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Stimulating productive mathematical noticing: developing a framework for exploring the affordances of task and talk

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This small scale action research doctoral study aims to deepen understanding of how children can be supported to notice and use mathematically relevant ideas in the course of their class-based mathematical activity. The focus is on supporting and encouraging ways to look rather than dictating what to see. I employ the theoretical lens of ecological psychology to develop a framework to analyse the affordances of children's tasks and of classroom dialogue in stimulating mathematical noticing. In this study, noticing is positioned as a particular type of mathematical engagement; this paper focuses on the development and early use of an analytic tool.

Key words: mathematical noticing; affordances; ecological psychology; mathematical tasks; dialogue

Introduction

In contrast to the significant research attention recently paid to teacher noticing (e.g. Mason, 2002; Sherin, Jacobs, & Philipp, 2011), relatively little research has focused specifically on what children notice in the course of their mathematical activity. However "what you do not notice, you cannot act upon" (Mason, 2002, p.7), applies just as much to children's noticing as it does to that of a teacher. Indeed, Lobato, Hohensee & Rhodehamel (2013) establish that "what students notice mathematically has consequences for their subsequent reasoning" (p.844). These consequences do not apply only in the immediate situation; Lobato et al. (ibid) posit that children become attuned to noticing the mathematical ideas to which they are frequently exposed and encouraged to use. These ideas, relationships and properties may, they suggest, become part of a 'noticing repertoire' influencing how students look and what they look for. Owens (2004) underlines the need to pay specific attention to developing children's noticing. Focusing on the properties of three dimensional shapes, she reports diversity of what was noticed and reasoned with and recommends that children have increased opportunities to look at objects and images and to talk about what they see.

In defining 'noticing' for the purposes of this study, it is important to distinguish it from the associated constructs of awareness and attention. We may be aware of a multiplicity of objects, sensations or stimuli at any one time without focusing on any one in particular; I identify 'awareness' thus as a more general notion. We may also attend to a particular object or idea without noticing any specific aspect of it. Mason identifies this as "holding wholes" (2011, p.47). However from amongst an array of stimuli we are able to focus on specific features that may be relevant to our current purposes or interests whilst others are diminished in the attention they receive. This ability to 'stress and ignore' (Gattegno, 1970) enables us to cope with an otherwise potential overload of stimuli. This is more than a capacity to maintain concentration amidst potential distraction; in the mathematics classroom (and elsewhere) this ability is particularly important in that it enables us to identify sameness amongst difference and invariance in the midst of change (Mason & Johnston-Wilder, 2006).

Mason defines noticing as a "movement or shift of attention" (2011, p.47), noting that such shifts may be fleeting or more sustained, productive or distractive. Lobato et al. interpret noticing as a more overtly deliberate act, defining it as "selecting, interpreting, and working with particular mathematical features or regularities when multiple sources of information compete for one's attention" (2013, p.809). Lobato's definition, in its inclusion of 'interpreting and working with' for me moves beyond noticing into the reasoning that may ensue; for me noticing is a necessary but not sufficient condition for reasoning. I view noticing as a particular way of attending, such that in Mason's terms one might "discern details", "recognise relationships" and "perceive properties" (2011, p.47). I also build on my observation that whilst at times children may select features apparently deliberately for sustained consideration, they may also at times be drawn to a particular feature, remark on it either verbally ('they're all odd numbers') or non-verbally (pointing at or circling a number on a page) and then apparently move on. Thus like Mason I include both sustained and passing attention to a feature as noticing. However, unlike Mason, and in order to focus this study I do not include noticing that is a distraction from the mathematical task in hand; I focus only on mathematical noticing. My working definition of noticing is: *'identifying particular mathematical features for sustained or passing focus'*.

The study

This is an action research study conducted during the 2014-15 academic year in three primary school classrooms in the UK. A baseline lesson followed by three research lessons in each of the classrooms generates data across 12 lessons in total. The mathematical content of each research lesson is jointly agreed between myself and the three class teachers. Based on Owens (2004), teacher emphasis on children looking and talking about what they notice is maintained as is a focus on considering how and whether what is noticed is significant or useful in subsequent reasoning (Lobato et al., 2013). In each class, data is drawn from video recordings of the activity of two pairs of focus children supplemented by audio recordings of whole class teacher-led dialogue. Following each research lesson a workshop meeting enables the researcher and class teachers to review samples of recorded data in order to refine approaches.

Theoretical framework

The theoretical framework guiding analysis in this study is drawn from Gibson's work on perceptual affordances (Gibson, 1986). Ecological psychology, building on this work asserts that aspects of an environment afford opportunities for particular types of activity; the extent to which an affordance is acted upon or realised being dependent on the effectivity of an individual in acting, together with their intentions. Whilst traditionally, affordances have been considered to be concerned with the physical opportunities arising in an environment, a broader interpretation of affordances, connected with the socio-cultural practices and higher order learning of humans is increasingly advocated (Greeno, 1994; Rietveld & Kiverstein, 2014). Indeed Watson argues that affordances provide a valuable lens for analysing and understanding mathematics learning in terms of ways that "learners might participate in what is available in the learning environment" (Watson, 2007, p.111).

The mathematics classroom is a complex environment which, through the lens of affordances, can be considered to afford a range of activity and behaviour both mathematical and non-mathematical. The specific features of this environment that I focus on in this study relate to the mathematical task and the interaction between teacher and child and between pairs of children as they work. Specifically the research questions under consideration are:

1. How can the affordances of task structure and teacher-led classroom dialogue combine in productive mathematical noticing?
2. How does children's interaction during mathematical activity contribute to mathematical noticing?
3. How does the realisation of affordances for noticing contribute to differences in task progress?

Developing an analytic tool

In this early stage of data analysis I have attempted to position noticing as a mathematical behaviour within a spectrum of mathematical behaviours or types of engagement. This will, I anticipate, enable analysis of noticing within the flow of mathematical activity and enable me to trace the emergence or absence of significant moments in task progress. Watson (2007) articulates the challenge in using established frameworks to analyse opportunities afforded for mathematical activity, devising a new tool to enable analysis of the teacher's contribution to shaping affordances. My own search for suitable frameworks to analyse affordances of task and talk has suggested the possibility of adapting an existing tool.

Gresalfi, Barnes & Cross (2012) examined the affordances of mathematical tasks for different types of mathematical engagement. They identified four types of engagement: procedural, conceptual, consequential and critical. Procedural engagement was defined as the accurate use of procedures; conceptual engagement as understanding procedures; consequential engagement as the recognition of the significance of particular approaches, and the connection of approaches to outcomes, and critical engagement as exercising agency in choosing tools and evaluating their impact. These definitions enabled analysis of the affordances of the particular style of tasks, described as 'problem based learning', which were employed in this study. In such tasks, real life scenarios provided a stimulus for investigation through the use of a range of statistical and computational approaches.

This framework suggested possibilities for me in positioning noticing as a type of mathematical engagement and I am currently exploring adaptations to this framework to suit my purposes. Firstly, I have included a fifth category, that of 'engagement for noticing'. I see this engagement for noticing as enabling of, but not part of, the 'consequential engagement' identified by Gresalfi et al. (2012), but I more strongly associate this latter category with reasoning about what is noticed and explaining why a noticed feature is significant. However whilst I view this fifth category as particularly enabling of consequential engagement, I currently consider that it may arise from engagement in any of the four original categories and thus I am currently not siting it in a particular position within Gresalfi et al.'s (2012) framework.

Secondly I have adjusted the descriptions of the behaviours which reflect each category of engagement, whilst maintaining the original intent, to better reflect engagement in the types of task in which children in the current study are engaged. In my study children's mathematical activity is based on mathematical problem solving within a defined set of rules and conditions and not linked as in Gresalfi et al.'s (2012) study, to a real life scenario. At this current stage of development and trial, the five categories and descriptions of behaviours associated are as follows:

Type of engagement	**Behaviours and activities associated with type of engagement**
Procedural engagement	Engaging in the processes of the activity, following rules of activity, performing procedures.
Conceptual engagement	Developing or demonstrating understanding of an idea or process, e.g. discussing what a square number is, using a resource to demonstrate how to find a remainder after division.
Consequential engagement	Reasoning about the task, e.g. recognising and articulating the significance of properties, patterns and structures that have been noticed, predicting and generalising.
Critical engagement	Exercising agency in investigative or problem solving processes, making decisions, reflecting on the impact of choices made.
Engagement for noticing	Focusing on specific features of the task or the numbers involved in the task. For example identifying a pattern, a relationship, noting a commonality in a set of numbers, noting a feature of the layout or organisation of the task.

Table 1: Descriptions of behaviours reflective of each type of mathematical engagement

Within the category of engagement for noticing I have thus far identified three areas within which noticing may focus: structure, patterns and properties of number, relationships. This further categorisation facilitates comparison of the focus of noticing across different classroom tasks. 'Structure' may relate to ways in which the mathematical organisational of the task may be seen, to the parameters of the task, or may relate to noticing features of a spatial layout, for example a grid structure. The sub category 'patterns and properties' relates to the specific numbers used or encountered within the task and noticing of their properties and any patterns revealed in findings. Both of these features come together in noticing 'relationships' which may exist, for example between two or more different properties, or between structure and patterns. I see this as the most sophisticated level within this category of engagement.

Early use of the analytic tool

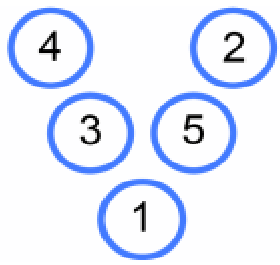

Figure 1, Magic V

Each task used in the study will be analysed in relation to its affordances for the five categories of mathematical engagement identified above. An example of this analysis in relation to one of the tasks is given in Table 2 (overleaf). The task in question in this example is called 'Magic V' (Nrich, 1997-2015); an activity in which children explore different ways to arrange 5 consecutive numbers into a V shape such that each arm of the V adds to the same total (see figure 1). The analysis of affordances in the table below builds on the teacher's guide given on the Nrich website, (http://nrich.maths.org/7821) and on my own analysis of the affordances of the task.

Video data will be used to identify specific points during which children are engaged in noticing and to identify the specific focus of their noticing. Tracing the flow of activity leading to and from such points will enable the identification of any patterns or trends in preceding and subsequent activity. Here both visual and audio evidence will be considered in analysing contributory influences on noticing. In relation to considering contributory affordances of speech, each teacher speech turn, characteristically asking a question or giving an instruction, can be considered to afford a particular type of mathematical engagement. Conversely, child-child pair dialogue consists of ideas, assertions, suggestions, refutations and many partial sentences as work progresses. Here analysis focuses on speech turns that introduce an idea or change the focus of activity. Thus child-pair speech is examined for affordances offered and realised in content or focus sections rather than line by line.

Procedural engagement	**Conceptual engagement**	**Consequential engagement**	**Critical engagement**	**Engagement for noticing**
Rehearsal and use of mental addition skills Manipulating cards in accordance with the activity rules to search for solutions Demonstrating flexibility and persistence in repeated search for solutions	Understanding that there are limits to permutations of the 5 cards Correctly using terminology and ideas of 'consecutive', 'odd', 'even' Being able to identify the outcome of adding odd and even numbers Recognising the equivalence of Vs created by swapping digit cards within one arm of the V	Explaining why only 3 totals are possible from any set of 5 consecutive numbers Predicting next set of arm totals when numbers used increased by set amount Explaining why only numbers with particular property can be base number Predicting the number range for a given arm total	Making decisions relating to how to approach the problem and record solutions/ patterns. Applying strategy for investigation Deciding how and/or deciding to manipulate number range	**Noticing Structure:** Seeing the V as consisting of two arms connected by a shared base number Defining/naming a V by a selected feature **Noticing patterns and properties:** Identifying that numbers at base of V have shared property Attending to property rather than number name or size Identifying patterns in arm totals **Noticing relationships:** Identifying relationship between base numbers and arm totals Identifying the multiplicative relationship between middle number of range and middle total

Table 2: Affordances of the Magic V task for different types of mathematical engagement

Analysis of data

At the time of writing, data collection is ongoing and analysis is at an early stage; this tool will doubtless require refinement. A small section of data (Table 3, below) provides an example of how two moments of noticing (lines 154-5) for focus child Jess, were afforded by immediately preceding teacher-child interaction. The teacher questions are of different types, one affording procedural engagement (line 152) and one specifically inviting *and focusing* engagement for noticing (line 153).

Line	**Speaker**	**Utterance**	**Engagement for noticing**
152	Teacher	Can you write the totals for each V?	
153	Teacher	Ok what I'd like you to do now is look at your Vs and	

		work out what you notice about them in general. So that's looking at all the magic Vs, can you say what they've got in common? Can you find some rules?	
154	Jess	Oh, when you add them all up together they go 8, 9, 10	Noticing patterns and properties
155	Jess	So we've got consecutive numbers,	
156	Mosie	Consecutive numbers	
157	Jess	..what else have we got…[pause] they've all got odd numbers at the bottom	Noticing patterns and properties

Table 3: Data sample from focus child pair engaged in Magic V task

In terms of what the data begins to reveal, the extract above provides two threads to explore: firstly the extent to which invitations or requirements to record results can, on their own, be sufficient to stimulate noticing. Secondly, the contribution to noticing of teacher open questions to say what has been seen (Owens, 2004), in comparison to the contribution of questions that provide some guidance as to the sphere in which looking should be focused.

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