Students' perceptions of A-level Further Mathematics as preparation for undergraduate Mathematics

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As part of a large project involving over 4,000 Science and Social Science undergraduates, 928 undergraduate mathematicians took part in an online questionnaire. Participants were surveyed regarding their experiences of studying Further Mathematics, motivations for doing so, and the extent to which A-level study had prepared them for their undergraduate course. Participants were positive about Further Mathematics, describing it more favourably as preparation for undergraduate study than the single A-level Mathematics. This research suggests that university admissions tutors and schools should consider at least *encouraging* prospective undergraduate mathematicians to take Further Mathematics – even if it is not a requirement for entry to their chosen universities – in order that they can be better-prepared for future study.

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A-level Mathematics and Further Mathematics

In England and Wales, the most commonly taken qualification awarded at age 18 is the A-level. The majority of students study three subjects of their choice (Gill, 2013) and are awarded separate grades in each. A-levels are typically studied over the course of the two final years of secondary schooling, and admission to university is usually dependent on students' grades. An Advanced Subsidiary (AS) level is the first year of the A-level, which current students may take as a stand-alone qualification. Alternatively they may continue into the second year to receive the full A-level.

A-levels are currently in Mathematics and Further Mathematics. Further Mathematics may only be taken in addition to Mathematics. A-levels in Statistics and Use of Mathematics are also currently available, although few students take them. A-level Mathematics and Further Mathematics each consist of six equally-weighted units which are individually assessed in 90 minute examinations.

Further Mathematics has been found to be the most (relatively) difficult A-level subject (Hillman, 2014); however, large proportions of candidates achieve the top grades in the subject, perhaps due to the high attainment of the cohort. In 2014, 56.0% of A-level Further Mathematics candidates achieved at least an A grade (Joint Council for Qualifications, 2014).

The current A-level Mathematics qualification affords students the opportunity to study a variety of units from Core Pure Mathematics, Mechanics, Statistics, and Decision Mathematics options. Four Core Pure Mathematics units are compulsory, and students may then take two applied units of their choosing. There are six possible routes through an A-level Mathematics qualification based on students' unit choices.

For Further Mathematics, students have more freedom. At AS-level, students must take one Further Pure Mathematics unit, and at A-level, two of those. They then may choose from up to four Statistics, five Mechanics, two Decision Mathematics and four Further Pure Mathematics units to make up three units at AS-level and six units

at A-level. Consequently, there is a lot of room for students to study a breadth of areas of Mathematics or to specialise in a particular area.

Uptake and reform

In 2014, A-level Mathematics was the most popular A-level subject, comprising 10.7% of all A-levels taken (Joint Council for Qualifications, 2014), and its popularity has been steadily increasing over the last ten years at both AS- and A-level. Further Mathematics, on the other hand, is by no means as popular (1.7% of candidates took it in 2014), but it is one of the fastest-growing A-level subjects.

A-levels in a variety of different subjects are currently undergoing reforms, with reformed A-level Mathematics and Further Mathematics qualifications planned for first teaching in 2017. Ofqual, the government department which regulates qualifications in England, has used the advice of their A-level Content Advisory Board to inform significant changes to the qualifications.

Specifically, A-level Mathematics will consist entirely of compulsory units, leaving students no opportunity for specialism in one area of applied Mathematics. This means that universities can be more certain of the mathematical background of their students (ALCAB, 2014). Further Mathematics will contain a compulsory Pure Mathematics core, with students able to choose from a wide variety of options for the remaining units. It is possible that these units may follow the current Further Pure/Statistics/Mechanics/Decision Mathematics theme. However, examination boards in England and Wales have the opportunity to develop innovative new applied options.

The Study

Rationale

It is important that the reformed A-levels in Mathematics and Further Mathematics are fit for purpose, in that they meet the needs of their key stakeholders. New undergraduates must have sufficient background knowledge, skills and understanding to be able to study at degree level. University staff must feel confident that they can assume a certain level of knowledge, skills and understanding when preparing and teaching their courses. A substantial study exploring stakeholders' perceptions was therefore conducted to inform the A-level redevelopments.

Overall, the views of both lecturers and students were sought regarding the motivations and mathematical preparedness of new undergraduates of Science and Social Science degrees who had taken A-levels in Mathematics and/or Further Mathematics. This article reports solely on some of the findings of a student questionnaire. Although it was completed by a total of 4,129 undergraduates of subjects with a mathematical component, the present article focuses on the 928 responses of those who were studying undergraduate Mathematics.

Method

Admissions tutors and heads of departments at Mathematics departments in all universities in the UK were contacted. They were asked to forward information to their students regarding an online questionnaire. Participants were required to have completed at least one year of undergraduate study in order that they would be able to reflect on their experiences of studying Mathematics at university. They also needed

to have taken a minimum of AS-level Further Mathematics. The online questionnaire comprised 21 questions, including three longer, open-response questions.

Results

The results given in this section are only a brief summary of the overall results, which will be published in due course.

Sample

In total, 928 students took part, 42% of whom were not required to have taken Further Mathematics in order to earn their place at university. Nonetheless, 87% had taken the full A-level.

The sample was heavily biased towards high-attainers: 73% of participants achieved an A* grade in Further Mathematics, and 45% achieved a first class result in their previous year's university examinations. This may be because participation was based on self-selection.

Motivations for studying Further Mathematics

Participants were presented with factors which could have motivated them in their decision to study Further Mathematics and asked to indicate the extent to which each factor influenced them. The factors were the options given in an earlier study regarding uptake of A-level Mathematics (Qualifications & Curriculum Authority, 2007).

Factor	% Participants		
	Influenced me a lot	**Influenced me a little**	**Didn't influence me**
I wanted to challenge myself.	44.8	29.5	25.7
I knew it was a requirement for the degree I wanted to do.	51.3	23.9	24.8
I was thinking of doing a Maths or Maths-related degree at university.	82.6	12.4	6.5
The topics covered looked interesting.	28.3	34.3	37.4
I wanted to do the same subject as my friends.	2.2	7.3	90.5
My teachers encouraged me to study it.	23.8	36.6	39.6
The teaching staff were good.	37.7	31.8	30.5
I was better at Maths than at other subjects.	80.9	14.6	4.5
I needed Maths for my future career.	47.0	29.6	23.4
I enjoyed school Maths.	85.0	11.8	3.2
It fitted well with my other subject choices.	53.2	26.8	20.0
I coped well with GCSE Maths.	66.8	22.7	10.5
I thought it would be a useful qualification to have.	62.0	29.5	8.5

The three reasons which participants most frequently described as influencing their decision to study Further Mathematics were: (1) they enjoyed school Mathematics; (2) they believed that they were better at Mathematics than at other subjects; and (3) they were considering doing a Mathematics or Mathematics-related degree.

Experiences of Further Mathematics

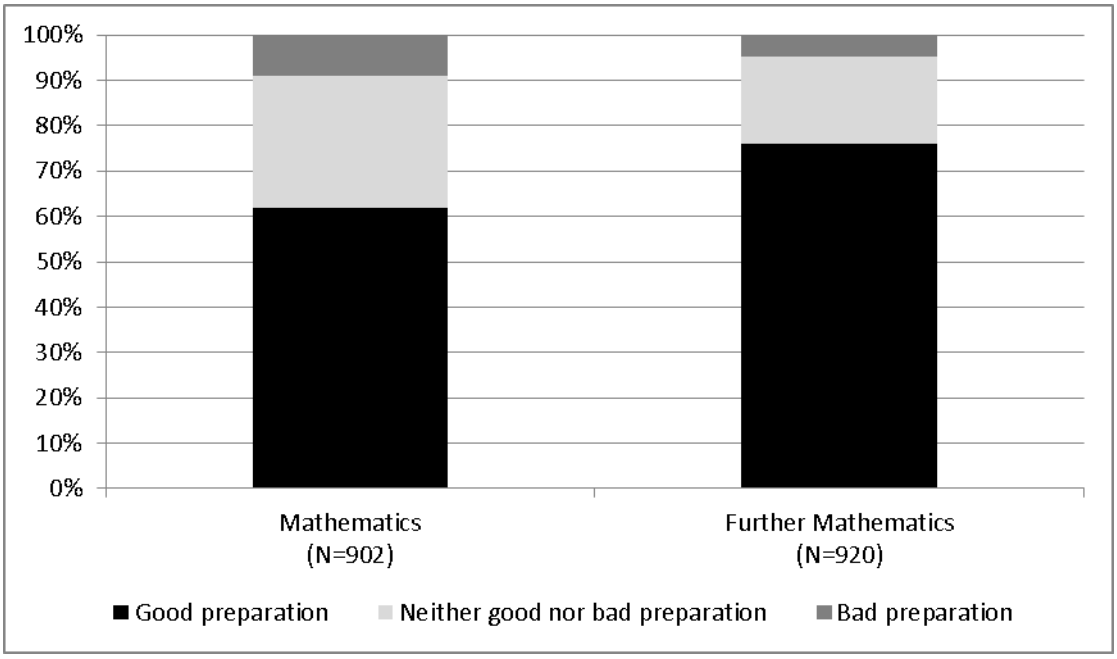
The participants' responses to a multiple choice Likert scale question regarding their experiences of studying Further Mathematics suggested that they were generally positive. Over 97% were glad they had taken the A-level, and over 91% had enjoyed it despite most of them finding it challenging.

	% Participants				
	Strongly Agree	**Agree**	**Neither Agree Nor Disagree**	**Disagree**	**Strongly Disagree**
Further Maths was my most difficult A-level.	20.6	15.6	14.0	32.7	17.1
I'm glad I did Further Maths.	73.2	24.1	2.1	0.4	0.2
I enjoyed Further Maths.	53.3	38.9	5.4	2.1	0.3
Studying Maths and Further Maths was sufficient preparation for my degree.	29.0	38.3	10.3	14.9	7.5
In my first year at university, we were taught material that I had learnt in Further Maths.	37.9	42.0	8.3	8.4	3.4
Further Maths was more demanding than Maths.	44.6	42.7	8.4	3.7	0.6
I found Further Maths challenging.	22.8	44.0	15.5	13.0	4.7

Additionally, almost 80% of participants thought that there was an overlap between Further Mathematics and the first year of undergraduate study.

Further Mathematics as preparation for university

Participants were asked to describe A-level Mathematics and Further Mathematics, individually, as either 'good', 'bad' or 'neither good nor bad' preparation for undergraduate Mathematics study. (No statistically significant differences were found between the responses of those who had taken AS-level Further Mathematics and those who had taken the full A-level.)



Overall, the majority of participants described each qualification as good preparation for tertiary Mathematics, although the proportion who gave this response was higher for Further Mathematics (76.0%) than for A-level Mathematics (61.9%). This suggests that there are definite benefits to studying Further Mathematics in addition to Mathematics when preparing for a degree in Mathematics.

Asked whether there were any areas in which the Mathematics and Further Mathematics qualifications could be improved to make them better preparation for undergraduate Mathematics, the participants gave a number of suggestions. The most-commonly described areas for improvement were:

- increasing depth rather than breadth in what is learnt;
- making both qualifications more challenging;
- making Further Mathematics in particular more representative of university-style Mathematics content and assessment; and
- increasing the amount of proof in the specification and assessment.

Furthermore, many participants complained that A-level examination questions are too predictable and formulaic, and that it was too easy to rely on past papers for revision. This was because they did not perceive the examinations to test mathematical understanding. A number of topics in Mathematics which the participants had not studied (be this because of their module choices or because the topics were not offered on the specification that they studied) are given in the table below.

Pure Mathematics	Mechanics	Other
Formal proof and analysis	Vector calculus	Notation
Logic	Vector spaces	Financial Mathematics
Number theory	Quantum mechanics	Probability theory
Set theory		
Eigenvalues and eigenvectors		
Gaussian elimination		
Partial differentiation		
Higher order differential equations		
Fourier and Taylor series		

Discussion and conclusion

The research briefly outlined in this article suggests that students make informed choices with regards to opting to take Further Mathematics. Decisions are made based on an enjoyment of the subject and a perception of its utility. Students enjoy Further Mathematics and, when studying undergraduate Mathematics, are glad that they took it. Participants reported that Further Mathematics is useful preparation for university Mathematics, but that it does not necessarily give an accurate impression of what this entails.

None of the findings reported here were surprising regarding students' perceptions of Further Mathematics and their motivations for studying it. However, they do reiterate the message of much literature in Mathematics education: new undergraduates are underprepared for its study. Existing research generally focuses on qualitative research with undergraduates or the views of lecturers, so this article enhances the literature with quantitative data from a large number of undergraduates.

This research suggests that it is important that students receive well-informed advice from their teachers and careers advisers if they are considering studying Mathematics at university. Further Mathematics should be a subject which students aim to take if they are considering pursuing the subject further, meaning that schools should take advantage of initiatives such as the Further Mathematics Support Programme[1] – a government-funded project which aims to foster and encourage the inclusion of Further Mathematics in all secondary schools through various factors such as teacher training and student support. Finally, admissions tutors on undergraduate Mathematics courses would be wise to consider strongly recommending prospective applicants take Further Mathematics, given how often the participants of this study described it as a useful preparation for tertiary study.

References

ALCAB (2014) *Report of the ALCAB Panel on Mathematics & Further Mathematics*. Retrieved from https://alevelcontent.files.wordpress.com/2014/07/alcab-report-on-mathematics-and-further-mathematics-july-2014.pdf

Gill, T. (2013) *Uptake of GCE A Level Subjects 2012*. Cambridge: Cambridge Assessment.

Hillman, J. (2014) *Mathematics After 16: The State of Play, Challenges & Ways Ahead*. London: Nuffield Foundation.

Joint Council for Qualifications (2014) *A-level Results*. Retrieved from www.jcq.org.uk/Download/examination-results/a-levels/a-as-and-aea-results-summer-2014

Qualifications and Curriculum Authority (2007) *Evaluation of participation in GCE mathematics: Final report*. Retrieved from http://webarchive.nationalarchives.gov.uk/20141031163546/http://www.ofqual.gov.uk/files/QCA_3388_Maths_GCE_eval_report.pdf

[1] http://www.furthermaths.org.uk/