Teaching postgraduate research students in mathematics about teaching mathematics to undergraduates: education or training?

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Postgraduate research students in university mathematics departments often teach undergraduates mathematics while they pursue their own research. This paper reports on a mathematics-specific 10 learning hour introduction to teaching for post-graduate mathematics research students. There are two strands to the paper: (1) presentation of the course design, delivery and data; (2) a discussion of the claim that such a course could not 'train' them but that it did offer them opportunities for 'education'.

Keywords: Graduate Teaching Assistants (GTAs); undergraduate mathematics teaching; education versus training.

Nowadays, at many research intensive UK universities, post-graduates in mathematics departments are 'graduate teaching assistants' (GTAs), contributing to the teaching of their departments' undergraduates; this has long been the standard situation in universities in North America (Park, 2004). The UK Higher Education Academy has in the past run workshops for maths GTAs but cuts in public expenditure led to the demise of this specialist mathematics provision. However, a local initiative, 'the IOE-UCL strategic partnership', in 2014-15, supported designing and running a mathematics-specific teaching course for postgraduate research students which was to take approximately ten hours of their time. This course provided a mathematics practitioner researcher context on which this paper reports.

Outline of the paper: The first part is an introduction to the context; firstly some background is given to post-graduate student preparation for university teaching, then a description of the course is presented together with some illustrative data. The second part of the paper is discussion on *What should feature in a 10 hour preparation for Teaching Undergraduates Mathematics course for postgraduate maths research students?* together with the claim: *a GTA cannot be 'trained' in 10 hours, but education is possible*.

Context

World-wide, young adults (and others) seek to enroll in undergraduate degree programmes and, in many educational jurisdictions, pay high fees for the privilege; the issue of 'who might be teaching me then?' asked when fees for the majority of home students increased (Castley, 2005) continues to be relevant. When choosing their programme, these potential undergraduates will get information from universities' websites and other publicity which, amongst other things, promotes the reputation of that university's academics. For example, a promotional video for potential mathematics undergraduates broadcasts that "anyone teaching you is a cutting edge researcher of modern mathematics, which is true for all our permanent staff" and also "all of them [faculty] are research active and all of them teach." (UCL, 2015). While the issue of the relationship between research expertise and teaching expertise is not addressed in this paper, it is the case that potential undergraduates are likely to have some tutorials (and possibly lectures as well) given by GTAs. The

GTAs have not yet become experts in their research area, though they are supervised by experts, who do also teach. If a distributed expertise (Pea, 1993) can be said to come from the parent mathematics department, not only in research but also in teaching, then inducting these postgraduates into university teaching as part of departmental practice is consonant with a notion of undergraduate experience given by the promotional video.

In the North American context, where GTA contribution to teaching is standard and includes lecture courses as well as tutorials, the issue of their preparation for teaching has been discussed for some decades (e.g., Carroll, 1980; Border, Speer, & Murphy, 2009). The models that have used to describe the GTAs induction include "orientation", "transitional" or "recurring" programmes (Border et al. 2009, pp.26-27). Orientation programmes, generally held prior to the GTA starting to teach, induct the graduate student to the ways of the department, including the syllabus of the course s/he is to teach; transitional programmes, lasting longer and typically meeting throughout a semester, included input on teaching styles, yet rarely on subject specific pedagogy; recurring programmes offered on-going support over the years of the GTA's graduate study. In terms of these categories the course described below was a transitional course that additionally addressed university mathematics subject-specific pedagogy.

As there is now a considerable body of research on learning to teach mathematics to undergraduates, the subject- and phase-specific pedagogy of the course discussed in this paper was informed by research across the decades, including, Tall and Vinner's 1980s notion of concept image (Tall & Vinner, 1981) as well as recent work on how young people transition to learning university mathematics (Grove, Croft, Kyle, & Lawson, 2015). This background was employed to design and deliver a course for postgraduate or postdoctoral researchers (n=16) from a range of mathematical disciplines; this was a considerable challenge especially as the course was intended to take less than 10 hours of their time! While initially the aim was to let participants choose whether to focus on teaching *real analysis*, *number theory* or *mathematical methods*, in practice it was not possible to give this amount of choice. So the course used *real analysis*–a foundational area of undergraduate mathematics with which all participants were familiar– as the content area from which to develop teaching. The course was oriented around three types of teaching task, namely: marking, lecturing and tutoring, from which observational data and reflections were collected, as shall be described:

Marking: From last year's *real analysis* exam scripts, parts of undergraduates' answers to a few questions were pasted onto a large sheet so several could be compared. The task was to mark these answers, firstly solo and then, after comparing marks awarded, in discussion with peers, teaching fellows and other tutors. Data were of the form: range of marks, participants' annotations to undergraduates' answers and notes of comments on topics including student errors, variation in marking, issues of precision, the nature of proof.

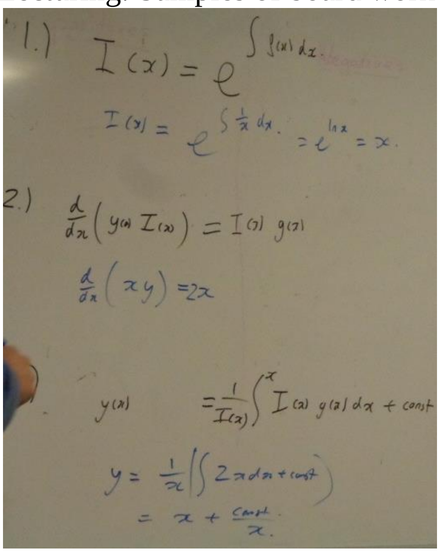
Lecturing: Participants were asked to prepare a mini-lecture (5-10 minutes) either on a topic that arose from the marking session (e.g., Rolle's theorem, or a fiendish counter-example to an intuitive 'truth') or another early undergraduate topic, to peers and tutors as if they were undergraduates; after each mini-lecture there was a discussion on the content and presentation. Data were of the form of notes and photos from the presentations and notes of points of discussion.

Tutoring: (a) Current first year maths undergraduates were invited for an extra tutorial on a topic of their choice from a neutral email address (so that they could respond and take advantage of the offer without being noticed by their usual lecturers/tutors); GTAs set up and taught the small group. Data were of the form of short completed proformas from tutor and tutees. (b) in some cases, the GTA was already a tutor. Data from these cases: a slightly longer proforma than (a) was completed. A post-tutoring discussion session has been planned.

Samples of data

Marking: Paraphrased from notes: one of the participants, R, argued that an undergraduate examinee should be penalised for not stating explicitly that 'N' should be an integer when using 'N' in a certain definition. The other participants and tutors, had a discussion about implicit meaning, given the practice (i.e. use of notation of the lecturer) and the pressure of an exam; the consensus was that R was being a bit harsh!

Lecturing: Samples of board work include:



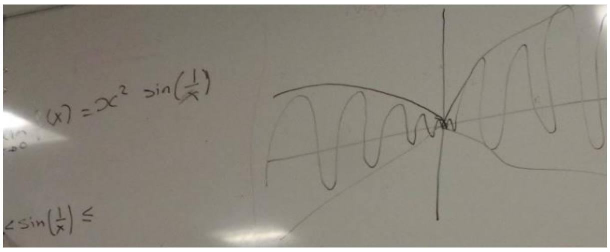


Fig 2. No graph plotter nor peer mention of lack.

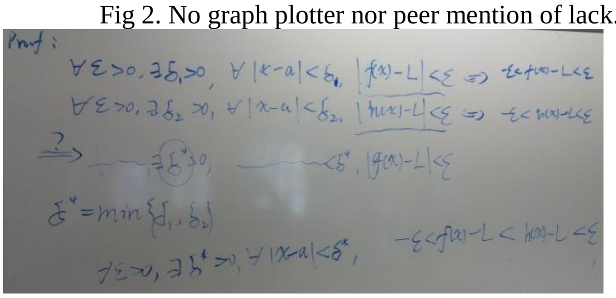


Fig 1. Peer praise for two colours.

Fig 3. No peer comment on readability.

Tutoring: Examples of GTA comments indicating reflection:

> GTA1: The students came like It was a lecture class, without questions and without their notes. So It was really bad to start to explain something. When I start the problems they start to remember and they ask me several questions.

> GTA2: I think I presented things more as a collection of tricks for how to tell when a series converges, rather than some motivated general techniques. Starting with sum(1/n) was probably a bad example, because the proof that it diverges *is* something of a one-off trick.

Undergraduate feedback, when given, was positive, for example:

> Student W: Explained things in a easy to understand way and gave tips for exams as well which was useful.

> Student N: ('GTA') did a great job by guiding and explaining us some tricks and concepts of orbital motion (mostly). however, there was a small question that she couldn't explain in details. overall, the tutorial session was awesome.

Reflective notes: It was satisfying to have been immersed in details of undergraduate mathematics with the participants, all of whom are potential university lecturers in mathematics, who contributed to group discussions lead by the project team enthusiastically and perceptively. In particular, their presentations embodied some of the challenges involved in giving mathematics lectures. Examples raised included: coordinating board writing with talking to students, helping students notice mathematical precision, consideration of suitable notation and the oral language associated with symbolic reasoning. The atmosphere of the feedback sessions was very collegial, which is a good environment for the mutually supportive and insightful peer comments to be taken in.

Discussion: education versus training

I was struck when we[1] started on this project how casually the word 'training' was used within the maths department for the induction into teaching we were planning for their postgraduates (I expect my surprise was because I come from a department of education where we aim to provide more than training!), yet the observation was a prompt to consider differences between education and training that were relevant to this context and lead to the claim ('not enough time for training') proposed above. It is worth noting that in all teacher preparation courses there are aspects that are more akin to training and other aspects deemed education. For example, in a year long PGCE course, based in a university with school practicum, the pre-service teacher could be said to be trained to take a register, know the content and progression of the National Curriculum, the current statutory requirements for 'special needs' etc. On the other hand, pre-service teachers on such a PGCE could be said to be educated into ways of thinking about students through being asked to interrogate different theories of learning within their school experience, and also educated into developing a reflective practice that includes personal and academic writing, reflection on literature and dialogue with peers and tutors that widen their views on relevant issues (such as what mathematics should be in the curriculum). It seems ironic, given the relative demands of training and education, as illustrated above, to say that we do not have enough time to train the GTAs, though we have an opportunity to educate. However, for our GTAs, I aim to explain why, firstly, by considering the meanings of the two key terms, then considering the participants and data from the course.

There is frequent confounding of education and training in common parlance; a quick internet search brings up a host of 'education&training' websites, for instance. This lack of discrimination between the terms 'education' and 'training' concerned Robert Dearden in the context of 'vocational education and training' schemes being set up in the 1980s.

> Training typically involves instruction and practice aimed at reaching a particular level of competence or operative efficiency. As a result of training we are able to respond adequately and appropriately to some expected and typical situation. … in every case what is aimed at is an improved level of performance … brought about by learning. (Dearden, 1984, pp.58-59)

> [Education] is very much a matter of conceptual insight, explanatory principle, justificatory or interpretative framework and revealing comparison. It also involves a degree of critical reflectiveness and hence autonomy of judgement, …Being concerned with understanding does not exclude from education any concern for feeling and desire, attitude, action or activity, but they will not be fostered apart from understanding. … A necessary condition of understanding many things is participation in them or experience of them. Education is not a purely intellectual affair. (*ibid*. p62)

[1] Author together with colleagues named in the acknowledgements.

Dearden's clarifications of the respective terms, training and education, can be applied to the case of our course for GTAs together with the observation that assessment or evaluation is also a part of a training cycle, which can be expressed as: find out the training needs, plan training, deliver training and assess whether participants have achieved the desired out come. If not, adapt and try again.

So a 'training' has not been achieved on the course because there was not opportunity to check performance and response in typical teaching situations and 'try again' if the performance was wanting. For example, part (1) of the course aimed to prepare a GTA to mark and to respond to undergraduates' written work by giving participants a range of undergraduate responses to some of last year's exam to mark. There was a great range in many of the marks given by participants and a lively discussion ensued justifying the marks given. This debate constituted (part of) the GTAs education, as it involved interpreting, developing autonomy of judgement and justification. But it was not a training, as there was no opportunity to have the participants mark another batch of exam questions and check that they had followed the principles which came from the discussion in the session and which conformed to international mathematical community represented by the maths department.

On the other hand, Dearden's characterisation of education suggests that understanding – in this case, how GTAs understand teaching mathematics to undergraduates – includes experience as well as affects like feeling and attitude. At this point a brief introduction to the course participants is appropriate: each of them has won a place at a prestigious mathematics department to do maths research, every one of the participants communicated within our sessions a genuine interest in teaching undergraduates mathematics, each one of them prepared thoroughly for a mini-lecture in front of peers and tutors in which all of them contributed supportive insightful comments. They were an exceptionally well-motivated group of people who came together with a common interest in mathematics and all of whom had studied undergraduate mathematics. Returning to Deardon's characterisation of education, the GTAs experience of participating in mathematical culture and their current career trajectory, positioned them to have, for instance, conceptual insight in the mathematics education domain. An example of this occurred following L's mini-lecture on integrating factors (illustrated in Fig. 1) when the notion was raised that a set of suitably designed problems could get students to discover/invent the integrating factor formula without the lecturer 'giving it'. This insight is in the spirit of Bob Burn's investigative approach to first year analysis (Burn, 2000) and illustrates that our course provided an opportunity for this insight to be realised thus educating them about different forms of mathematical instruction in context. Another aspect of education offered by the course was in post-discussion consolidation. For example, in post mini-lecture session feedback I summarised points from the session and offered some general principles, for instance, on different roles of examples in mathematics learning. Thus education might well have taken place as the participants had a contextual entry to general principles, but training did not as performance was not monitored.

What then should feature in such a 10 hour course? Opportunities for practicing marking and lecturing with peer feedback and tutor guidance; opportunities for reflection on their practice both in their mini-lectures and their tutorials; a safe environment to try out ideas; skilled tutor input within discussions to pin down and possibly offer a conceptual framework for their thinking (e.g. such as in Tall and Vinner 1981). What do we not have time for? Examples include: training in educational technologies, video recording tutorial sessions and debriefing, asking them to read educational research on learning maths at undergraduate level, lesson studies. What we saw can be achieved in 10 hours is a lively collegiate atmosphere of postgraduates engaging with teaching undergraduates mathematics through addressing key areas of marking, lecturing and tutoring. Is it enough?

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