child does not get the necessary competence and intellectual flexibility.

I have now, through reading the work of Thompson (2005) and Richards (2014) become aware of what Richards calls 'additive chunking'. Additive chunking involves working towards the dividend rather than away from it as in subtractive chunking. They suggest this method as many learners feel more secure with addition rather than subtraction. Richards (2014) states that the main advantage of additive division is that it is linked to mental methods and therefore aids the consolidation of a learner's understanding. Richards goes on to comment that this method fits naturally into progression allowing a child to draw upon known facts, place value and divisibility rules.

My next area of thinking is, could 'additive chunking' be a way forward for the lower achievers rather than the 'subtractive' form used in most schools? Would it give them the understanding and fluency that they need? However, it has also made me realise all the more about how important it is that a child understands place value, has basic computational skills and also has a secure knowledge of times tables.

This study is part of a larger study where I look into the implementation of the new National Curriculum in Mathematics and the changes and effects it provokes both at school and in the home. There are many unanswered questions here that warrant a further study and these are concerned with the effective teaching and learning of all algorithms. Some more questions to be considered are:

- 1) Are we overloading our learners rather than enabling them to be successful?
- 2) Can children digest all the strategies they are introduced to?
- 3) Are all primary practitioners and parents equipped to fully support children in their learning?

Limitations

There are limitations to this study. With such a small sample of children, judgements cannot be made in respect of all primary aged children. However, this study has given an insight into possible ways forward with regard to coverage and progression. It has also given a better understanding of the issues faced in the classroom with the division algorithm.

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