Teaching Knowledge-in-Practice: An introduction

Angeliki Mali
Loughborough University

Following research in school mathematics teaching, there is an increasing interest in recent years in understanding university mathematics teaching and the knowledge that is used to teach in this level. This paper reports an ongoing doctoral study on the teaching of various mathematics lecturers at an English University. I present *Teaching Knowledge in Practice* (*TKiP),* a theoretical construct which has emerged from the data through a synthesis related to teaching practice and knowledge. I then exemplify its use to date providing a teaching episode.

Keywords: teaching practice; teaching knowledge; small group tutorials; university mathematics teaching;

Introduction

Despite major professional societies' call for increasing attention to university mathematics teaching and the creation of professional development resources, empirical research on teaching that could inform these efforts is limited; thus the design of most existing programs and resources is not informed by research findings (Speer, Smith & Horvath, 2010). Nevertheless, there are some studies of relevance and an example is the *Approach to Teaching Inventory* (Prosser & Trigwell, 1999). This is a scale to measure the 'teacher-focused approach' and the 'student-focused approach' in a particular context. The data here consisted of lecturers' self-reports of their experiences rather than an observational study of their teaching; suggesting potential inconsistency between teaching practice and self-reports. Furthermore, published scholarship analyses aspects of undergraduate teaching practice and knowledge using and adapting elements of extremely popular frameworks developed for school teachers (e.g. Jaworski, 2003; Rowland, 2009). Lecturers at university, however, differ from schoolteachers in terms of their perspectives for teaching.

My doctoral research is a descriptive empirical study of 26 lecturers' mathematics teaching at an English university which addresses how lecturers' knowledge is revealed in their teaching practices with first year undergraduate mathematics courses. It has shown that lecturers bring practices from their own research in mathematics into their mathematics teaching (Mali, Biza & Jaworski, 2014) in agreement with other studies in university mathematics teaching (e.g. Petropoulou, Potari, & Zachariades, 2011). In my study, I am working towards a framework which for the moment I am calling *Teaching Knowledge-in-Practice (TKiP)* because it relates teaching knowledge in mathematical, didactical and pedagogic domains to teaching practice. In my analysis, I am drawing on Vygotskian theory and its conceptualisation of tools to reveal the full complexities of teaching and issues deriving from teaching decisions, such as the aims, challenges and pitfalls lecturers face. In this paper, I offer *TKiP* as a contribution to the current literature. This is significant since it is a research-based framework of teaching knowledge and practice specific to the undergraduate mathematics context.

Context

I selected the small group tutorial setting to investigate university mathematics teaching since opportunities of tutor-student dialogue emerge there. Students are in their first year of a straight or joint degree in mathematics. They are expected to attend lectures in analysis and linear algebra modules and allocated to small tutorial groups of five to eight students. Tutors are lecturers in modules offered by the mathematics department and conduct research in mathematics or mathematics education. Tutorials are fifty-minute weekly sessions. The work is on the lecture materials and students are expected to work on them in advance and bring questions.

Method

In order to address how lecturers' knowledge is revealed in their teaching practices, I collaborated with 26 tutors: 20 mathematicians and 6 mathematics educators. The collaboration involved observing, audio-recording and transcribing at least one of their small group tutorials and follow up interviews with them about their underlying considerations. Subsequently, I studied in depth the tutorials of 3 of the 26 tutors; observing, recording and transcribing tutorial dialogue and interviewing the tutors for more than one semester. In this paper, I present analysis of data from one of these 3 tutors, Phanes[6], who conducts research in mathematics. I take a grounded analytical approach to the data for the purpose of generating theory. My analysis leads to emergent findings which I then relate to relevant studies in order to build on what I find in the literature in terms of further analysis. I thus consider if my findings are giving me new insights in the theoretical perspectives in literature or they are a consequence of the theory. So there is a cyclic process between my emergent findings, consulting literature and feeding what I find in the literature back into my analytic process. The unit of analysis is the teaching episode in which aspects of tutors' actions that seemed to be informed by their teaching knowledge are coded. I consider tutors' actions as what the tutor says -I can read in transcripts-, does in tutorials -gestures, body language- and intentions -I can ask in interviews, hear in the classroom. I treat my data from a sociocultural perspective so that I consider the holistic nature of teaching practice and knowledge rather than some tutors' actions. I select the sociocultural theory of Vygotsky (1978) which considers the overall social and cultural context, which frames mathematics teaching and learning in its complexity. It proposes that concepts and meanings pre-exist and the individual first develops knowings (e.g. of mathematics, didactics, pedagogy) on the social plane and then appropriates these knowings to herself.

Overview of findings: The categories

In order to analyse my data, coding led to six categories: a tutor's *epistemology*, *didactics* and *pedagogy*, which constitute her teaching knowledge; and *tools*, *strategies* and *characteristics* of teaching, which define a tutor's teaching practice. These are the six categories of *TKiP*. So, a teaching episode can be understood in terms of the tutor's teaching practice and knowledge. Accounts of the six categories of *TKiP* now follow.

Epistemology, didactics and pedagogy. In my study, a tutor's epistemology is her view of the nature of mathematics and/or teaching and can influence her didactics

[6] The lecturers suggested their pseudonyms and Phanes is the abbreviation of Aristophanes.

and pedagogy (Mali, Biza & Jaworski 2014). By didactics I mean her aims for and design of the lesson which is mathematics-specific. By pedagogy I mean the tutor’s actual implementation of her teaching to the particular students and the associative decision-making. Pedagogy is not mathematics-specific but relates to general theories of teaching and learning.

Tools, strategies and characteristics. The *characteristics* of teaching are patterns in the ways that tutors teach in the tutorials. As I code transcriptions, I identify these patterns repeatedly in the data and altogether provide an insight into a tutor’s teaching. Within *characteristics*, I recognise the tutor using a range of *tools* and *strategies* for teaching. A tutor can use a *tool* for teaching so as to enable students to make meaning of mathematics. I consider *tools* for teaching to be physical (e.g. board, problem sheet) and psychological (e.g. examples, graphs, exposition). I also investigate how this consideration relates to the Vygotskian frame.

In his theorisation of *tools*, Vygotsky (1978) focused on psychological (mental/ideal) *tools* as opposed to technical (material) *tools* (Wertsch, 1991, p. 28). Within the Vygotskian perspective, material *tools* are “used to bring about changes in other objects” (Daniels, 2001, p.15) transforming “the process of a natural adaption by determining the form of labour operations” (Vygotsky, 1981, p.137); for instance, prehistoric stone tools. In contrast, psychological *tools* are “devices for influencing the mind and behaviour of oneself or another” (Daniels, 2001, p.15) transforming “the entire flow and structure of mental functions” (Vygotsky, 1981, p.137). Daniels (2001, p. 22) asserts that a *tool* carries its material and psychological nature inherently making the separation of *tools* into material and psychological ones impossible. In *TKiP*, I use the term *tools* to refer to their dual material and psychological nature. For example, a graph can be used as a *tool* for teaching and it has a dual nature: that of a signifier and of a signified. The signifier is material, and in the case of a graph it has a “clearcut materiality”; it is a “physical object that can be touched and manipulated” (Wertch, 1998, p. 30) since it is sketched on the board. On the other hand, the signified is ideal; for example, the mathematical concept of function referred to by the signifier.

In *TKiP*, I define *strategies* for teaching to be another category of teaching practice. The *strategies* are the ways tutors use the *tools* for teaching and include the tutors’ decisions about the mathematics and the particular students in the context. My analysis suggests that the categories of teaching practice can show a consistent variation; for example, a *tool* or a *strategy* can be regarded as a *characteristic* of a tutor’s teaching if it is identified repeatedly in her teaching. I offer a teaching episode below to exemplify the categories of *TKiP*. In my analysis of this episode, I draw on literature in scaffolding since I think of it as a *strategy* for teaching. In particular, I use the term scaffolding to refer to analytic scaffolding (Williams & Baxter, 1993), which relate to the tutor’s support to students’ cognitive development.

Analysis of a teaching episode using *TKiP*

The episode below belongs in one of the first tutorials of the year and starts by the tutor, Phanes, reading an exercise of a problem sheet in Analysis:

Phan: “Re-write the formula $||x|-1|$ without the modulus signs using several cases
where necessary. You do not need to provide lengthy derivations.” Ok, what
is this? What is $||x|-1|$? How can we write this? Maybe what we can do is we
can just sketch the graph of the function. Let’s sketch the graph of this function.
If I give you a minute to sketch the graph as a function of x. $f(x)$ is this thing,
of course. You see, to get rid of the modulus sign of mod x, you need to make

7 sure that *x* is positive or negative. You have to consider cases. But there is
8 another outer modulus. It's external. Again, to get rid of it, you need to either
9 consider the case whether the expression inside it is positive or not. So, there
10 will be several cases here. Maybe I'll give you a small hint. What is the graph
11 of mod x^3, for instance? Try to sketch the graph of mod x^3. Ok. So, let's sketch
12 x^3. It's this. This is x^3. So, what is the mod x^3? Here, it is bigger than 0. x^3 is
13 positive, actually, so mod x^3 just coincides with x^3. But here, x^3 is negative, so
14 we have to reflect it about the *x* axis, and it will be something like this. So, mod x^3
15 – is this a positive expression? So, in other words, if you know the graph of f,
16 to sketch the graph of mod *f*, you just sketch all the negative bits of the graph
17 about *x*'s, because modulus is always positive, isn't it? So, sketch the mod
18 graph of this function. [He refers to $f(x)=||x|–1|$.]

This episode captures the way Phanes usually starts approaching exercises he wishes students to solve. He utilises several *tools* for teaching (*tools*/table 1) some of which are *tools* students can use to solve the exercise (graph, know-how exposition); whereby mostly providing students with scaffolding (*strategies*/table 1) before they start solving. In this episode, scaffolding constitutes Phanes' support to students in order to enable them to start solving. So in order to support students, he treats the formula of the exercise as a function and suggests the students to sketch the graph of $f(x)=||x|–1|$ [lines 3-4]; uses know-how exposition for the work on modulus signs [lines 6-10]; sketches the graph of $f(x)=|x^3|$ which is more familiar to students than the graph of $f(x)=||x|–1|$ and relevant to it in terms of procedures for the modulus sign [lines 10-15]; and uses know-how exposition for sketching the graph of mod *f* [lines 15-17]. He then offers students time to work on their scripts while he is circulating and supporting them (*characteristics*/table 1).

Episode lines	Tools for teaching	Strategies for teaching	Characteristics of teaching
3–4:	Graph of $f(x)= ||x|–1|$	Scaffolding; Use of different representations.	Use of problem solving techniques; Use of graphical representations to provide a visual intuition for formal representations.
10–15:	Example (including graph of $f(x)=x^3$); board.	Scaffolding; Use of a more accessible relevant example.	Use of problem solving techniques; Use of graphical representations to provide a visual intuition for formal representations.
6– 10:	Know-how exposition.	Scaffolding.	Know-how exposition about procedures or techniques for the work on mathematics; Use of problem solving techniques.
15–17:		Scaffolding; Generalisation of the specific task.	
5, 17, 18:	Students' work.	Time management; Assessment of students' work; Scaffolding.	Provision of time to students to work on their scripts while tutor is circulating and supporting.

Table 1: Tools, strategies and characteristics of Phanes' teaching episode.

I focus now on the psychological nature of the *tools* for teaching in table 1 in order to explain why they are *tools* for teaching. In Vygotskian terms, a *tool* is a device for influencing the mind and behaviour transforming the flow and structure of mental functions. The graphs, example and know-how exposition (*tools*/table 1) are the tutor's intellectual *tools* which he uses to influence students' behaviour to be mathematical. The associative *strategies* to these *tools* are scaffolding and problem solving techniques (use of different representations, use of a more accessible relevant example, generalisation of the specific task) (*strategies*/table 1). These are *strategies* for teaching since they constitute the modes of *tool* use and the tutor's decisions of how to deal with the mathematical task and the particular students in the context. I code Phanes' use of know-how exposition [lines 15-17] as generalisation of the specific task, because he generalises the procedure of producing the graph and as such it is a problem solving technique. Other problem solving techniques are the use of different mathematical representations (the graphical and the symbolic representation of $f(x)=||x|-1|$) and the use of the graph of $f(x)=|x^3|$ as a more accessible relevant example. So, Phanes uses the graphs, example, know-how exposition (*tools*/table 1) and problem solving techniques (*strategies, characteristics*/table 1) in order to enculturate students into "being mathematical" (Jaworski, 1995) and this enculturative aim reveals his didactics. Furthermore, the students' work on their scripts (*tools*/table 1) is the tutor's *tool* to engage as well as to assess and support students. This *tool* is different in nature from the others since it is oriented more precisely to the particular students and can be used in the teaching of other subjects as well as mathematics. So, it is a pedagogical *tool* for teaching and so are its relevant *strategies* and *characteristics* in table 1.

In the process of coding, I identified patterns of teaching for each tutor repeated several times in their tutorials and these are the *characteristics* of their teaching. In table 1, I present the *characteristics* of Phanes' teaching which emerged in the above episode. I focus now on the characteristic *use of graphical representations to provide a visual intuition for formal representations* which is common among the three lecturers observed for more than one semester. In the above episode, the graphical representation is the graph of $f(x)=x^3$ and the visual intuition concerns the construction of the graph of $f(x)= ||x|-1|$ so that it gives insight into the solution of the exercise in a symbolic form. Phanes, after circulating and supporting students, sketched the graphs of $|x|$ and $|x|-1$ reflecting the negative part of $|x|-1$ to construct $||x|-1|$, all three in the same axes. He then connected the four linear functions with the intervals in which they were defined. In this way, the two graphs provided a visual intuition for the formal representation

$$||x|-1| = \begin{cases} x-1, x \in [1,+\infty) \\ -x+1, x \in [0,1) \\ x+1, x \in (-1,0) \\ -x-1, x \in (-\infty,-1] \end{cases}$$

In my discussion with Phanes and in response to a question of why he chose a geometric solution when some mathematicians avoid doing so, he connected his choice with mathematicians' research practices.

> It depends on your research area. If you are a geometer [as Phanes is], you are happy with geometric solutions; it depends on your background I think. […] You see to me it is easier to see the graph. […] For instance if you are a programmer writing computer programs, then it is more convenient to you to give an algorithm.

From this interview excerpt as well as interviews with Phanes as a whole, I conclude that Phanes approaches mathematics teaching putting emphasis on the mathematics and geometric thinking. His epistemology of mathematics seems to be related to geometers' research practices and in this episode, his didactics is within his thinking about the graphs. So in this episode as well as other episodes, from his geometric thinking about mathematics, Phanes draws out his teaching practice recognised through the specific *tools*, *strategies* and *characteristics* of teaching.

Conclusions

TKiP is a theoretical construct emerging from the data to characterise a tutor's mathematics teaching practice and link it with her teaching knowledge. The tutor's pedagogy, didactics and epistemology are domains of teaching knowledge which are significant since they inform teaching practice and the latter can be effective or ineffective for students' meaning making of mathematics. Through *TKiP* a tutor's teaching practice can be understood in terms of *tools*, *strategies* and *characteristics* of her mathematics teaching. These three categories of teaching practice emerged through the process of coding and categorising aspects of tutors' actions that seemed to be informed by their teaching knowledge. A first layer of analysis is the identification of a tutor's *characteristics* of teaching in the data and then a second layer is the identification of the tutor's *tools* and *strategies* for teaching within *characteristics*. Another layer of complexity is the link between these categories and the domains of teaching knowledge, which are revealed through coding and categorising the information that came from interviews with tutors. So far, I have identified different tutor's *characteristics* of teaching and the associative *tools* and *strategies* for teaching. Another layer of my analysis is the juxtaposition of these *characteristics* that look similar but are implemented in different ways by the different tutors. In a final form, *TKiP* is going to include its six categories of teaching practice and knowledge and codes assigned to each one of the categories. The codes are going to be the ones that led to these categories in my analysis. In future, *TKiP* will contribute as a researcher's tool for the analysis of university mathematics teaching and a tutor's/lecturer's developmental tool for her professional development of her own teaching practice and knowledge.

References

Daniels, H. (2001). *Vygotsky and Pedagogy*. London: Routledge.

Jaworski, B. (2003). Sensitivity and challenge in university mathematics tutorial teaching. *Educational Studies in Mathematics*, *51*, 71-94.

Jaworski, B. (1995). Being mathematical within a mathematical community. In M. Selinger (Ed.) *Teaching mathematics*. London: Open University Press.

Mali, A., Biza, I., & Jaworski, B. (2014). Characteristics of university mathematics teaching: Use of generic examples in tutoring. *Proc. of PME 38 Conf.* (Vol. 4, pp. 161-168). Vancouver, Canada: PME.

Petropoulou, G., Potari, D., & Zachariades, T. (2011). Inquiring mathematics teaching at the university level. In Ubuz, B. (Ed.), *Proc. 35th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol.3, pp. 385-392). Bergen, Norway: PME.

Prosser, M. & Trigwell, K. (1999). *Understanding Learning and Teaching: The Experience in Higher Education*. Buckingham: Open University Press.

Rowland, T. (2009). Beliefs and actions in university mathematics teaching. *Proc. of PME 33 Conf.* (Vol. 5, pp. 17-24). Thessaloniki, Greece: PME.

Speer, N. M., Smith, J. P., & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *The Journal of Math. Behavior*, *29*, 99- 114.

Vygotsky, L. (1981). The development of higher forms of attention. In J. V. Wertsch (Ed.), *The Concept of Activity in Soviet Psychology* (pp. 189–240). Armonk, NY: M.E. Sharpe.

Vygotsky, L. (1978). *Mind in society: The development of the higher psychological processes*. Cambridge, MA: Harvard University Press.

Wertsch, J.V. (1998). *Mind as action*. New York: Oxford University Press.

Wertsch, J.V. (1991). *Voices of the Mind: A sociocultural Approach to Mediated Action*. Cambridge, MA: Harvard University Press.

Williams, S.R. & Baxter, J. (1993). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal*, *97*(1), 21–38.