Ratio and proportional thinking: a study in an Irish context

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The concepts of *ratio* and *proportional thinking* can be problematic for students, teachers and also prospective teachers. As part of a project examining the meanings ascribed to ratio and the representations offered for it especially by the latter group, data were collected from prospective primary teachers in Irish teacher education institutions. In this paper, graduates taking an eighteen-month teacher education course are considered. A curriculum and textbook analysis was undertaken to examine the treatment of ratio-related concepts for primary classes. Findings show that curricular and textbook support for teaching ratio are too sparse to support the prospective teachers' rather patchy knowledge adequately; further work in initial education is indicated.

Keywords: ratio; proportional thinking; initial teacher education

Introduction

The concepts of *ratio* and *proportional thinking* can be problematic for school students, teachers and also prospective teachers. To explore the extent of the problem especially for the latter group, members of the Research and Development Community "Science and Mathematics Education" of the Association for Teacher Education in Europe (ATEE) initiated the "ATEE Ratio Project;" this examines the meanings ascribed to ratio, and the representations used for it, by prospective teachers (taking teacher education or related courses) and other student groups. An initial round of data collection in three countries led to the identification of a framework that could be used for analysing further sets of data, as described below.

The second author's participation in the ATEE Ratio Project suggested to her that ratio and proportional thinking have been somewhat overlooked in Irish mathematics education. This led to an Irish focus on curriculum and textbook analysis as well as to the collection of data from different groups of prospective teachers in Ireland. In this paper, data collected from prospective primary teachers are analysed using the ATEE Ratio Project framework, and the findings are considered in the light of an analysis of the primary mathematics curriculum and commonly used textbooks.

Literature review

The review of literature considers three topics: knowledge required for teaching; ratio; and representations. In each case, roots for the research go back at least to the 1980s, but work is ongoing. Treatment here is brief; further work in the context of the ATEE Ratio Project is reported by Oldham and Ni Shuilleabhain (2014).

As regards teachers' and prospective teachers' *knowledge for teaching*, seminal work is due to Shulman (1986), who introduced the idea that content knowledge and pedagogical content knowledge are two distinct areas. More recent work, such as that of Ball, Thames and Phelps (2008), focuses on the former area and examines the kind of content knowledge that teachers require in order to help their

students learn with understanding – that is, with relational understanding, or understanding "why," as defined by Skemp (1976); teachers' own relational understanding is one of several important factors here.

Development of work on students' understanding of *ratio* can be traced from the studies initiated in the late 1970s by Hart and her colleagues (Hart, 1984). Other important contributions were made, for instance by Lesh, Post and Behr (1987) and Lamon (1993), with major summaries offered by Behr, Harel, Post and Lesh (1992) and Lamon (2007). The ratio concept itself is somewhat problematic; Clark, Berenson and Cavey (2003) discuss whether fractions belong within the concept, and show some preference for two-variable meanings (say, emphasising comparisons or relationships) over those dealing with part of a whole (as in the case of a fraction).

The work by Lesh et al. (1987) deals especially with *representations* of ratio and related topics; Ball et al. (2008) also refer to representations. The importance of *multiple* representations in relational understanding is familiar from the work of Dienes (Gningue, 2014). Pape & Tscoshanov (2001) draw together research from the late 1980s and 1990s and declare emphatically that "the use of multiple representations facilitates students' development of mathematical concepts" (p.120).

The ATEE Ratio Project

As indicated above, the work reported here was done for the ATEE Ratio Project (Berenson, Oldham, Price, & Leite, 2013). In 2011, a one-page, four-item instrument was devised to collect data on meanings and representations of ratio, typically in ten to fifteen minutes at the beginning or end of a lecture. After the first round of data collection in 2011-2012, the wording of some items underwent modifications with the aim of improving their power to elicit relevant responses from participants; however, their basic thrust remained unaltered. The items in the forms used for this paper are:

1. What does the term "ratio" mean to you?
2. A) When do you use ratios? B) Who else uses ratios?
3. How do you represent a ratio using mathematical symbols? If relevant, indicate clearly which is/are the **main** symbol(s) but list others as well. *You may write expressions that include the symbols, rather than just the symbols themselves.*
4. What representations – drawings, charts, graphs, words, and so forth – might you provide to illustrate how ratio is used?

The initial round of data collection involved 158 student participants in four institutions, located in three countries. Samples were not intended to be representative; the focus was on similarities rather than comparisons. Initial analysis focused mainly on Item 1 (meaning) and Items 3 and 4 (representations). Using a grounded theory approach, three emergent themes were identified in the responses to Item 1: *two variables* (typically comparison or relationship), *uses/applications* (such as rate, scale and odds), and *part-whole* (involving fractions). Based on these themes and the literature described above, analysis of the meanings and representations led to the conjecture that *participants who offered meanings reflecting two variables, and who provided many, varied, and relevant representations, possessed relational understanding of ratio* (Berenson et al., 2013; Oldham & Ni Shuilleabhain, 2014).

Subsequent data collection has been carried out independently in different countries, allowing for varying emphases. In Ireland, attention was paid initially to prospective second-level teachers (of students aged typically 12 to 18 years); the conjecture above provided the framework for analysis. As mentioned already,

participation helped to highlight the lack of focus on ratio and proportional thinking in the Irish second-level curriculum (Oldham & Ni Shuilleabhain, 2014). The work outlined in this paper extends the Irish investigations to primary level.

Methodology

The research questions addressed in the component of the project reported here are:

1. What meanings and representations of ratio are offered by participating students from primary teacher education courses in Ireland?
2. How do these relate to coverage of ratio-related topics in the Irish primary curriculum and textbooks?

Three institutions involved in teacher education were involved. The students whose responses are used in this paper were undertaking an eighteen-month Postgraduate Diploma in Primary Teaching (PDPT 2014); they had varied academic backgrounds and very varied levels of achievement in mathematics. Ethical clearance was obtained, and data collected as usual in the Ratio Project by asking respondents to complete the instrument during a lecture period. The data from Items 1, 3 and 4 were coded by one author and checked by a second, using the categories for meaning and representation developed in earlier rounds of the project. Minor discrepancies were resolved by discussion. Data from Item 2 are not used in this paper.

The curriculum analysis involved examination of the official mathematics curriculum for Irish primary schools (National Council for Curriculum and Assessment [NCCA], 1999a) and the accompanying guidelines for teachers (NCCA, 1999b) to identify explicit and implicit occurrences of ratio-related material. For textbooks, which are produced commercially, four widely-used series were selected and the relevant books compared with the curriculum material. Ethical issues obviate naming the texts.

Findings

Student data

Responses were made by 59 students. The results in comparison with those from other Irish cohorts are reported elsewhere (Oldham, Stafford, & O'Dowd, 2014); the focus here is on the results in relation to the framework and the Irish curriculum.

The results for Item 1, addressing the meanings ascribed to ratio, are shown in Table 1. Three-quarters of the students were judged to have presented one of the preferred two-variable meanings, though in some cases the formulation was not very clear. The total percentage is more than 100 because some students made multiple responses within or across themes. Four students did not supply a meaning.

Table 1. PDPT 2014 group referring to themes in responses to Item 1: numbers and percentages

Meanings	PDPT 2014 N			PDPT 2014 %
Two variables	Comparison	16		
	Relationship	13		
	Other two-variable	23		
	Total no. students		45	76.3
Uses etc.			13	22.0
Part-whole			6	10.2

For item 3, asking for symbolic representations, the results are shown in Table 2. All students provided the colon symbol, sometimes with or in examples such as "4 : 3" or "*x* : *y*." Other types of representation – such as fractions (fully correct in this context) or descriptions in words (not meeting the requirement for a symbolic answer) – were comparatively scarce. Multiple types of representation were necessarily scarce also, while only about two-fifths provided multiple examples (including those using the same basic representation, as for the two "colon" examples given above).

Table 2. PDPT 2014 group using each main type of representation in responses to item 3: numbers and percentages

Representations		**PDPT 2014 N**	**PDPT 2014 %**
Type of representation	Colon	59	100
	Fractions	3	5.1
	Percent / other symbolic	1	1.7
	Non-symbolic	9	15.3
Multiple types		12	20
Multiple examples		25	42

For Item 4, data are shown in Table 3. Aspects of interest are the provision of multiple examples, examples reflecting the preferred two-variable meaning of ratio, and pictorial examples (diagrams, graphs/charts, or drawings, providing alternatives to symbols and words). The most conspicuous finding is that only around three-quarters of the participants made any response to the item. Moreover, for those who did respond, their representations were often too rudimentary to allow firm categorisation as whether or not they represented two variables; for instance, a circle with one quadrant shaded – and without clear labels – may have been intended to represent the fraction ¼ (hence, a part-whole understanding) rather than the ratio 3:1 or 1:3. Less than half of the group offered a pictorial example, despite the prioritisation of such representations in the wording of the item.

Table 3. PDPT 2014 group using various kinds of representation in responses to Item 4: numbers and percentages

Representations	**PDPT 2014 N**	**PDPT 2014 %**
Examples given	45	76.3
Multiple examples	23	39.0
Clear two-variable representation(s)	23	39.0
Pictorial representation(s)	28	47.5

Curriculum and textbook analysis

The primary curriculum has five content "strands," the content in each strand being presented in the curriculum document via learning outcomes. Within the strands, ratio is mentioned explicitly only once: in the Fractions strand unit of the Number strand for Sixth Class children (typically aged 11 to 12), where it is stated that "The child will be enabled to … understand and use simple ratios – *explore and record the relationship between the natural numbers and their multiples*" (NCCA, 1999a, p.90). The colon symbol is not used there. However, it occurs in one entry in the Algebra strand: "The child should be enabled to … identify relationships and record symbolic rules for number patterns … [such as] *4:1, 8:2; 16:4*" (NCCA, 1999a, p.95). Incidentally, the colon symbol also appears in the Data strand of the curriculum in

relation to scale in Fourth class, before ratio has been introduced. Other Sixth Class topics that implicitly involve proportional thinking – for example, scale, data and probability – do not mention ratio. Ratio appears in the glossary for both the curriculum (NCCA, 1999a, p.125) and the teacher guidelines (NCCA, 1999b, p.77) – its only occurrence in the latter. A two-variable meaning is given: "Ratio: the relationship between two numbers of the same kind; e.g. the ratio of 2 kg to 6 kg is 2 : 6" (which may appear to exclude, say, finding the ratio of boys to girls in a class). The words "proportion" and "proportional" do not occur in either document.

In the Sixth Class textbooks, ratio occurs only in chapters on fractions, where it is introduced via definitions or perhaps explanations. Examples include phrases such as "the relationship between one quantity and another" (a two-variable meaning) and (obscurely?) "the way in which amounts are divided." The contexts – rather sketchily presented – include fractions, sharing, unequal sharing, dividing, measures and unitary method. The emphasis is mainly on procedures; applications are limited and genuine problem-solving challenges almost absent. Mirroring the curriculum, there is no mention of ratio with regard to scale, data, probability, and so forth. The colon symbol is utilised for scale, but no connection is made to ratio; somewhat confusingly, it is also used (without clarification) for representing time. Altogether, ratio-related work typically occurs explicitly in only three to five pages of the books.

Discussion and conclusion

This paper reports on a study done as part of the ATEE Ratio Project examining the meanings and representations of ratio offered by prospective teachers. The Irish prospective primary teachers sampled here provided a limited range of meanings and representations, suggesting that their own relational understanding of ratio is not strong enough to develop relational understanding in their students. Moreover, analysis of the Irish curriculum documents and textbooks indicates that these do not provide sufficient support for such prospective teachers in their classroom teaching. The curriculum documents clearly promote the use of linkage across the strands of mathematics and explicitly state that these strands are "interrelated units in which understanding in one area is dependent on, and supportive of, ideas and concepts in other strands. Such linkage within the subject is essential" (NCCA 1999a, p.3). Yet, opportunities to make these connections are not availed of, either in the curriculum document, or in the textbooks, which have more scope for providing suitable examples. Thus, more attention needs to be paid to this topic in teacher education courses to strengthen prospective teachers' relational understanding. Also, future revisions of the curriculum and textbooks should make more explicit linkage to ratio and proportional thinking across the strands and strand units; textbooks should consider wider applications and provide genuine problem solving opportunities.

The ATEE Ratio Project instrument used in data collection might be criticised for eliciting an immediate rather than a reflected response, and hence underestimating the respondents' knowledge. However, in a classroom setting, an immediate response may be required. Moreover, "correct" responses may reflect recall rather than understanding – thus overestimating the respondents' knowledge. The next phase of this study includes the use of this instrument in teacher education classes in think-pair–share mode to allow students to critique and further develop their own relational understanding.

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