Using a Single-Subject Design to Examine the Effectiveness of a Mathematical Instructional Activity

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This article examines a single-subject research design as a means to assess the effectiveness of an instructional activity in mathematics. Research from the special educational needs field commonly uses a single-subject research design to examine the effectiveness of instruction because the unique nature of individuals' special educational needs means that there are a limited number of comparable participants. This article examines the strengths and weaknesses of this research design for use in mathematics education research by referring to a research project in which I used this design to assess the effectiveness of an instructional activity for developing base-ten concepts in students with dyslexia. I then propose additions to this design that make it more explanatory and therefore more useful for examining students' learning.

Keywords: Single-subject research design; special educational needs; place value; dyslexia

Single-subject research design

Approximately 10% of primary school children have special educational needs, but the vast majority of the research on the mathematics learning of this population comes from the perspective of the special educational needs field rather than that of mathematics education (Karp, 2013). Therefore in order for members of the mathematics education field to join in this conversation, they need to understand the research designs used in special education.

Single-subject research designs (SSRD) are commonly used in special educational needs research because these methodologies allow for experimental, quantitative evaluation of an instructional intervention when there are few participants (Rakap, 2015). There are generally a limited number of participants available for special educational needs research because so few students fit in any particular category. Even a high incidence disability such as dyslexia accounts for only 10% of the population (BDA, 2015) so there may be only 6 students with dyslexia in any year group at each school, which makes it difficult to create equivalent control groups for typical experimental designs (van Daal, 2015).

SSRD examine the effect of an independent variable, often an instructional activity, on the dependent variable over time with respect to an individual participant (Rakap, 2015). The individual acts as their own control because the research design compares their performance on the dependent measure, an instrument that is taken repeatedly, at baseline with their performance during and after the instruction. There are several possible designs that can improve the rigour of SSRD, one of which is the multiple-baseline design in which the intervention is given to the different participants at different times, so that some participants continue in the baseline condition while others are in the intervention condition (Rakap, 2015). This helps to make sure that changes are due to the intervention rather than due to external conditions.

Visual inspection of the data, in which the researcher makes a graph of the data and then visually analyses the data with respect to the trend of the line, the level of performance achieved and the variability, was traditionally the main form of analysis (Rakap, 2015). However, this form of analysis makes it hard to make comparisons between studies or to measure the impact of an intervention, so a statistical measure was needed (Scruggs, Mastropieri, & Casto, 1987). While there are several possible statistical measures to measure effect sizes, the percentage of non-overlapping data (PND) is easily calculated and correlates well with visual inspection of data (Rakap, 2015). The PND is calculated by converting into a percentage the number of data points from the intervention and maintenance phases of the study that were higher than the highest data points from the baseline (Scruggs, Mastropieri, & Casto, 1987).

Current research

In order to illustrate how this type of research design can be used to examine the mathematics learning of students, I will examine a research project that used a SSRD to examine the effectiveness of an intervention designed to teach students with dyslexia about base-ten numeration.

Base-ten numeration is a special case of unitizing, in which one can count sets of tens as well as counting by tens, and flexibly transfer between these two modes of counting (Van de Walle, Karp, & Bay-Williams (2010). In a previous study it had been found that most of the 9-10 year-old students with dyslexia were fluent with base-ten numeration up to 100, but struggled with it with numbers greater than 100 (Thouless, 2014). So this instructional study was designed to teach these children about base-ten numeration for numbers greater than 100, using the instructional activity of counting collections (Schwedtfeger & Chan, 2007).

Research questions

The main research question for this study was: is counting collections an effective instructional activity to increase the base-ten numeration understanding of students with dyslexia?

Methods

In this study there were four participants aged 9-10 years old, three were girls and one was a boy. All four participants had been diagnosed with dyslexia and had previously shown that they had difficulties with base-ten numeration in numbers greater than 100 (Thouless, 2014).

The study design was a multiple baseline SSRD, with a dependent variable that consisted of five place-value problems given weekly. The structure of the place-value problems stayed the same across the study, but the numbers in the problems and the order of the problems varied each week. The independent variable was an instructional activity called counting collections (Schwedtfeger & Chan, 2007) that was taught daily for six weeks.

Results

Single-subject design

Student performance on the place-value problems is represented in Figure 1. Results of this intervention study were examined through visual inspection of the data and the effectiveness of the intervention was examined using the percentage of non-overlapping data.

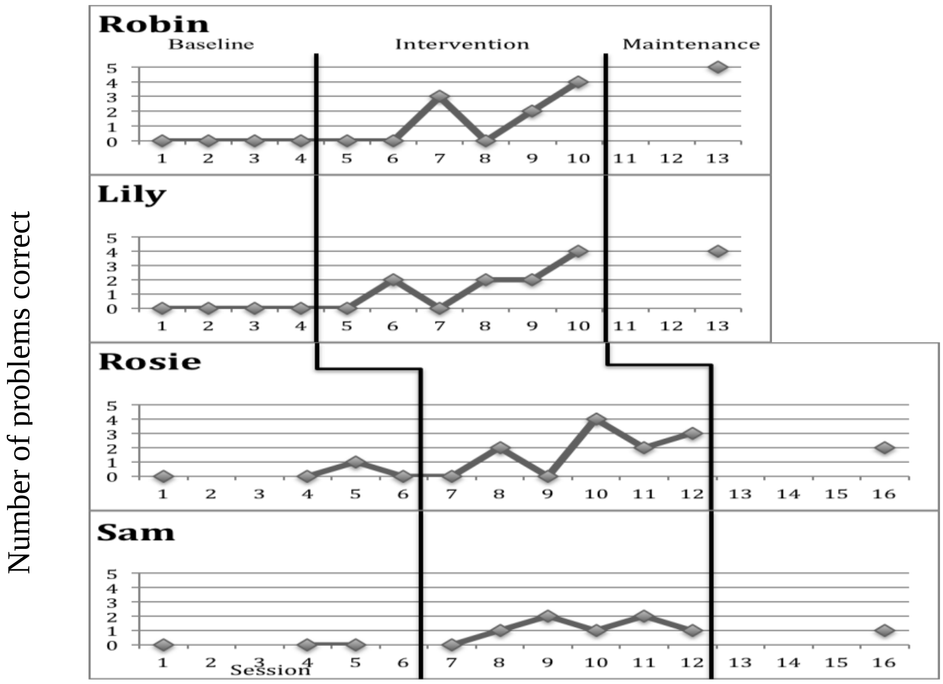

Figure 1: Place-value accuracy

Visual inspection of the data revealed that all the students made progress in their understanding of place-value, but only Robin[6] and Lily reached the criterion of 4/5 problems correct and maintained this knowledge. Rosie reached criterion but did not maintain it. While Sam made some progress he did not meet criterion. There was not an immediate change between baseline and the intervention data for any of the students; the intervention took a couple of weeks to show changes in the accuracy data.

It was possible to use PND to analyse this data because there were no outliers and there was no upwards trend at baseline (Rakap, 2015). The PND for Robin was 57%, Lily 71%, Rosie 71%, and Sam 86%, with a median PND of 71%, which indicates that the intervention was effective (Rakap, 2015).

Strategy use

Although investigating the accuracy data from the SSRD gave a numerical answer about the effectiveness of the intervention for students with dyslexia, it left many unanswered questions: 1) Why did all the participants have at least one week during instruction with no problems correct? 2) How did the students' responses change across the period of instruction? 3) Why did the students' progress differ? In order to

[6] All names are pseudonyms.

address these questions I applied techniques from mathematics education research to the data; I investigated which strategies the students used to solve problems.

I analysed the students' strategies, breaking them into three broad categories: invalid, beginning-level, and more mature strategies. Invalid strategies were strategies that could not produce the correct answer. Beginning-level strategies consisted of direct modelling by tens and counting by tens (Carpenter, Fennema, Franke, Levi, & Empson, 1999), both of which have numerous steps but can produce the correct answer if approached carefully. More mature strategies were efficient strategies with a limited number of steps. There were five types of more mature strategies: in *repeated addition* the student used addition to solve a grouping problem; when using *partial products* the student broke the number into parts and multiplied each section separately; for *recall/rule* the student immediately knew the answer or used a rule for multiplying by ten or hundred; when using *standard algorithm* the student used a standard written algorithm; and when using *direct place-value* the student immediately knew how many groups of tens were in the number.

Figure 2 shows the distribution of strategies for each student during each session, as well as the number of problems that the students did not attempt because they ran out of time. During baseline and at the beginning of the intervention the students were unable to complete all the questions during the half an hour set aside for the tests. They completed as many questions as they could during the allotted time period, but there were several questions that they did not have time to attempt.

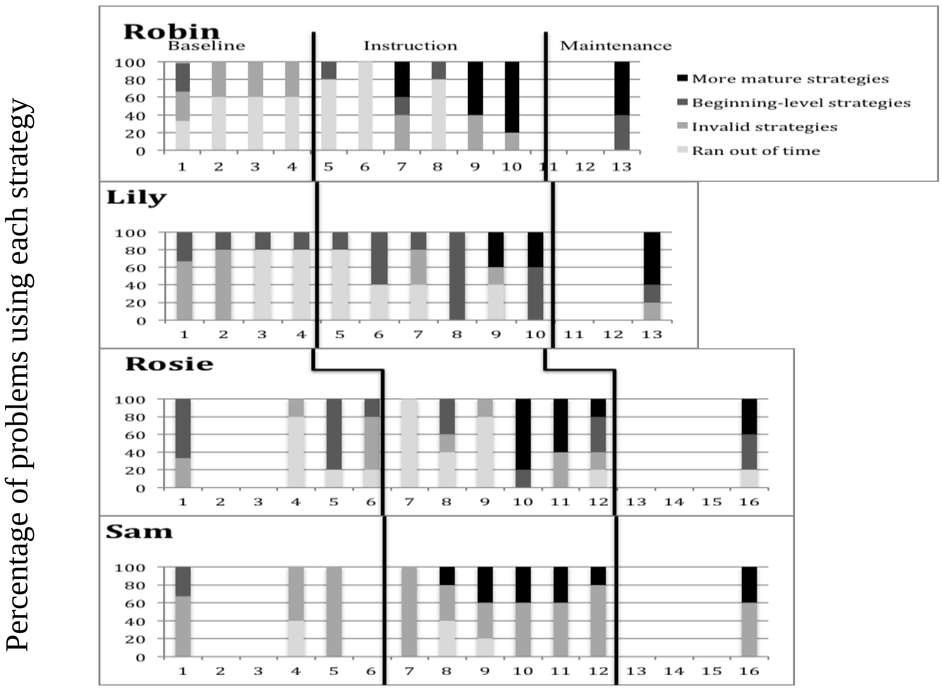

Figure 2: Percentage of valid and invalid strategies used over time

The three girls usually ran out of time when they had been using a beginning-level strategy, as these methods were very time-consuming, taking approximately 8 times as long to complete as the more mature strategies. They also made multiple errors when using beginning-level strategies, whereas they were usually accurate when using more mature strategies. The weeks when these students had no problems correct occurred because they were relying on slow and inaccurate beginning-level strategies, whereas Sam had no answers correct when he used invalid strategies to answer every question.

As the intervention progressed Robin and Lily used more valid and more mature strategies to solve the place-value problems (see Figure 2). Robin progressed from using only one valid strategy after the first week of instruction to using valid strategies for all of the questions during the follow-up interview. She also progressed from using no mature strategies until week 3 of the instruction to using mature strategies for all but one problem in the final week of instruction. Lily also progressed from using only one valid strategy after the first week of instruction to using valid strategies for all of the questions after the last week of instruction. She also progressed from using no mature strategies until week 5 of the instruction to using mature strategies for all but two problems during the maintenance phase of the instruction.

Rosie's use of valid strategies was more variable than the other students (see Figure 2). On one of the baseline days she used no valid strategies and on another she used four. Despite this variability, Rosie showed progress in her use of valid strategies. During the first half of the intervention she used a mean of 1 valid strategy per week, whereas from the second half of the intervention onwards she used a mean of 4 valid strategies per week. Her use of more mature strategies did develop over the course of the instruction, developing during week 4 of the instruction, but like her use of valid strategies her use of more mature strategies was also variable.

Sam also started to use more valid strategies as the instruction progressed, but unlike the other students he never used a valid strategy for all of the problems, and he never used a beginning-level strategy during the instructional time-period (see Figure 2).

Table 1
Variety of strategies

Student	***Number of types of valid strategies used***	***Number of correct answers at maintenance***
Robin	6	5
Lily	5	4
Rosie	4	2
Sam	2	1

The students who made the most progress across the course of the intervention were the students who tried a variety of strategies during the problems (see Table 1), using strategies they had learned during the instruction. Both Lily and Robin used a variety of valid strategies throughout the intervention, with Robin using six valid strategies and Lily using five, each of them trying all but one strategy multiple times. When these students first used a new strategy they often solved the problem incorrectly or ran out of time, but once they had used the same strategy several times their accuracy and speed improved so that they no longer made errors with that strategy. For example, the first time that Robin used the partial products strategy she said, "Ten times sixty-one then it's … six hundred and one but if there's five singles left over then I would actually have six hundred and six." The following week she corrected her error when multiplying by ten by saying, "Since it's ten times the other number if it's a two-digit it's just the same with the zero at the end plus the one single then that would mean it would equal a hundred and eleven", and then correctly used partial products twice more. These students benefited from trying multiple strategies and from using them multiple times so that they could refine their technique.

Rosie used five valid strategies, and she used each of them multiple times. Unlike Lily and Robin, her errors were not towards the beginning of her use of a certain strategy, but the third or fourth time she used a strategy. The exception to this is her use of the standard multiplication algorithm, which she used incorrectly both times. In solving how much candy is in 24 packs of candy she wrote 24x10=20 because she solved 2*1=2 and 4*0=0 and put these answers together to make 20. Except for her use of the standard multiplication algorithm, Rosie's errors when using valid strategies seemed to be related to her level of focus on the day, rather than her understanding of the strategy.

Sam only used two valid strategies: recall and partial products. From week 2 of the intervention Sam was either using a rule or recall to correctly solve multiples of hundreds, as he said "because like I said anything times ten … just add the like two or one two or three zeroes to it to make it comes the right number. That's how I know it. I learned it last year." In the maintenance phase of the testing he could generalize this strategy to solve 16 boxes of 100 and say that this contained 1,600 pieces of candy.

Discussion

SSRD can be useful for determining the effectiveness of an intervention on improving a students' accuracy, particularly when the potential population is small, but this type of design is not explanatory. It cannot show how or why the responses changed. In order to get a deeper understanding of what is occurring during an intervention it is useful to examine both accuracy and strategy use. When examining strategy use it is useful to examine the types and variety of strategies used, and to examine their speed and accuracy.

References

British Dyslexia Association. (2015). Dyslexia and Specific Difficulties: Overview. Retrieved from www.bdadyslexia.org.uk.

Carpenter, T., Fennema, E., Franke, M., Levi, L, & Empson, S. (1999). *Children's mathematics: Cognitively Guided Instruction*. Portsmouth, NJ: Heinemann.

Karp, K. (January, 2013). *The invisible 10%: Preparing teachers to teach mathematics to students with special needs.* Judith Jacobs Lecture at the AMTE 17th Annual Conference, Orlando, FL.

Rakap, S. (2015). Effect sizes as result interpretation aids in single-subject experimental research: description and application of four nonoverlap methods, *British Journal of Special Education, 42* (1), 7-10.

Scruggs, T.E., Mastropieri, M. A. & Casto, G. (1987). The quantitative synthesis of single subject research: Methodology and validation. *Remedial and Special Education, 8* (2), 24- 33.

Schwedtfeger, J., & Chan, A. (2007). Counting Collections. *Teaching Children Mathematics*, 356-361.

Thouless, H. (2014). Whole-Number Place-Vale Understanding of Students with Learning Disabilities. (Unpublished doctoral dissertation). University of Washington, Seattle, WA.

van Daal, V. (2015). Can educational practice be based on single-subject designs?, *British Journal of Special Education, 42* (1), 11-33.

Van de Walle, J.A., Karp, K.S., & Bay-Williams, J.M. (2010). *Elementary and Middle School Mathematics: Teaching Developmentally—7th Ed*. Boston, MA: Allyn & Bacon.