Investigating approaches to develop secondary mathematics student teachers' rich mathematical understanding

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My research investigated how students on a Subject Knowledge Enhancement (SKE) course at Manchester Metropolitan University engage in discussions about basic calculation processes. The theoretical foundation is Skemp's (1976) notions of procedural and conceptual understanding and related research. Through a series of whole class deliberations, informal questioning and semi-structured interviews, my study discovers and attempts to explain shifts in mathematical thinking. The debate considers how the groups' development of a deeper view of mathematical procedures can enhance learning and it seeks to explore an emerging insight into how our learning as children affects our perception of knowledge and understanding as adults (Costello, 1991). My findings suggest that through revisiting school mathematics from an advanced perspective, the students actually gain much more than merely competence in mathematical procedures. The findings suggest that the students appear to develop a deeper understanding and appreciation of the underlying mathematical concepts. This relational learning enhances their own conceptual mathematical knowledge and subsequently strengthens their pedagogical awareness.

Keywords: Secondary; understanding; procedural; conceptual

Introduction

My initial desire to consider the relationship between procedural methods of learning mathematics compared to conceptual methods was grounded by Skemp's research (1976) and the deliberation of how a six-month enhancement course can develop the thinking of prospective secondary mathematics teachers. My investigation was focused towards how the course can ensure that the students begin to improve their own understanding of mathematics and embark on pedagogical enquiry in order to question the understanding behind their learnt procedures. The foundations of their learning lay in the experiences they had as children and the way this may affect their thoughts about teaching. The suggestion of adopting the procedures and practices that they were taught when at school (Kennedy, 1999) and their philosophies about mathematics, as Ernest (2001) suggests, could influence the way in which they teach.

Consideration of Literature

Since Skemp (1976) first proposed the discussion of procedural and conceptual learning much has been written concerning these issues (e.g. Hiebert & Lefevre 1986; Silver 1986; Halai 1998; Orton 2004). I am particularly interested in an article published in the Journal of Mathematics Teacher Education (Adler et al., 2014) that investigated the feelings of eighteen students, across three institutions about the idea of their development of deep understanding in mathematics. Adler et al. interpreted this "deep understanding" as promoting thought within the SKE students, "...about concepts and processes as they learn new mathematics and as they revisit concepts located in elementary mathematics, which they may previously have taken for granted" (2014, p.132). This was also the essence behind my investigation, as the course considers the way children learn and understand mathematics by allowing for and exposing misconceptions. During the first term, time is spent discussing and observing how children and adults engage in the basic rules of calculation and then unpicking why the four operations can cause distress to children and adults alike. In Peterson and Williams' article (2008), the mentoring experience of one student teacher stands out, as they discuss what she has learned from her time as a trainee in the classroom. She talks about learning mathematics including addition, subtraction, multiplication and division but not in the sense of subject knowledge but in a pedagogical sense. She explains, "...what it actually means, how to show it with pictures....what you're actually saying when you're saying four divided by two" (Peterson & Williams, 2008, p.472). Although a secondary mathematics teacher can comfortably calculate using the four operations, perhaps not every teacher can explain why the rule they are using works. The experiences generated in my classroom were an attempt to address this type of thinking, just as Peterson and Williams (2008) comment upon one student's chance to think about her growing perception of mathematics both in content knowledge and pedagogical approach.

A similar scenario is described by Kinach (2002, p.155), as she discusses her experiences of attempting to shift future teachers' endorsement of "learning without meaning", towards a more important stance, in her eyes, of "teaching for understanding". As a teacher-educator, she investigates the effect mathematical explanations can have on deepening children's understanding. Kinach (2002) proposes two possible questions to consider, one focused upon pedagogy and the belief system of the prospective teacher, surrounding mathematics and the teaching of mathematics; the other is the type of content a course must offer in order to address, "...the kinds of epistemological knowledge new teachers need to teach mathematics for understanding" (Kinach, 2002, p.155). This type of perception includes the 'why' and not just the 'how' of knowing. This point is supported by Shulman (1986, cited in Adler et al., 2014) when he talks about the importance of the teacher knowing and understanding why a mathematical process leads to the correct answer and not just knowing how to apply that particular process. My research with the SKE students has enabled me to begin to explore this understanding. I was aware that they learn more than just mathematics during their time on the course but how much pedagogical shift takes place during the six months was difficult to measure in such a limited period of time.

Research methodology

The most appropriate methodology for the purposes of investigating the SKE students shift in knowledge and understanding was that of action research (Cohen, Manion & Morrison, 2011).

The study design that fit with my research question was the one Kumar (1999, p.83) described as "the before-and-after study design" as it aimed to quantify the impact of a particular phenomenon, in this case, my SKE unit, by analysing observations collected between two set points in time. The data collection, semi-structured interviews, are analysed and the difference between the two sets of observational variables noted. These are considered to be the main identifiers of impact for the unit.

The course may not be able to change any opinions or have any effect on mathematical thinking. Many of the students involved have built on their knowledge from their formative years, through primary school, into secondary school and now into higher education. They all come to the course with prior experiences that have moulded their thinking and as Sanders (1994, cited in Halai, 1998, p.307) states, “the way teachers teach depends strongly on their own personal view of what constitutes mathematics”. For this reason, I am extremely aware of the limitations of this study and the findings my paper presents.

Data Collection

This was undertaken with the six-month subject knowledge enhancement students from January 2014 to June 2014 inclusive. It involved class discussions; informal questioning; thought capturing on postcards during sessions and semi-structured interviews at the beginning and end of the course. In University sessions, my aim was to investigate and interrogate the SKE students own mathematical understanding of several, straight-forward calculations in order to understand the difficulties children experience. If felt important to discuss their own understanding of mathematical concepts.

Data Analysis

There were several mathematical processes and procedures to investigate. For this paper I focus upon the discussion of a decimal multiplication question. I invited the group to attempt the multiplication, 1.83 x 47 and below is a selection of responses.

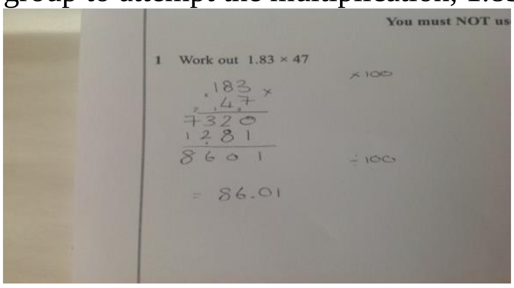

Fig. 1. Use of a recognised ‘traditional approach’

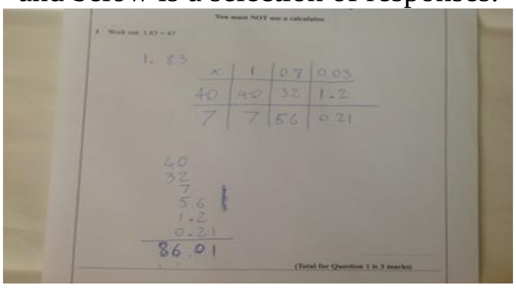

Fig. 2. Use of the grid method of multiplication

The method of calculation in figure 1 was chosen by 55% of the group whereby one hundred and eighty three was multiplied by forty seven. The final answer was then obtained by dividing by one hundred. This is an example of a procedural calculation but with a degree of conceptual reasoning as confirmed through discussion. A further 18% used the grid method of multiplication and kept the numbers as the original decimals before splitting into their component parts (Figure 2) and another 18% demonstrated a confidence with number (Figure 3) and their own ability to manipulate calculations (Thompson, 1994).

The final example as shown below (Figure 4) used several flawed attempts of the traditional approach before finally gaining success with the grid method. The most interesting point for me was the discussion that took place following this activity. The grid form of multiplication could indicate an understanding of mathematical distribution to split the numbers using partitioning and Ma (1999, cited in Fosnot & Dolk, 2001) in her research would advocate this as a conceptual teaching and learning strategy. However, the actual drawing of the grid shows no attempt at a rough scale which could indicate that the true meaning for this form of visualisation had been lost along the way as proposed by Fosnot & Dolk (2001).

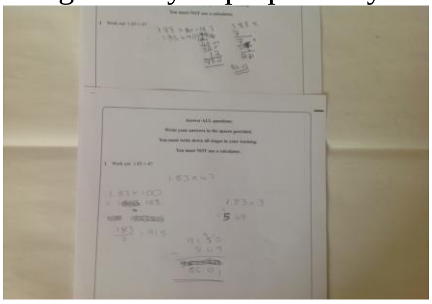

Fig 3. Use of a mixed method approach

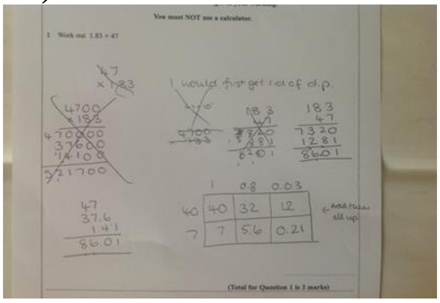

Fig 4. Use of traditional and grid methods

I am not stating that this shows lack of understanding but possibly the familiarity of the approach has led to the development of another type of algorithm.

The responses gathered during our discussions (see below) gave me an indication into the students' thinking.

> I used the grid method for multiplication but I was in top set, so when I used that method it was almost like I couldn't use the real method, I was using the cheat's method of multiplication. I really felt there was a stigma to using the grid method.
>
> So when I did that in your session, I was hiding the way I did it, I was actually hiding it from other people as I was doing it! I was thinking, I can't remember how to do it properly, because I couldn't remember long multiplication.

These admissions indicate embarrassment and Evans (2000, p.4) in his study of adult mathematical thinking explains this belief. He discusses the popular idea that there is only "one right method" and how belief in this has led to a great deal of apprehension and embarrassment. Reinforcing this, Sewell (1981, cited in Thompson, 1994) explains that both children and adults often choose to use an adapted method for solving calculations rather than the taught standard algorithm. However, they feel uncomfortable and self-conscious, that they are unable to recall and use the 'traditional' method. Therefore, the determination to use the traditional long multiplication method is an illustration of what Ebby (2005) discovered in her case study, where the power of the procedure won out over common sense.

On several occasions throughout the research period, the explanation of the actual method of calculation was limited and the SKE students began to question their own understanding.

> It makes you realise that perhaps we don't understand as much as we thought we did. On certain occasions I just did not understand why we used that method but I thought, it doesn't matter, I can apply it effectively and answer the questions...

The comment above may well suggest a confidence in executing a known algorithm in a procedural manner but it highlights a lack of conceptual understanding as to why the method actually works as Skemp (1976), Kamii & Dominick (1997) and Lee (2007) all discuss in their research.

Summary of the emerging themes from the data analysis

Context

The postcards and interviews indicated the desire for mathematics to be given a context to enable learners to develop 'transferable skills'. Many were aware of how the different concepts linked but the course provided a scaffold to enable the group to discover and therefore understand these links allowing for a deeper understanding of mathematics, as Sullivan, Tobias & McDonough (2006) explain.

Confidence

The second theme was focused on the development of their confidence as teachers. Although they had been successful at mathematics the skill of passing this to others was, at times, daunting and one student admitted

> My understanding has come on leaps and bounds!

Misconceptions

One of the biggest surprises for the group was that children have mathematical misconceptions and these are many and varied. One student's realisation that children may not understand mathematics when it was taught was quite astonishing and she described her feelings of being in a "bubble" and never realising that there were different techniques to learning and teaching. At the final interview she exclaimed, "It's like a new beginning to life!"

Conclusion

After completing my research, I now believe, as Silver (1986, p.184) suggested, that procedural and conceptual understanding are "inextricably linked". The confidence issue that emerged throughout the six-month project was not surprising either but has led me to question the difficulties that prospective teachers have in changing their epistemologies (Kinach, 2002).

If I was to verbalise one key finding from my research, it would be to reinforce what Adler et al. (2014) suggested from the research they undertook and that is, there has been a significant shift in the way the SKE students talk about mathematics. There is a focus on pedagogy, on mathematical context and on teaching for understanding. As Artzt & Armour-Thomas (1998) comment on, it is the academic environment that encourages time to reflect upon their own understanding and by building in this time, the students want to develop both conceptual approaches to problem solving accompanied with efficient and appropriate procedural techniques.

Consequently, it can be proposed that linking subject knowledge together with flexibility of thought can lead to a much deeper understanding of mathematics. Although the data cannot provide evidence of change in all members of the group, perhaps it would be appropriate to conclude that the course has attempted to challenge the way some of the prospective teachers think and given them space to consider their own mathematical beliefs, as suggested by Kinach (2002).

References

Adler, J., Hossain, S., Stevenson, M., Clarke, J., Archer, R. and Grantham, B. (2014). Mathematics for teaching and deep subject knowledge: voices of Mathematics Enhancement Course students in England. Journal of Mathematics Teacher Education, 17 (2), 129–148.

Artzt, A. and Armour-Thomas, E. (1998). Mathematics teaching as problem solving: A framework for studying teacher metacognition underlying instructional practice in mathematics. Instructional Science, 26, 5–25.

Cohen, L., Manion, L. & Morrison, K. (2011). Research methods in Education. 7th Edition. Oxon: Routledge.

Costello, J. (1991). Teaching and Learning Mathematics 11–16. London: Routledge.

Ebby, C, B. (2005). The powers and pitfalls of algorithmic knowledge: a case study. Journal of Mathematical Behavior, 24, 73–87.

Ernest, P. (2001). Critical mathematics education. In P. Gates, (Ed.), Issues in Mathematics Teaching (pp, 277–293). London: RoutledgeFalmer.

Evans, J. (2000). Adults' Mathematical Thinking and Emotions. A Study of Numerate Practices. London: RoutledgeFalmer.

Fosnot, C, T., and Dolk, M. (2001). Young Mathematicians At Work. Constructing Multiplication and Division. Portsmouth: Heinemann.

Halai, A. (1998). Mentor, Mentee, and Mathematics: A Story of Professional Development. Journal of Mathematics Teacher Education, 1, 295–315.

Hiebert, J. and Lefevre, P. (1986). Conceptual and Procedural Knowledge in Mathematics: An Introductory Analysis. In J. Hiebert (Ed), Conceptual and Procedural Knowledge: The Case of Mathematics.(pp, 1-27). Lawrence Erlbaum Associates, Inc: New Jersey.

Kamii, C. & Dominick, A. (1997). To Teach or Not to Teach Algorithms. Journal of Mathematical Behavior, 16 (1), 51–61.

Kennedy, M. M. (1999). The role of preservice teacher education. In L. Darling-Hammond and G. Sykes (Eds.), Teaching as the Learning Profession: Handbook of Teaching and Policy (pp, 54-86). San Francisco: Jossey Bass.

Kinach, B. (2002). Understanding and Learning-to-Explain By Representing Mathematics: Epistemological Dilemmas Facing Teacher Educators In The Secondary Mathematics 'Methods'Course. Journal of Mathematics Teacher Education, 5, 153–186.

Kumar, R. (1999). Research Methodology. A Step-By-Step Guide For Beginners. London: SAGE Publications.

Lee, J-E. (2007). Making sense of the traditional long division algorithm. Journal of Mathematical Behavior, 26, 48–59.

Orton, A. (2004). Learning mathematics. 3rd edition. London: Continuum.

Peterson, B. and Williams, S. (2008) Learning mathematics for teaching in the student teaching experience: two contrasting cases. Journal of Mathematics Teacher Education, 11, 459–478.

Skemp, R. (1976). Relational Understanding and Instrumental Understanding. Mathematics Teaching, the journal of the Association of Teachers of Mathematics, 77, 20-26.

Silver, E, A. (1986). Using Conceptual and Procedural Knowledge: A Focus on Relationships. In J. Hiebert (Ed), Conceptual and Procedural Knowledge: The Case of Mathematics. (pp, 181 - 198). Lawrence Erlbaum Associates, Inc: New Jersey.

Sullivan, P., Tobias, S. and McDonough, A. (2006). Perhaps the decision of some students not to engage in learning mathematics in school is deliberate. Educational Studies in Mathematics, 62, 81-99.

Thompson, I. (1994). Young Children's Idiosyncratic Written Algorithms For Addition. Educational Studies in Mathematics, 26, 323–345.