Affective aspects of mathematical resilience

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I link the concept of mathematical resilience, as introduced by Johnston-Wilder & Lee (2010), to the emotional and affective issues that a student might have with mathematics. My main research questions are:

1) What can we do as teachers to engender the positive emotions associated with mathematical resilience?

2) Does having more time explicitly ring fenced for the development of learning and thinking skills have a positive impact on mathematical resilience?

3) Is there a case for making a distinction between long-term and short-term mathematical resilience?

I gained insight into these questions by focusing on a case study of 2 of the classes I taught in an 11-18 school in the UK. I videoed each class looking for evidence of the use of thinking skills and mathematical resilience (or its absence) and then selected 3 students from each class to interview. Thus my research methods fell into the case-study model described by Burgess (1990) as the 'multi-site case-study' (p.5), whilst trying to describe some 'paradigmatic cases' as described by Freudenthal (1981, p.135).

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Introduction

I first became interested in mathematical resilience whilst working with undergraduate students at Oxford in tutorials on complex analysis. One of my highest attaining students preferred applied mathematics and went as far as to state that he "couldn't do pure mathematics". This seemed ridiculous to me as he was a very strong mathematician (who would in fact go on to complete a DPhil. in quantum mechanics). Yet even at this level, the experience of failure in one single problem had convinced him that he "couldn't do" half of the subject. I tried to convince him that he could do pure mathematics, and succeeded in as much as he passed his examination in complex analysis that year, but failed in that he only took applied options in his third year. This experience got me started thinking about the importance of resilience in mathematical thinking.

Several years passed, however, before I came across an attempt to formalise the concept. Through work as part of my initial teacher training and the start of a Masters degree I had become very interested in the "New Skills Curricula" which were appearing at the time. These included Claxton's "Building learning Power" or BLP (2005), Microsoft & Futurelab's "Enquiring Minds" (2009), the Royal Society for the Arts' "Opening Minds" (RSA, 2009), the Cabot Competency Curriculum

(CCC) developed by John Cabot City Technology College, and finally the Personal, Learning and Thinking Skills (PLTS) curriculum, introduced as a statutory component of the National Curriculum for England by the then Labour government (QCDA, 2010). Some general form of learning resilience was part of all of these. It was Johnston-Wilder and Lee (2010), however, who first tried to refine this notion to apply specifically to mathematics, and pull out some of the specific mathematical attributes and strategies that learners need to have in order to be resilient in doing mathematics. I decided that I wanted to investigate this concept further, and explore how different implementations of the PLTS curriculum might influence the development of mathematical resilience (MR) in learners.

Methodology

At the time of this investigation my school had just changed their year 7 curriculum from one where thinking skills were taught in separate ring-fenced lessons for 6 hours a week to one where PLTS was to be largely integrated into subject teaching. I taught a middle set year 8 class who had experienced the earlier system, and a top set year 7 class who were in the new one, giving me the opportunity to compare the results of the two. Because the sets they were also following the same scheme of work I could compare their performances on the same tasks. I decided to start keeping a research diary to make note of any incidents I saw as having a bearing on my students' level of MR, as well as videoing key lessons where it seemed MR would be required. As I went about this data collection I realised that it would only be my first level of data. I really wanted to dig down into the emotional reasons why my learners might or might not show resilient behaviours.

As I shifted away from using video as my primary data source and my focus turned more to affective issues, reading the work of Ginsberg (1981) allowed me to realise that perhaps there were other ways to measure, or at least investigate, students' feelings and beliefs. Ginsberg describes how clinical interviews can be used in cognitive research, and points out their advantages over both standardised tests and naturalistic observation. He states that research into mathematical thinking has three basic aims: discovery, identification and evaluation of competence of mathematical cognitive processes. None of these aims cover the affective issues I now wished to focus on, but which clearly form a central part of mathematical behaviour. But the ability Ginsberg describes, to get information from children using "exploitation of the one-to-one relationship" the "flexible construction and presentation of tasks" and "persistence and repetition" (p.10) is one I was to go on to use.

The methods of research described in articles by Freudenthal (1981) and Mason (2002) describing "paradigmatic cases" and "brief but vivid accounts" were key to my own methodology. These ideas seemed close to what I had been doing – trying to draw conclusions from one or two cases that I saw as in some way "typical", or at least demonstrating a problem, issues or solutions that I thought common.

Having done this reading, I decided to structure my research as follows:

1) Identify students of whom I had both positive and negative evidence regarding their mathematical resilience, on video or in my journal.

2) Interview the students by giving them a task and seeing which behaviours they displayed. Then question them regarding these behaviours, and their feelings when they are enacting them. Try to establish what factors contribute

to the positive emotions connected with increased resilience and what factors contribute to the negative emotions connected with its absence.

Clearly, if I could find a way to encourage the emotions associated with increased mathematical resilience then this would constitute a step forward towards being able to help students become more resilient and, thus, stronger and more independent learners.

As my research questions solidified, and my methodology along with them, it became clear what I was looking for in my interviews, namely:

a) any similarities between the language used by my learners regarding their emotions as they struggled with new mathematical concepts;

b) any similarities between learner responses with reference to what a teacher could do in order to make these emotions positive;

c) any marked differences between the responses of the year 7 students and year 8 students in the above;

d) any differences or similarities in students whom I had identified as having strong short term resilience and long term resilience.

In each interview, I started by explaining that I was interested in how the students felt in mathematics lessons, as they worked on their mathematics, and the different feelings they went through when they were either successful or unsuccessful in gaining understanding or problem solving. I then presented to them contrasting occurrences, either on video or just from my diary, when I felt that they had shown and not shown mathematical resilience, and asked them how they had felt at each time. I then gave them a mathematical problem to work on briefly, gave them some time to think about it, and then asked them about their feelings as they had attempted to solve the problem.

Data: A vignette and interview responses

Here I present one story from my research diary and some of the responses to my first interview question in order to inform the conclusions in the next section. The story involves a pair of quotes from a year 8 girl who often had the plaintive "I don't *get* it!" on her lips. Early in the year she came up with the memorable question:

"How do you expect me to learn if I don't get it in the first place?"

I decided to bring this question to the attention of the class to point out that if they "got it in the first place" then they really would never learn anything new. The girl said that she expected to understand it after I had explained it, but before they started the task. I explained that we learn by *doing* something, rather than just passively listening to someone talk about it. So, their understanding would only be useful if they constructed it *through* doing the task, rather than just being told it. Later in the year, after a couple of disappointing test scores, and a parents' evening where I expressed my concerns, a miraculous change seemed to come over the student. The near permanent sight of her hand in the air became a thing of the past, and she started really trying hard to start tasks without asking for help. And, to both her and my joy,

once she had started a task, using the information I had given her, she often found that she could complete it. The following quote after 3 weeks of working like this seemed to sum up her thoughts on the benefits mathematical resilience:

> "I'm really enjoying maths now… I didn't realise that I'd start to understand if I actually started trying."

With this girl, increasing her MR seemed a simple conscious decision to "try harder before asking for help".

Interview question: How do you feel when you are trying to do a problem in maths and at first you can't do it?

Responses to this question were varied, and many students needed a bit of extra prompting before they 'got' what I was asking. They would immediately describe to me some strategies of what they would do (e.g. "ask a friend" or "look on the board for a clue") when what I really wanted to know was how they felt. After clarification from me, there seemed to be some agreement in feelings. Here is a selection of the words used in response to the question: worried, frustrated, stressed, irritated, annoyed. The strength of these feelings seemed to be in direct correlation with the students' mathematical attainment, i.e. the stronger students would feel more annoyed. The student who did not mention any feelings of this kind is the one who showed least resilient behaviour in class. She sums up her feelings:

> It depends how I feel that day…..Sometimes, if it's got x in it or something, and I just don't get it I just go "Nah, can't do that" and give up. But sometimes I think "I want to get this", so I ask the person next to me.

When asked what (apart from the appearance of x) could shape how she felt and her subsequent behaviour, she said that she tended to work harder in morning lessons, or when there was a test coming up ("because I want to get a high level 6 – I really want to do well in this test"). This raised two issues for me. Firstly, was it possible for a student to behave in a strongly resilient way in the morning and, as they became more fatigued by the school day, become less resilient? The answer, as I think I was aware from my own experience of doing mathematics, is yes. And secondly, is the external motivation of test scores and levels of attainment counter-productive in mathematics, where understanding is key. The other students all seemed to be motivated by the mathematics itself, to some degree at least, whereas this student did not mention it at all.

Conclusions and further questions

In conclusion I have identified five feelings, beliefs and behaviours which came through from the majority of the interviews as being positive indicators of strong mathematical resilience:

1) A *sense of frustration and annoyance* when first faced with a problem which one cannot do.
2) An *expectation* that they will have to spend time thinking about the problem before they solve it, and that it might be hard.
3) A *belief* that they will be able to 'get there in the end'.
4) A strong *want or need* to work the problem out themselves.
5) A selection of *strategies* to help them to get there (e.g. 'draw a diagram', 'ask a friend what they did', 'list what you know and need to know', etc…)

The next obvious question would be how to encourage these feelings, beliefs and behaviours in our students in order to engender higher levels of mathematical resilience in the future. Answers I have come across when working with my students and talking to other teachers are:

a) Encourage them by giving them tasks that they can do at first, so that they experience success in mathematics and learn to trust themselves to get to a sensible answer if they try for long enough. This also allows a trust to develop between the teacher and student that the teacher will not ask the impossible.
b) Develop this self-trust by trying to avoid giving answers to problems so that students do not develop a 'learned helplessness' (Dweck, 2000).
c) Focus on teaching the strategies, and being explicit in this. Several teachers I spoke to mentioned that whenever they stressed that the students should be 'being organised' in their work, the students would produce better work.
d) Try not to compare students' abilities or tell them that you expect them to be able to do things easily, or find them hard.

It is difficult to discern any significant difference between the responses which would allow me to draw any conclusions on my second research question. I feel that there were simply too many other factors at work confusing the issue, not to mention the lack of any recognised objective way of measuring progress in thinking skills. I think that I would have been able to gain data more pertinent to this second question from two groups which were more similarly matched with respect to their mathematical ability.

An interesting point relating to my second research question was that of how coherent the students were in discussing their learning and conscious of their own learning styles and what they were actually doing when they learned. Disregarding for a moment any thought of mathematical prowess, the year 8s seemed much more confident and fluent in talking and thinking about their own learning than the year 7s, who required much more prompting and were much less able to discuss their own actions and reactions. I put this down to the increased time that the year 8 students spent discussing exactly this kind of thing in their 'learning to learn' lessons. I believe that this must have a positive impact on their mathematical resilience, but there is no significant data I can point to that I have come across in the course of this investigation in order to put this belief on a more solid foundation.

Interviewing teachers, as well as my own experience and the data from my student interviews, informs my answer to the third of my research questions, concerning the distinction between long and short term resilience. It seems clear that a distinction can be made as there are students who show a good deal of long term resilience, but much less short term. One student provided a good example of this, bouncing back from bad test results, but on a day-to-day basis being all too willing to give up before she had tried a problem for long. Finally, I'd like to conclude that we should all bear in mind that many children, and some adults, tend to be far more resilient after a good night's sleep than after 5 hours of concentration and a heavy lunch!

The 'elephant in the room' throughout the research was the problem of measuring mathematical thinking or even mathematical resilience in an objective manner. This problem currently remains unsolved. In any further research I would like to look at this problem more closely, possibly using more quantitative methods to compare results from two entire school cohorts. The question of measuring a learners "learning power" in particular seems of importance to me, as there is still much effort going into the promotion of such skills in my school and across the UK. Without any

way of measuring these skill sets there can be no way of measuring the comparative success different ways of promoting them.

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