Using the Singapore Bar Model to support the interpretation and understanding of word problems in Key Stage 2

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The research project was conducted in a large junior school where children, normally confident with calculation, experience difficulties with the interpretation of word problems. The Singapore Bar model was chosen to provide a clear visual representation in order to support all children identifying the underlying structure of word problems, and would hopefully narrow the gap between the genders. The areas in which it was most commonly used were problems involving fractions, 2-step money problems and division. It was found that children valued the model more in areas of mathematics that were difficult and new to them, and where they felt less confident.

Keywords: Singapore bar; bar modelling; problem solving; fractions; key stage 2 mathematics

Introduction

In a 1981 diagnostic test, the Ministry of Education in Singapore found that only 46% of students in grades 2-4 could solve word problems that were presented without such key words as 'altogether' or 'left' (Hong, Mei, and Lim, 2009 in Englard, L. 2010, p.157). However, the results from the Trends in International Mathematics and Science Study (TIMSS 2007) "show Singapore students to be among the best mathematical problem solvers in the world." (Englard, 2010, p157). The DFE also reports that Singapore students are 1.6 in years' progress ahead of English students in a PISA 2009 study (DFE, 2012, p.20). So what caused this amazing turn around in the Singapore students' performance in mathematics? Englard, 2010, p.157 argues that it is down to the 'Model Method' or 'Singapore Bar'. It is not a method for solving problems per se, but it reveals the mathematical structure underpinning the problem and enables children to gain an insight into how they can solve it. It provides links between real life problems and their mathematical form and "can bridge the gap between concrete mathematical experiences and abstract representations" by providing a pictorial (iconic) stage (Bruner 1961, NCETM, 2014a). It can be incorporated into established problem solving approaches, enabling children to interpret the problem effectively.

Hoven and Garelick, 2007, state that Singapore Bar 'is a specific variant on the Draw a Picture problem solving strategy. Because it uses one variant consistently, students know what kind of picture to draw… It communicates graphically and instantly the information that the learner already knows, and it shows the student how to use that information to solve the problem' (Hoven and Garelick, 2007 p. 28).

Four basic types of Singapore Bar model were used during this study and they are designed to represent the underlying mathematical structure of the four operations and make explicit the connections between them.

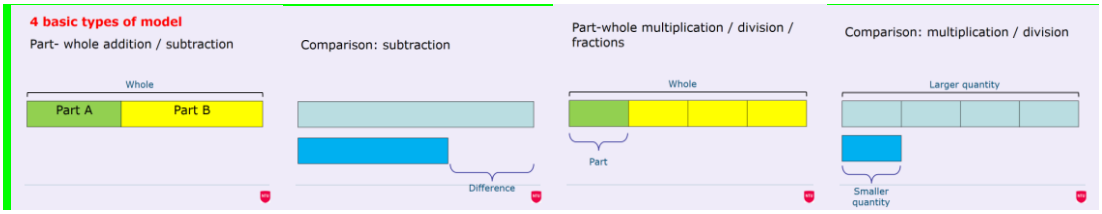


Figure 1 Four Models of the Singapore Bar (adapted from Erie 2 Math, 2012).

Below is an example of how the comparison: multiplication / division model can be used to solve a word problem.

Peter has four books. Harry has five times as many books as Peter. How many more books does Harry have? Harry has 16 books (NCETM, 2014b).

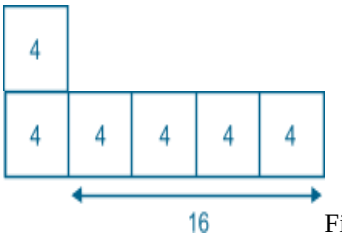


Figure 2

The model clearly demonstrates the idea of scaling up, and how this is related to multiplication. It could just as easily help a student answer a 'How many more books…' as a 'How many books altogether…' question. It also shows that multiplying 4 by 5 is the same as dividing 20 by 4.

As the Singapore Bar clearly gives such depth of mathematical insight and has had such success in Singapore, it was postulated that the Bar model could help in this school where students, normally confident with calculation, experience difficulties with the interpretation of word problems.

Method

The research began with a pre-test for all children (years 3 – 6) which included word problems on fractions, and all four operations, including two step problems. Then the Singapore Bar was introduced to teachers and subsequently taught to students in years 4, 5 and 6 and the higher attaining children in year 3. To support the teachers, there were staff meetings, as well as collaborative teaching in years 4 and 5. There was also collaborative planning in all year groups. Data was gathered through pupil interviews, staff questionnaires, work scrutiny and pre and post-test responses. There were approximately 5 months between the pre and post-test.

Results and discussion:

Pre-test

In the pre-test, children generally had not experienced the Singapore Bar model. Girls, particularly in lower school, were found to use a wider range of pictorial representations. Some representations were more mathematically sound than others.

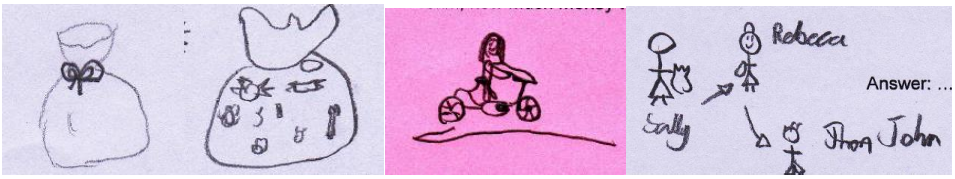


Figure 3

These first three diagrams are not terribly mathematically helpful, but are from girls, who needed pictorial representations to understand the questions. It was the regular occurrence of these types of diagrams, which made it clear that the children needed a straightforward, visual method for interpreting word problems.

Some children had devised, or continued to use, mathematically viable options, but which lacked the efficiency of Singapore Bar.

This is an example from a year 5 girl. She understands how to divide by sharing, but it is not an efficient method. It is time consuming, and the more dots required, the more likely it is that mistakes will occur.

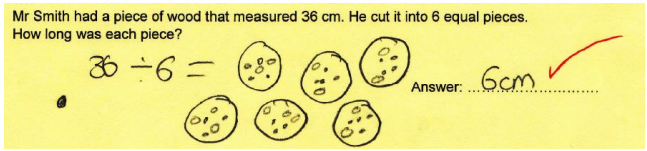


Figure 4 (Question from NCETM, 2014 b)

Interestingly, some children had already devised their own 'bar' models. The year 3 child cut 36 cm (represented by circles) into 6 equal lengths and the year 6 girl has cut a line, representing the wood, into 6 equal pieces. Both of these are very similar to the representation Singapore Bar would give to this question.

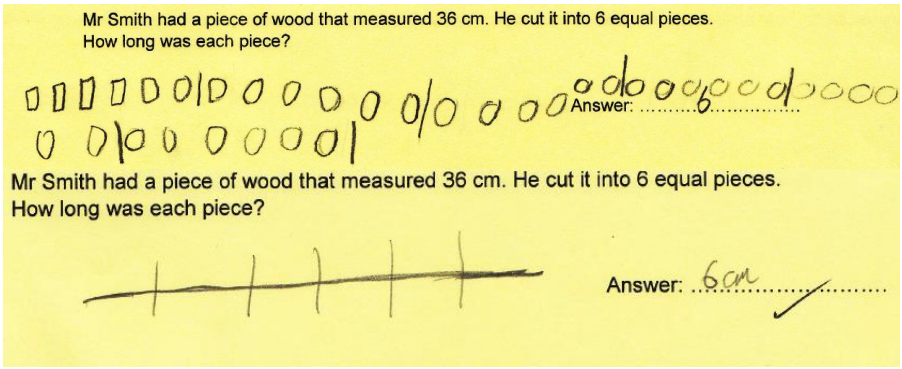
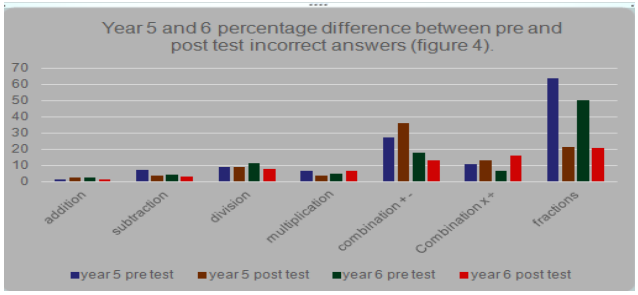


Figure 5 (Question from NCETM, 2014 b)

In the post-test, a relatively small number of children chose to use the Singapore Bar model overall. Gender differences were evident in the post-test: in lower school, boys were found to use Singapore Bar more frequently, whereas in upper school, girls were using it more often. (In year 6: 81% of girls used the Singapore Bar model, as opposed to 46% of boys). 63% of children in Year 6 chose to use it for at least one question. The areas in which it was most commonly used were problems involving fractions, 2-step money problems and division.

Figure 6 shows Year 5 and Year 6 pre and post test incorrect answers where children got none correct in that category.



Whilst we would expect progress in all areas over this period of time, it is difficult to establish what could be attributed to the Singapore Bar model. However, there were particularly positive results in the fractions questions, where the largest decrease in incorrect answers, where children got no answers correct, between pre and post-test was seen. In year 4, 68% had none correct in the pre-test compared with 32% in the post-test. Year 5 had 64% in the pre-test, but 22% in the post-test. Year 6 had 50% in the pre-test, but 3.3% in the post-test, although 17.7 % of cohort from one class didn't have time to complete these questions.

Examples of children's work

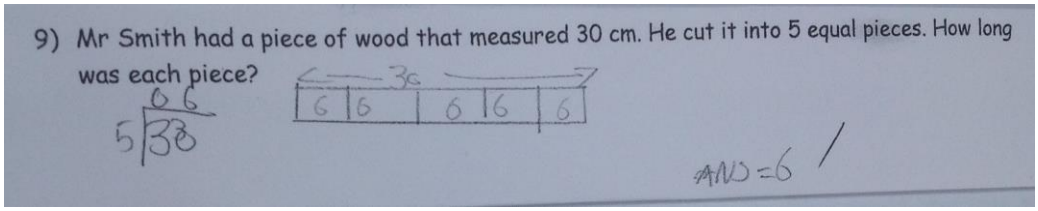


Figure 7 (Question from NCETM, 2014 b)

This example, from question 9, a division question with fractional language ('equal pieces') shows that the children have used the bar to model the underlying structure of the problem, but have still recognised the need for formal written methods to calculate the answers.

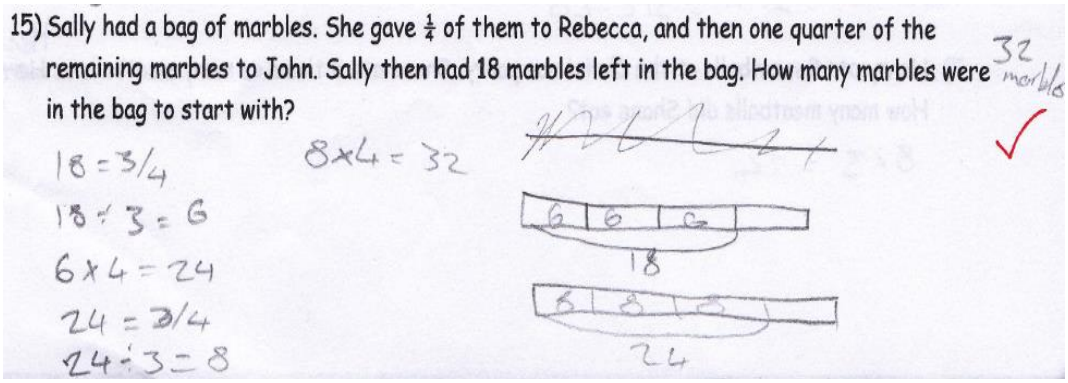


Figure 8 (Question adapted from NCETM, 2014 c)

Above is a successful example of the use of the Singapore Bar, which has proved very effective for finding fractions of amounts and reunitising to solve a multi-step question. It has enabled a year 4 child to successfully solve the complexity of this question.

Children's views

Children expressed a range of views about Singapore Bar. They ranged from "I hate this, please don't make me do it again!" to "I find it easy, but I wish I had been taught it in year 2 because I find it useful" (Year 5 boys). And "I've got it with fractions!!! It's really helping me!!" (Year 5 girl). Many children noticed how useful it is for fractions ("Singapore Bar makes fractions easy" (Year 4 boy)), and as one Year 5 child put it during an interview - "Easier with fractions – can see what it means." Their comments reflect the overall success in solving fraction problems observed in the work scrutiny.

Introducing the Bar in Key Stage 2

There was an inherent difficulty with introducing the Bar in Key Stage 2 where methods and approaches towards problem solving have already been established. Sometimes, children did find having to use the Singapore Bar model very frustrating and saw it as an extra step, especially in areas they were already confident with: "I wouldn't use it without being told to use it because it's hard – extra step I don't need" (Year 6 boy)." It was interesting, however, how the Year 5 boy said he wished he'd been taught it in Year 2, as it would seem that introducing the Singapore Bar model in a junior school, when other strategies are already established, has been a limiting factor in its usefulness for the children; this is perhaps why they have found it most useful in 'new' topics such as fractions. "Singapore Bar helped me with fractions, but not with anything else because it got in the way with subtracting and adding" (Year 5 boy). The introduction in Key Stage 2 contrasts with Singapore, where according to Englard, 2010 p.163, bar modelling is used as a natural part of understanding operations: "The method builds throughout the grade levels to form a developmentally appropriate and vertically aligned concrete-to-pictorial-to-abstract strategy."

Year 6 children have also found it useful for ratio and algebra. Again showing that, when a new area of mathematics is being introduced, how useful the bar can be to explain and represent its underlying structures. "I think using Singapore bar in algebraic equations which involve adding and multiplying is a good idea. It is a simple, straightforward method to understand and it makes algebra easy." "Best for division and percentages, ratio" (Year 6 boys).

Also due to its introduction in Key Stage 2, where it had not been used previously, there was a clash between teachers trying to teach it, starting with the basic addition and subtraction models, and the students finding this frustrating because they could work out the answers without needing to use it. However, without the teachers introducing it from 'the basics', the children wouldn't have necessarily had the appropriate bar modelling skills to attempt more difficult areas of mathematics.

Teachers' perceptions

Within such as large school, there were inevitable differences in teachers' confidence and appreciation of the value of the approach: the extent to which it was used by each class, tended to reflect this. The introduction of the Singapore Bar model along with the new curriculum and S.A.T.s looming for Year 6, meant time was constrained for teachers when embedding this in their established practice. For some teachers this was a new way of representing aspects of maths, which challenged their established

approaches. Although predominantly positive, they held contrasting views about the effectiveness of the approach, which ranged from: "It has really helped to embed conceptual understanding of topics children thought they understood. Love it!" to, "I can see the huge benefit it has/will have, I just need to get my head round it." And, "Personally, I struggle to see how it helps".

Implications

The above analysis of the children's responses to word problems demonstrates that the Singapore Bar model has had a positive impact on their understanding of fractions in particular and has been invaluable. This approach will continue to be taken. The Singapore Bar model has been integrated into the school's calculation policy. Although it is not in itself a 'written calculation method', it is a vital step in problem solving and the power of this should be recognised in policy and future practice. Also, as the findings suggest that introducing Bar Modelling at an earlier stage would be beneficial, in Year 3 and in liaison with the infant feeder schools. Further, now it has been trialled in the school, and there are clear positive outcomes in some areas, this should encourage teachers to persevere with the Singapore Bar Model.

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