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A good foundation for early years mathematics education in England?

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The most recent number curricula for three to seven year olds in England have raised expectations which are not supported by research. A focus on number sense would provide a better foundation, especially if supported by clear exemplification of progression, and this could be appropriately integrated with creative processes identified in the curricula.

Keywords: Early years number learning; mathematics curriculum

Introduction

This paper addresses the research basis for the current English curricula for three to seven year olds, particularly focusing on number and arithmetic, with potential revisions suggested from current research. It draws on a previous examination of the research basis for the three to five curriculum (Gifford, 2014) with additions concerning the actual and potential curriculum for six and seven year olds.

The Early Years Foundation Stage

In 2008, England became the first country to introduce a curriculum from birth to five years, the ‘Early Years Foundation Stage’ (EYFS) (Department for Children, Schools and Families, 2008) with mathematics goals, which were supported by Suggate, Aubrey and Pettit’s (1997) findings about the number knowledge of children in English reception classes. Following concerns about the number of goals, a review and public consultation (Tickell, 2011) indicated that practitioners, academics and early years professionals thought that some mathematics goals were pitched too high, alongside children’s low achievement in solving number problems. The resulting single Numbers Goal included most of the previous goals, plus a new introduction by Tickell (2011) of counting on and back to add and subtract. This item had previously been part of a higher level than the Goals, which was achieved by only 7% of five year olds according to the Department for Education (DfE, 2012b). According to Ryan and Williams (2007), 48% of five year olds can count on to add, but only 51% of six year olds. No research supports counting back to subtract as a goal for the majority of five year olds: in New Zealand only 21% of six year olds achieved this (Gifford, 2014). A further expectation, to solve doubling, halving and sharing problems, was added later by the government, with no explanation (DfE, 2012a, p.9) With regard to sharing problems, research indicates that young children can tackle these practically, but may not understand about equal shares (Nunes & Bryant, 2009). Doubling and halving is more challenging to understand because it involves scaling: according to Ryan and Williams (2007, p.184) only 14% of seven year olds “recognise one half of a small number of objects”. In summary, there was no research support for the new expectations.

It is interesting to consider what rationale or underlying theories of teaching and learning underpinned these raised expectations, since no additional guidance was offered (Thompson, 2013). The implicit theory seemed to be that if teachers were set higher targets and told that five year olds should achieve them, within a high accountability framework, teachers would manage to get children to achieve them. Government statistics suggest this theory is valid: whereas just over 70% achieved the previous goals in 2012 (DfE, 2012b), just over 70% achieved the raised expectations in 2014 (DfE, 2014). This may have been due to the changed method of assessment, to a judgement of 'best fit'. This might seem preferably holistic to atomising the curriculum, in line with current thinking as in the Irish early years curriculum review, (Dunphy, Dooley & Shiel, 2014). Unlike previously, the exemplification for assessment was presented as a collection of children's work samples, screenshots and written observations, with no commentary. Not all of the Goal content was exemplified: apparently no examples of doubling and halving were 'forthcoming' (Gifford, 2014). Interestingly, a training pack was provided for Ofsted inspectors, including examples of doubling and halving (Ofsted, 2013) but this was not shared with teachers. Some examples were potentially misleading: for instance in relation to a child's picture where they had written numerals 1 to 5 on castle turrets, the practitioner's comment suggests this is evidence of 'one more than', 'one less than' and placing numbers in order, rather than simply writing the numerals in sequence. Previously, individual concepts had been explained and exemplified by assessments derived from research (Suggate et al., 1997). As assessment takes up teachers' time and provokes valuable discussion in moderation, it seems a missed opportunity to not provide research based exemplification to support teachers' mathematics knowledge.

Similarly, in providing a goal with many elements of very different levels, there is a missed opportunity to outline a research based progression of pre-requisite skills and understandings, as provided for instance, by the New Zealand government website (New Zealand Ministry of Education, 2010). Therefore, not only is the Numbers Goal itself, not research based, but the new curriculum provided less professional support based on research.

The National Curriculum

The curriculum for five to seven year olds in England comprises Key Stage One of the current National Curriculum (NC) (DfE, 2013). A previous draft primary NC had been researched and developed based on a progression of big ideas, in line with current thinking worldwide, focusing on addition and subtraction in the early years, multiplication and division in the middle years and proportional reasoning in the later years (Qualifications and Curriculum Authority, 2009). This was rejected by a new incoming government, who developed the 2013 curriculum with raised requirements. For instance, one expectation for six year olds is: "solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? - 9" (DfE, 2013, p.7). This kind of equation is extremely difficult for young children: according to Ryan and Williams (2007, p.180), only 37% of six year olds 'begin to use the signs for subtraction and equals in a number sentence" and Clements and Sarama (2009, p.78) suggest that 'start unknown' addition calculations such as ? + 6 = 11 are only achievable by some six year olds by trial and error. The NC example not only is a 'start unknown' problem involving subtraction, but places the unknown in the middle of an equation following the equals sign, making it yet more complex.

Some expectations for seven year olds are similarly advanced. The 2013 NC requirement, “recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100” (DfE, 2013, p.12) is virtually the same as the 1999 NC attainment target for nine year olds: “Pupils use mental recall of addition and subtraction facts to 20 in solving problems involving larger numbers” (DfE/QCA, 1999, p.13). However research by Cowan (2011) had shown that the 1999 NC expectation was unrealistic for nine year olds in year 4, who were otherwise achieving well:

> ..The current English National Curriculum goal for knowledge of addition and subtraction facts was achieved by none of the 259 children in the study when they were assessed either in Year 3 or Year 4. Despite ignorance of many facts, their mathematical achievement was slightly above average. (p.1)

The expectations for seven year olds also include: “Recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity” (DfE, 2013, p13). According to Nunes and Bryant (2009, p.4), fractions are very challenging for primary pupils, and ‘even after the age of 11 many students have difficulty in knowing whether two fractions are equivalent’. Ryan and Williams (2007) state that only 39% of eight year olds can identify one quarter of 8 in a pictorial context. The sample national test question to assess this item is “3/4 of 20”. The commentary suggests that the selection of the highest difficulty level for the question is deliberate: “This question demonstrates the increased demand in the new curriculum when working with fractions. Pupils will have to apply a learned procedure to answer this question” (Standards and Testing Agency, 2014, p.8). This implies that children are not expected to understand the abstract problem sufficiently to find their own methods, but will need to be ‘taught to the test’. However, teaching procedures without understanding is likely to prevent learners from being able to solve problems, as Nunes and Bryant (2009) point out. This focus on procedures seems indicative of the overall lack of emphasis on understanding in the 2013 NC, which is of general concern, as is the absence of clear conceptual progression towards the raised expectations, according to the Advisory Committee for Mathematics Education (ACME, 2014).

It seems that the NC content is not based on research, but rather on the curricula of high performing jurisdictions (Thompson, 2012). Apart from issues of transferring curricula from very different cultures the later school starting age of many jurisdictions makes adopting their curricula for young children even more questionable. As with the EYFS, the lack of clear guidance as to how these raised expectations might be achieved, while making schools accountable for doing so, seems to risk undermining teachers’ subject knowledge and children’s security of understanding.

What should we be teaching and how?

From current research, it seems clear that a priority for early years mathematics should be to develop number sense, or a ‘feel for numbers’ and the relationships between them. The early big ideas, which have been shown to predict later achievement, are understanding the cardinal value of numbers (eg ‘the nineness of nine’), comparing numbers by relative size, and part-whole concepts for numbers, which involve seeing numbers as made up of other numbers (Gifford, 2014). These key ideas are also endorsed by research as important for older learners. An understanding of the relative size of numbers as shown by the ability to place numbers appropriately on an empty number line is predictive of more general arithmetical achievement (Siegler & Booth, 2004). Part-whole concepts for numbers, which depend on understanding of cardinal numbers, also involve knowing number combinations and factors, as well as understanding inverse operations and place value. Part-whole concepts underpin the ability which Boaler found characteristic of high-achieving students who “engage in flexible thinking when they work with numbers, decomposing and recomposing numbers” (Boaler, 2009, p.139). This suggests that a curriculum might emphasise ideas such equivalence, with activities involving arranging and rearranging numbers of objects in different ways, including grouping in tens. Research also suggests developing young children’s intuitive skill of subitising, or recognising a number of items without counting, as shown with fingers or dice. This key skill has also been found to be predictive of later arithmetical performance (Young-Loveridge, 1991). It not only helps children learn cardinal number values, but builds knowledge of number combinations, such as three and three making six, without involving equations, in playful activities such as number rhymes and games which are appropriate for the age group.

While research may indicate mathematical content, the challenge for curriculum designers, as identified by Dunphy et al. (2014), is to integrate content with mathematical thinking processes. While both the EYFS and the NC identify processes endorsed by research (eg. Dunphy et al, 2014) these are not integrated: however, they suggest curriculum opportunities. The EYFS generic “characteristics of effective learning”, which include “playing and exploring”, “active learning” and “creating and thinking critically”, have been exemplified mathematically in the Ofsted (2013) training pack as “playing with what they know, having their own ideas, finding ways to solve problems, finding new ways to do things, making links and noticing patterns, testing ideas, developing ideas of grouping and sequencing”. This guide to identifying early mathematical thinking might usefully be shared with teachers. Similarly, one of the concerns about the 2013 NC is the dislocation between the aims, which emphasise processes such as “mathematical reasoning and competence in solving increasingly sophisticated problems”, and the statutory requirements in the programmes of study, which contain few references to reasoning or problem solving strategies (ACME, 2014). However, the NC describes mathematics as a “creative and highly inter-connected discipline”, “in which pupils need to be able to move fluently between representations of mathematical ideas” (DfE, 2013, p.3) which suggests that teaching approaches could encourage children to develop methods and representations and to discuss these. Research clearly shows that justifying reasoning develops understanding (Dunphy et al., 2014), while Harries, Barmby and Suggate (2008) suggest that connecting representations defines reasoning and understanding.

Research therefore suggests that developing number sense, combined with an emphasis on understanding and reasoning, including creative approaches to problem solving and representation, would be a productive focus for a mathematics curriculum for three to seven year olds.

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